Mathematics and Statistics 10(1): 251-261, 2022
DOI: 10.13189/ms.2022.100124 http://www.hrpub.org

Fuzzy EOQ Model for Time Varying Deterioration and Exponential Time Dependent Demand Rate under Inflation

K.Geetha*, **S.P.Reshma**

Department of Mathematics, Emerald Heights College for Women, Udagamandalam, The Nilgiris, Tamil Nadu, India

Received September 8, 2021; Revised January 15, 2022; Accepted January 25, 2022

Cite This Paper in the following Citation Styles

***(a)**: [1] K.Geetha1, S.P.Reshma, "Fuzzy EOQ Model for Time Varying Deterioration and Exponential Time Dependent Demand Rate under Inflation," Mathematics and Statistics, Vol.10, No.1, pp. 251-261, 2022. DOI: 10.13189/ms.2022.100124*

***(b)**: K.Geetha1, S.P.Reshma, (2022). Fuzzy EOQ Model for Time Varying Deterioration and Exponential Time Dependent Demand Rate under Inflation. Mathematics and Statistics, 10(1), 251-261. DOI: 10.13189/ms.2022.100124*

Abstract In this study, we have discussed a fuzzy eoq model for deteriorating products with time varying deterioration under inflation and exponential time dependent demand rate. Shortages are not allowed in this fuzzy eoq model and the impact of inflation is investigated. An inventory model is used to determine whether the order quantity is more than or equal to a predetermined quantity for declining items.The optimal solution for the existing model is derived by taking truncated taylor's series approximation for finding closed form optimal solution. The cost of deterioration, cost of ordering , cost of holding and the time taken to settle the delay in account are considered using triangular fuzzy numbers. In this study, the fuzzy triangular numbers are used to estimate the optimal order quantity and cycle duration. Furthermore, we have used graded mean integration method and signed distance approach to defuzzify these values. To validate our model, numerical examples are discussed for all cases with the help of sensitivity analysis for different parameters. Finally, a higher decay rate results in a shorter ideal cycle time as well as higher overall relevant cost is established. The presented model can be used to predict demand as a quadratic function of time,stock level time dependent demand, selling price, and other variables.

Keywords Inventory, Inflation, Exponential Time Dependent, Triangular Fuzzy Number, Defuzzification

1 Introduction

An inventory control systems principal goal is to determine when and how much to order.Inventory management is complicated by deterioration. Everything deteriorates over time. Deterioration might be gradual or rapid, thus it's crucial to factor it into your EOQ model. Many scholars are interested in building trade credit-based mathematical models. Goyal, who looked at inventory models with a pre-authorized payment delay. According to numerous research, the cost does not alter over the planning horizon. This assertion may not be accurate, despite the fact that many countries have significant inflation rates. Inflation has an impact on the demand for particular goods. As the value of money decreases, the inflation rate rises. As a result, when selecting the best inventory policy, the impact of inflation and the time worth of money cannot be overlooked.

Dutta and Kumar [1] used a fuzzy trapezoidal number and the Signed distance method to create a fuzzy inventory model without shortages. Jaggi et al [8] created a fuzzy inventory model with deterioration that took demand into account as time-varying. Kumar and Rajput [12] proposed a fuzzy inventory model for deteriorating products with time-dependent demand and partial backlog for deteriorating items with time-dependent demand and partial backlog. Economic order quantity in Fuzzy sense for Inventory without backorder Fuzzy sets was developed by Huey –Ming Lee and Jing-shing [6]. Harish Nagar and Priyanka Surana [5] developed a fuzzy inventory model for deteriorating objects that used pentagonal fuzzy numbers as parameters. In a fuzzy world, Dutta and Pavan Kumar[2] built an inventory model without shortages by taking into account keeping costs, ordering costs, and demand. Jershan Chiang-Shing Yao and Huey-Ming Lee [9] deal with backordered inventory. S.K.Indirajit Sinha, P.N.Samantha, and U.K.Mishra [7] use the signed distance approach to solve a fuzzy inventory model of shortages under completely backlogged conditions. Jershan Chiang-Shing Yao and Huey-Ming Lee [9] deal with backordered inventory. S.K.Indirajit Sinha, P.N.Samantha, and U.K.Mishra [7] use the signed distance approach to solve a fuzzy inventory model of shortages under completely backlogged conditions. G.Michael Rosario and R.M.Rajalakshmi [13] used different fuzzy numbers and defuzzified using the signed distance approach to examine an inventory model of allowable shortage. As supplier credit is related to order quantity, Tripathi, R.P., and Mishra, T. [19] developed an eoq model with exponential time-dependent demand rate under inflation.

In this paper an fuzzy eoq model for deteriorating products with time varying deterioration under inflation and exponential time dependent demand rate is adopted for consideration.The main objective is to estimate the optimal order quantity and optimal cycle time using triangular fuzzy numbers. Further more for defuzzification these quantities we use graded mean integration and signed distance method.Finally the model is illustrated by numerical example.

2 Notations and Assumptions

This paper makes use of the following notations:

$\tilde{h}$: fuzzy keeping cost rate per unit time

r : inflation rate is stable over time., where $0 \leq r < 1$

pe^{rt} : selling price per unit at time, where p is the unit selling price at time zero.

$\tilde{C}e^{rt}$: fuzzy purchase cost per unit at time t, where c is the unit purchase cost at time zero and $p > c$

$\tilde{A}e^{rt}$: fuzzy ordering cost per order at time t

H : length of planning horizon

$\tilde{m}$: the maximum amount of time that can be allowed before settling an account

I_c : Interest paid per \$ in stock per year

I_d : Interest gained per unit

Q : order quantity

Q_d : minimum order quantity for which payment delays are allowed

T : interval between refills

T_d : the time period due to time-dependent demand in which Q_d units are depleted to zero.

$I(t)$: stock at time t

$R(t)$: fuzzy yearly demand as a $R(t) = \lambda e^{\alpha t}$, $\lambda > 0$, $0 < \alpha = 1$

$\tilde{Z}(T)$: fuzzy total relevant cost over $(0, H)$

3 Assumptions

1. The rate of inflation remains constant.
2. No shortages are permitted.
3. As time passes, the market for the item increases exponentially.
4. Instantaneous replenishment.
5. If $Q < Q_d$, payments for obtained goods must be produced as soon as possible..
6. If, $Q = Q_d$, a payment delay up to m is allowed.

The generated sales revenue is deposited in an interest-bearing account because the account is not settled within the allowed time frame.. The customer pays off all units bought at the end of the credit period and starts paying the interest on the products in stock.

4 Mathematical formulation

Assume that the horizon length is $H = nT$, with n indicating the amount of replenishments to be produced during the period H and T indicating the time interval between replenishments. To meet demand, the inventory level $I(t)$ is gradually reduced. As a consequence, the variance in inventory over time can be calculated as

$$\frac{dI(t)}{dt} + \theta t I(t) = -\lambda e^{\alpha t},\ 0 \leq t \leq T = \frac{H}{n};\ 0 \leq \alpha \leq 1 \quad (1)$$

With boundary conditions $I(0) = Q$ and $I(T) = 0$. Solution of (1) is given by

$$I(t) = \lambda[(T-t) + \frac{\alpha}{2}(T^2 - t^2) + \frac{\theta}{6}(T^3 - t^3)]e^{\frac{-\theta t^2}{2}}, \quad 0 \leq t \leq T \quad (2)$$

And order quantity is

$$Q = \lambda[T + \frac{\alpha}{2}T^2 + \frac{\theta}{6}T^3] \quad (3)$$

Since the time interval have equal lengths, we have

$$I(kT + t) = \lambda[(T-t) + \frac{\alpha}{2}(T^2 - t^2) + \frac{\theta}{6}(T^3 - t^3)]e^{\frac{-\theta t^2}{2}}, 0 \leq t \leq T \quad (4)$$

By using the order quantity, we determine the time period during which Q_d units are depleted to zero due to demand. Then we have

$$T_d = \frac{Q_d}{\lambda[1 + \frac{\alpha}{2}T_d + \frac{\theta}{6}{T_d}^2]} \text{ (approximately)} \quad (5)$$

To calculate the overall applicable cost in $[0, H]$, we add the ordering, buying, and keeping costs together.:

$$(a)\ OC = \sum_{k=0}^{n-1} A(kT) = A(\frac{e^{rH} - 1}{e^{rT} - 1}) \quad (6)$$

$$(b) \text{ Purchasing cost } PC = \sum_{k=0}^{n-1} I(0)C(kT)$$

$$= C\lambda[T + \frac{\alpha}{2}T^2 + \frac{\theta}{6}T^3](\frac{e^{rH} - 1}{e^{rT} - 1}) \quad (7)$$

$$(c) HC = h\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT + t)dt$$

$$= hC\lambda[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{12}T^4](\frac{e^{rH}-1}{e^{rT}-1}) \quad (8)$$

Case(i) $0 < T < T_d$

Payment delays are not allowed in this case since the cycle time interval $T < T_d$. When the customer receives the goods, the supplier must be paid immediately. Because interest is paid on all unsold products from the beginning,

$$IC_1 = I_c\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT+t)dt$$
$$= I_cC\lambda[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{12}T^4](\frac{e^{rH}-1}{e^{rT}-1}) \quad (9)$$

$$\begin{aligned}Z_1(T) &= OC+PC+HC+IC_1\\ &= [A+C\lambda[T+\frac{\alpha}{2}T^2+\frac{\theta}{6}T^3]+hC\lambda[\frac{T^2}{2}+\frac{\alpha}{3}T^3\\ &+\frac{\theta}{12}T^4]+I_cC\lambda[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{12}T^4]](\frac{e^{rH}-1}{e^{rT}-1})\\ &= [\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2]+(h+I_c)C\lambda[\frac{T}{2}\\ &+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3]-\frac{Ar}{2}-\frac{rC\lambda}{2}(T+\frac{\alpha}{2}T^2+\frac{\theta}{6}T^3)\\ &-(h+I_c)\frac{C\lambda r}{2}[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{12}T^4]](H(1+\frac{rH}{2}))\end{aligned} \quad (10)$$

Case(ii) $T_d \le m < T$

Permissible delay m occurs which is more than the interval between cycles T, since $T_d \le m < T$. As a result, no interest is paid ie $IC_2 = 0$, but the interest received in $(0, H)$ is calculated as follows:

$$IE_2 = I_d\sum_{k=0}^{n-1} P(kT)(\int_0^T \lambda e^{\alpha t}tdt+(m-T)\int_0^T \lambda e^{\alpha t}dt)$$
$$= \frac{I_dP\lambda}{\alpha}((e^{\alpha T}-1)(m-\frac{1}{\alpha})+T)(\frac{e^{rH}-1}{e^{rT}-1}) \quad (11)$$

$$\begin{aligned}Z_2(T) &= OC+PC+HC-IE_2\\ &= [\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2]+hC\lambda[\frac{T}{2}\\ &+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3]-\frac{Ar}{2}-\frac{rC\lambda}{2}(T+\frac{\alpha}{2}T^2\\ &+\frac{\theta}{6}T^3)-\frac{hC\lambda r}{2}[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{12}T^4]\\ &-I_dP\lambda(m+\frac{T}{2}(m\alpha-1))+\frac{I_dP\lambda r}{2}(mT\\ &+\frac{T^2}{2}(m\alpha-1))](H(1+\frac{rH}{2}))\end{aligned} \quad (12)$$

Case(iii) $T_d \le m \le T$

Delays in payments are allowed since $T \ge m \ge T_d$, and the overall applicable expense includes both the interest paid and the interest received. In $(0, H)$, the interest rate is

$$\begin{aligned}IC_3 &= I_c\sum_{k=0}^{n-1} C(kT)(\int_m^T I(kT+t)dt)\\ &= I_cC\lambda[\frac{T^2}{2}+\frac{\alpha}{3}T^3+\frac{\theta}{8}T^4-\frac{\theta}{24}T^4\\ &-\frac{m}{2}(2T-m)-\frac{\alpha m}{6}(3T^2-m^2)\\ &-\frac{\theta m}{24}(4T^3-4Tm^2+2m^3)](\frac{e^{rH}-1}{e^{rT}-1})\end{aligned} \quad (13)$$

The interest gained in $[0, H]$ is given by

$$IE_3 = I_d\sum_{k=0}^{n-1} P(kT)(\int_0^m \lambda e^{\alpha t}tdt)$$
$$= \frac{I_dP\lambda}{\alpha}(me^{\alpha m}-\frac{1}{\alpha}(e^{\alpha m}-1))(\frac{e^{rH}-1}{e^{rT}-1}) \quad (14)$$

$$\begin{aligned}Z_3(T) &= OC+PC+HC+IC_3-IE_3\\ &= [\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2+\frac{hT}{2}+\frac{h\alpha}{3}T^2+\frac{h\theta}{12}T^3\\ &-\frac{rT}{2}-\frac{r\alpha}{4}T^2-\frac{r\theta}{12}T^3-\frac{hrT^2}{4}\\ &-\frac{hr\alpha}{6}T^3-\frac{hr\theta}{24}T^4]+I_cC\lambda[\frac{T}{2}+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3\\ &-m+\frac{m^2}{2T}-\frac{\alpha mT}{2}+\frac{\alpha m^3}{6T}-\frac{m\theta}{6}T^2\\ &+\frac{\theta m^3}{6}+\frac{\theta m^4}{12T}-\frac{r}{4}T^2-\frac{r\alpha}{6}T^3-\frac{r\theta}{24}T^4+\frac{rmT}{2}\\ &-\frac{rm^2}{4}+\frac{rm\alpha T^2}{4}-\frac{r\alpha m^3}{12}+\frac{rm\theta}{12}T^3-\frac{r\theta m^3}{12}T\\ &-\frac{r\theta m^4}{24}]-\frac{Ar}{2}-\frac{I_dP\lambda}{\alpha T}(m^2(m\alpha+1))\\ &+\frac{I_dP\lambda r}{2\alpha}(m^2(m\alpha+1))](H(1+\frac{rH}{2}))\end{aligned} \quad (15)$$

Case (iv) $m \le T_d \le T$

Case (iv) is similar to case (iii), since T $\ge T_d \ge$ m ,
As a result, in (0,H), the total applicable cost is

$$\begin{aligned}Z_4(T) &= [\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2+\frac{hT}{2}+\frac{h\alpha}{3}T^2\\ &+\frac{h\theta}{12}T^3-\frac{rT}{2}-\frac{r\alpha}{4}T^2-\frac{r\theta}{12}T^3-\frac{hrT^2}{4}\\ &-\frac{hr\alpha}{6}T^3-\frac{hr\theta}{24}T^4]+I_cC\lambda[\frac{T}{2}+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3\\ &-m+\frac{m^2}{2T}-\frac{\alpha mT}{2}+\frac{\alpha m^3}{6T}-\frac{m\theta}{6}T^2\\ &+\frac{\theta m^3}{6}+\frac{\theta m^4}{12T}-\frac{r}{4}T^2-\frac{r\alpha}{6}T^3-\frac{r\theta}{24}T^4+\frac{rmT}{2}\\ &-\frac{rm^2}{4}+\frac{rm\alpha T^2}{4}-\frac{r\alpha m^3}{12}+\frac{rm\theta}{12}T^3-\frac{r\theta m^3}{12}T\\ &-\frac{r\theta m^4}{24}]-\frac{Ar}{2}-\frac{I_dP\lambda}{2T}(m^2(m\alpha+1))\\ &+\frac{I_dP\lambda r}{2\alpha}(m^2(m\alpha+1))](H(1+\frac{rH}{2}))\end{aligned} \quad (16)$$

Fuzzy Model:

Let $\tilde{A} = (a_1, a_2, a_3)$, $\tilde{C} = (C_1, C_2, C_3)$, $\tilde{h} = (h_1, h_2, h_3)$, $\tilde{m} = (m_1, m_2, m_3)$, $\tilde{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$, $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$.

The total cost per unit time in a fuzzy context is calculated using the above triangular fuzzy numbers.

Case(i) $0 < T < T_d$

$$\begin{aligned}\widetilde{Z_1(T)} &= [\frac{A}{T} + C\lambda[1 + \frac{\alpha T}{2} + \frac{\theta}{6}T^2] + (h + I_c)C\lambda[\frac{T}{2} \\ &+ \frac{\alpha}{3}T^2 + \frac{\theta}{12}T^3] - \frac{Ar}{2} - \frac{rC\lambda}{2}(T + \frac{\alpha}{2}T^2 + \frac{\theta}{6}T^3) \\ &- (h + I_c)\frac{C\lambda r}{2}[\frac{T}{2} + \frac{\alpha}{3}T^2 + \frac{\theta}{12}T^3]](H(1 + \frac{rH}{2}))\end{aligned} \quad (17)$$

Now defuzzifing the total cost $\widetilde{Z_1(T)}$ by using Signed distance method, we have

$$\begin{aligned}&Z_{1\widetilde{(sd)}}(T) = \frac{1}{4}\{Z_{1\widetilde{(sd_1)}}(T) + 2Z_{1\widetilde{(sd_2)}}(T) + Z_{1\widetilde{(sd_3)}}(T)\} \\ &= \{\{\frac{1}{4}[\frac{a_1}{T} + C_1\lambda_1[1 + \frac{\alpha_1 T}{2} + \frac{\theta_1}{6}T^2] + (h_1 + I_c)C_1\lambda_1 \\ &\times [\frac{T}{2} + \frac{\alpha_1}{3}T^2 + \frac{\theta_1}{12}T^3] - \frac{\alpha_1 r}{2} - \frac{rC_1\lambda_1}{2}[T + \frac{\alpha_1}{2}T^2 \\ &+ \frac{\theta_1}{6}T^3] - (h_1 + I_c)\frac{C_1\lambda_1 r}{2}[\frac{T}{2} + \frac{\alpha_1}{3}T^2 + \frac{\theta_1}{12}T^3]]\} \\ &+ \{\frac{1}{2}[\frac{a_2}{T} + C_2\lambda_2[1 + \frac{\alpha_2 T}{2} + \frac{\theta_2}{6}T^2] + (h_2 + I_c)C_2\lambda_2 \\ &\times [\frac{T}{2} + \frac{\alpha_2}{3}T^2 + \frac{\theta_2}{12}T^3] - \frac{\alpha_2 r}{2} - \frac{rC_2\lambda_2}{2}[T + \frac{\alpha_2}{2}T^2 \\ &+ \frac{\theta_2}{6}T^3] - (h_2 + I_c)\frac{C_2\lambda_2 r}{2}[\frac{T}{2} + \frac{\alpha_2}{3}T^2 + \frac{\theta_2}{12}T^3]]\} \\ &+ \{\frac{1}{4}[\frac{a_3}{T} + C_3\lambda_3[1 + \frac{\alpha_3 T}{2} + \frac{\theta_3}{6}T^2] + (h_3 + I_c)C_3\lambda_3[\frac{T}{2} \\ &+ \frac{\alpha_3}{3}T^2 + \frac{\theta_3}{12}T^3] - \frac{\alpha_3 r}{2} - \frac{rC_3\lambda_3}{2}[T + \frac{\alpha_3}{2}T^2 + \frac{\theta_3}{6}T^3] \\ &- (h_3 + I_c)\frac{C_3\lambda_3 r}{2}[\frac{T}{2} + \frac{\alpha_3}{3}T^2 + \frac{\theta_3}{12}T^3]]\}\}(H(1 + \frac{rH}{2}))\end{aligned} \quad (18)$$

The necessary condition for minimizing the total cost is $\frac{\partial Z_{1\widetilde{(sd)}}(T)}{\partial T} = 0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 Z_{1\widetilde{(sd)}}(T)}{\partial T^2} > 0$

$$\begin{aligned}&\{\{\frac{1}{4}[\frac{-a_1}{T^2} + C_1\lambda_1[\frac{\alpha_1}{2} + \frac{\theta_1}{3}T - \frac{r}{2} - \frac{r\alpha_1}{2}T + \frac{r\theta_1}{4}T^2] \\ &+ (h_1 + I_c)C_1\lambda_1[\frac{1}{2} + \frac{2\alpha_1}{3}T + \frac{\theta_1}{4}T^2 - \frac{rT}{2} - \frac{r\alpha_1}{2}T^2 \\ &- \frac{r\theta_1}{3}T^3]]\} + \{\frac{1}{2}[\frac{-a_2}{T^2} + C_2\lambda_2[\frac{\alpha_2}{2} + \frac{\theta_2}{3}T - \frac{r}{2} - \frac{r\alpha_2}{2}T \\ &+ \frac{r\theta_2}{4}T^2] + (h_2 + I_c)C_2\lambda_2[\frac{1}{2} + \frac{2\alpha_2}{3}T + \frac{\theta_2}{4}T^2 - \frac{rT}{2} \\ &- \frac{r\alpha_2}{2}T^2 - \frac{r\theta_2}{3}T^3]]\} + \{\frac{1}{4}[\frac{-a_3}{T^2} + C_3\lambda_3[\frac{\alpha_3}{2} + \frac{\theta_3}{3}T - \frac{r}{2} \\ &- \frac{r\alpha_3}{2}T + \frac{r\theta_3}{4}T^2] + (h_3 + I_c)C_3\lambda_3[\frac{1}{2} + \frac{2\alpha_3}{3}T + \frac{\theta_3}{4}T^2 \\ &- \frac{rT}{2} - \frac{r\alpha_3}{2}T^2 - \frac{r\theta_3}{3}T^3]]\}\}(H(1 + \frac{rH}{2})) = 0\end{aligned} \quad (19)$$

Now defuzzifing the total cost $\widetilde{Z_1(T)}$ by using Graded mean integration method, we have

$$\begin{aligned}&Z_{1\widetilde{(gm)}}(T) = \frac{1}{6}\{Z_{1\widetilde{(gm_1)}}(T) + 4Z_{1\widetilde{(gm_2)}}(T) + Z_{1\widetilde{(gm_3)}}(T)\} \\ &= \{\{\frac{1}{6}[\frac{a_1}{T} + C_1\lambda_1[1 + \frac{\alpha_1 T}{2} + \frac{\theta_1}{6}T^2] \\ &+ (h_1 + I_c)C_1\lambda_1[\frac{T}{2} + \frac{\alpha_1}{3}T^2 + \frac{\theta_1}{12}T^3] - \frac{\alpha_1 r}{2} \\ &- \frac{rC_1\lambda_1}{2}[T + \frac{\alpha_1}{2}T^2 + \frac{\theta_1}{6}T^3] - (h_1 + I_c)\frac{C_1\lambda_1 r}{2}[\frac{T}{2} \\ &+ \frac{\alpha_1}{3}T^2 + \frac{\theta_1}{12}T^3]]\} + \{\frac{2}{3}[\frac{a_2}{T} + C_2\lambda_2[1 + \frac{\alpha_2 T}{2} + \frac{\theta_2}{6}T^2] \\ &+ (h_2 + I_c)C_2\lambda_2[\frac{T}{2} + \frac{\alpha_2}{3}T^2 + \frac{\theta_2}{12}T^3] - \frac{\alpha_2 r}{2} \\ &- \frac{rC_2\lambda_2}{2}[T + \frac{\alpha_2}{2}T^2 + \frac{\theta_2}{6}T^3] - (h_2 + I_c)\frac{C_2\lambda_2 r}{2}[\frac{T}{2} + \\ &\frac{\alpha_2}{3}T^2 + \frac{\theta_2}{12}T^3]]\} + \{\frac{1}{6}[\frac{a_3}{T} + C_3\lambda_3[1 + \frac{\alpha_3 T}{2} + \frac{\theta_3}{6}T^2] \\ &+ (h_3 + I_c)C_3\lambda_3[\frac{T}{2} + \frac{\alpha_3}{3}T^2 + \frac{\theta_3}{12}T^3] - \frac{\alpha_3 r}{2} \\ &- \frac{rC_3\lambda_3}{2}[T + \frac{\alpha_3}{2}T^2 + \frac{\theta_3}{6}T^3] - (h_3 + I_c)\frac{C_3\lambda_3 r}{2}[\frac{T}{2} \\ &+ \frac{\alpha_3}{3}T^2 + \frac{\theta_3}{12}T^3]]\}\}(H(1 + \frac{rH}{2}))\end{aligned} \quad (20)$$

The necessary condition for minimizing the total cost is $\frac{\partial Z_{1\widetilde{(gm)}}(T)}{\partial T} = 0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 Z_{1\widetilde{(gm)}}(T)}{\partial T^2} > 0$

$$\begin{aligned}&\{\{\frac{1}{6}[\frac{-a_1}{T^2} + C_1\lambda_1[\frac{\alpha_1}{2} + \frac{\theta_1}{3}T - \frac{r}{2} - \frac{r\alpha_1}{2}T + \frac{r\theta_1}{4}T^2] \\ &+ (h_1 + I_c)C_1\lambda_1[\frac{1}{2} + \frac{2\alpha_1}{3}T + \frac{\theta_1}{4}T^2 - \frac{rT}{2} - \frac{r\alpha_1}{2}T^2 \\ &- \frac{r\theta_1}{3}T^3]]\} + \{\frac{2}{3}[\frac{-a_2}{T^2} + C_2\lambda_2[\frac{\alpha_2}{2} + \frac{\theta_2}{3}T - \frac{r}{2} - \frac{r\alpha_2}{2}T \\ &+ \frac{r\theta_2}{4}T^2] + (h_2 + I_c)C_2\lambda_2[\frac{1}{2} + \frac{2\alpha_2}{3}T + \frac{\theta_2}{4}T^2 - \frac{rT}{2} \\ &- \frac{r\alpha_2}{2}T^2 - \frac{r\theta_2}{3}T^3]]\} + \{\frac{1}{6}[\frac{-a_3}{T^2} + C_3\lambda_3[\frac{\alpha_3}{2} + \frac{\theta_3}{3}T - \frac{r}{2} \\ &- \frac{r\alpha_3}{2}T + \frac{r\theta_3}{4}T^2] + (h_3 + I_c)C_3\lambda_3[\frac{1}{2} + \frac{2\alpha_3}{3}T + \frac{\theta_3}{4}T^2 \\ &- \frac{rT}{2} - \frac{r\alpha_3}{2}T^2 - \frac{r\theta_3}{3}T^3]]\}\}(H(1 + \frac{rH}{2})) = 0\end{aligned} \quad (21)$$

Case(ii) $T_d \le m < T$

$$\begin{aligned}\widetilde{Z_2(T)} &= [\frac{A}{T} + C\lambda[1 + \frac{\alpha T}{2} + \frac{\theta}{6}T^2] + hC\lambda[\frac{T}{2} + \frac{\alpha}{3}T^2 \\ &+ \frac{\theta}{12}T^3] - \frac{Ar}{2} - \frac{rC\lambda}{2}(T + \frac{\alpha}{2}T^2 + \frac{\theta}{6}T^3) - \frac{hC\lambda r}{2}[\frac{T^2}{2} \\ &+ \frac{\alpha}{3}T^3 + \frac{\lambda}{12}T^4] - I_d P\lambda(m + \frac{T}{2}(m\alpha - 1)) \\ &+ \frac{I_d P\lambda r}{2}(mT + \frac{T^2}{2}(m\alpha - 1))](H(1 + \frac{rH}{2}))\end{aligned} \quad (22)$$

Now defuzzifing the total cost $\widetilde{Z_2(T)}$ by using Signed distance method, we have

$$\begin{aligned}&Z_{2\widetilde{(sd)}}(T) = \frac{1}{4}\{Z_{2\widetilde{(sd_1)}}(T) + 2Z_{2\widetilde{(sd_2)}}(T) + Z_{2\widetilde{(sd_3)}}(T)\} \\ &= \{\{\frac{1}{4}[\frac{a_1}{T} + C_1\lambda_1[1 + \frac{\alpha_1 T}{2} + \frac{\theta_1}{6}T^2] + h_1C_1\lambda_1[\frac{T}{2} \\ &+ \frac{\alpha_1}{3}T^2 + \frac{\theta_1}{12}T^3] - \frac{\alpha_1 r}{2} - \frac{rC_1\lambda_1}{2}(T + \frac{\alpha_1}{2}T^2 + \frac{\theta_1}{6}T^3) \\ &- \frac{h_1\lambda_1 r}{2}[\frac{T^2}{2} + \frac{\alpha_1}{3}T^3 + \frac{\theta_1}{12}T^4] - I_d P\lambda_1(m_1\end{aligned}$$

$$\begin{aligned}
&+\frac{T}{2}(m_1\alpha_1-1))+\frac{I_dP\lambda_1 r}{2}(m_1T+\frac{T^2}{2}(m_1\alpha_1-1))]\}\\
&+\{\frac{1}{2}[\frac{a_2}{T}+C_2\lambda_2[1+\frac{\alpha_2T}{2}+\frac{\theta_2}{6}T^2]+h_2C_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2\\
&+\frac{\theta_2}{12}T^3]-\frac{\alpha_2 r}{2}-\frac{rC_2\lambda_2}{2}(T+\frac{\alpha_2}{2}T^2+\frac{\theta_2}{6}T^3)\\
&-\frac{h_2C_2\lambda_2 r}{2}[\frac{T^2}{2}+\frac{\alpha_2}{3}T^3+\frac{\theta_2}{12}T^4]-I_dP\lambda_2(m_2\\
&+\frac{T}{2}(m_2\alpha_2-1))+\frac{I_dP\lambda_2 r}{2}(m_2T+\frac{T^2}{2}(m_2\alpha_2-1))]\}\\
&+\{\frac{1}{4}[\frac{a_3}{T}+C_3\lambda_3[1+\frac{\alpha_3T}{2}+\frac{\theta_3}{6}T^2]+h_3C_3\lambda_3[\frac{T}{2}+\frac{a_3}{3}T^2\\
&+\frac{\theta_3}{12}T^3]-\frac{\alpha_3 r}{2}-\frac{rC_3\lambda_3}{2}(T+\frac{\alpha_3}{2}T^2+\frac{\theta_3}{6}T^3)\\
&-\frac{h_3\lambda_3 r}{2}[\frac{T^2}{2}+\frac{\alpha_3}{3}T^3+\frac{\theta_3}{12}T^4]-I_dP\lambda_3(m_3+\frac{T}{2}(m_3\alpha_3\\
&-1))+\frac{I_dP\lambda_3 r}{2}(m_3T+\frac{T^2}{2}(m_3\alpha_3-1))]\}(H(1+\frac{rH}{2}))
\end{aligned}\tag{23}$$

The necessary condition for minimizing the total cost is $\frac{\partial \widetilde{Z_{4(gm)}(T)}}{\partial T}=0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 \widetilde{Z_{4(gm)}(T)}}{\partial T^2}>0$

$$\begin{aligned}
&\{\{\frac{1}{4}[\frac{-a_1}{T^2}+C_1\lambda_1[\frac{\alpha_1}{2}+\frac{\theta_1}{3}T-\frac{r}{2}-\frac{r\alpha_1}{2}T+\frac{r\theta_1}{4}T^2]\\
&+h_1C_1\lambda_1[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3]\\
&-\frac{I_dP\lambda_1}{2}[(m_1\alpha_1-1)(1-rT)-m_1r]]\}+\{\frac{1}{2}[\frac{-a_2}{T^2}\\
&+C_2\lambda_2[\frac{\alpha_2}{2}+\frac{\theta_2}{3}T-\frac{r}{2}-\frac{r\alpha_2}{2}T+\frac{r\theta_2}{4}T^2]+h_2C_2\lambda_2[\frac{1}{2}\\
&+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{rT}{2}-\frac{r\alpha_2}{2}T^2-\frac{r\theta_2}{3}T^3]\\
&-\frac{I_dP\lambda_2}{2}[(m_2\alpha_2-1)(1-rT)-m_2r]]\}+\{\frac{1}{4}[\frac{-a_3}{T^2}\\
&+C_3\lambda_3[\frac{\alpha_3}{2}+\frac{\theta_3}{3}T-\frac{r}{2}-\frac{r\alpha_3}{2}T+\frac{r\theta_3}{4}T^2]+h_3C_3\lambda_3[\frac{1}{2}\\
&+\frac{2\alpha_3}{3}T+\frac{\theta_3}{4}T^2-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\alpha_3}{3}T^3]\\
&-\frac{I_dP\lambda_3}{2}[(m_3\alpha_3-1)(1-rT)-m_3r]]\}\}(H(1+\frac{rH}{2}))=0
\end{aligned}\tag{24}$$

Now defuzzifing the total cost $\widetilde{Z_2(T)}$ by using Graded mean integration method, we have

$$\begin{aligned}
&\widetilde{Z_{2(gm)}}(T)=\frac{1}{6}\{\widetilde{Z_{2(gm_1)}}(T)+4\widetilde{Z_{2(gm_2)}}(T)+\widetilde{Z_{2(gm_3)}}(T)\}\\
&=\{\{\frac{1}{6}[\frac{a_1}{T}+C_1\lambda_1[1+\frac{\alpha_1T}{2}+\frac{\theta_1}{6}T^2]+h_1C_1\lambda_1[\frac{T}{2}\\
&+\frac{\alpha_1}{3}T^2+\frac{\theta_1}{12}T^3]-\frac{\alpha_1 r}{2}-\frac{rC_1\lambda_1}{2}(T+\frac{\alpha_1}{2}T^2+\frac{\theta_1}{6}T^3)\\
&-\frac{h_1\lambda_1 r}{2}[\frac{T^2}{2}+\frac{\alpha_1}{3}T^3+\frac{\theta_1}{12}T^4]-I_dP\lambda_1(m_1\\
&+\frac{T}{2}(m_1\alpha_1-1))+\frac{I_dP\lambda_1 r}{2}(m_1T+\frac{T^2}{2}(m_1\alpha_1-1))]\}\\
&+\{\frac{2}{3}[\frac{a_2}{T}+C_2\lambda_2[1+\frac{\alpha_2T}{2}+\frac{\theta_2}{6}T^2]\\
&+h_2C_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2+\frac{\theta_2}{12}T^3]-\frac{\alpha_2 r}{2}-\frac{rC_2\lambda_2}{2}(T+\\
&\frac{\alpha_2}{2}T^2+\frac{\theta_2}{6}T^3)-\frac{h_2C_2\lambda_2 r}{2}[\frac{T^2}{2}+\frac{\alpha_2}{3}T^3+\frac{\theta_2}{12}T^4]\\
&-I_dP\lambda_2(m_2+\frac{T}{2}(m_2\alpha_2-1))+\frac{I_dP\lambda_2 r}{2}(m_2T\\
&+\frac{T^2}{2}(m_2\alpha_2-1))]\}+\{\frac{1}{6}[\frac{a_3}{T}+C_3\lambda_3[1+\frac{\alpha_3T}{2}+\frac{\theta_3}{6}T^2]\\
&+h_3C_3\lambda_3[\frac{T}{2}+\frac{\alpha_3}{3}T^2+\frac{\theta_3}{12}T^3]-\frac{\alpha_3 r}{2}-\frac{rC_3\lambda_3}{2}(T\\
&+\frac{\alpha_3}{2}T^2+\frac{\theta_3}{6}T^3)-\frac{h_3\lambda_3 r}{2}[\frac{T^2}{2}+\frac{\alpha_3}{3}T^3+\frac{\theta_3}{12}T^4]\\
&-I_dP\lambda_3(m_3+\frac{T}{2}(m_3\alpha_3-1))\\
&+\frac{I_dP\lambda_3 r}{2}(m_3T+\frac{T^2}{2}(m_3\alpha_3-1))]\}(H(1+\frac{rH}{2}))
\end{aligned}\tag{25}$$

The necessary condition for minimizing the total cost is $\frac{\partial \widetilde{Z_{2(gm)}(T)}}{\partial T}=0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 \widetilde{Z_{2(gm)}(T)}}{\partial T^2}>0$

$$\begin{aligned}
&\{\{\frac{1}{6}[\frac{-a_1}{T^2}+C_1\lambda_1[\frac{\alpha_1}{2}+\frac{\theta_1}{3}T-\frac{r}{2}-\frac{r\alpha_1}{2}T-\frac{r\theta_1}{4}T^2]\\
&+h_1C_1\lambda_1[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3]\\
&-\frac{I_dP\lambda_1}{2}[(m_1\alpha_1-1)(1-rT)-m_1r]]\}+\{\frac{2}{3}[\frac{-a_2}{T^2}\\
&+C_2\lambda_2[\frac{\alpha_2}{2}+\frac{\theta_2}{3}T-\frac{r}{2}-\frac{r\alpha_2}{2}T-\frac{r\theta_2}{4}T^2]\\
&+h_2C_2\lambda_2[\frac{1}{2}+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{rT}{2}-\frac{r\alpha_2}{2}T^2-\frac{r\theta_2}{3}T^3]\\
&-\frac{I_dP\lambda_2}{2}[(m_2\alpha_2-1)(1-rT)-m_2r]]\}+\{\frac{1}{6}[\frac{-a_3}{T^2}\\
&+C_3\lambda_3[\frac{\alpha_3}{2}+\frac{\theta_3}{3}T-\frac{r}{2}-\frac{r\alpha_3}{2}T-\frac{r\theta_3}{4}T^2]+h_3C_3\lambda_3[\frac{1}{2}\\
&+\frac{2\alpha_3}{3}T+\frac{\theta_3}{4}T^2-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\theta_3}{3}T^3]\\
&-\frac{I_dP\lambda_3}{2}[(m_3\alpha_3-1)(1-rT)-m_3r]]\}\}(H(1+\frac{rH}{2}))=0
\end{aligned}\tag{26}$$

Case(iii) $T_d \le m \le T$

$$\begin{aligned}
&\widetilde{Z_3(T)}=[\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2+\frac{hT}{2}+\frac{ha}{3}T^2+\frac{h\theta}{12}T^3\\
&-\frac{rT}{2}-\frac{r\alpha}{4}T^2-\frac{r\theta}{12}T^3-\frac{hrT^2}{4}-\frac{hr\alpha}{6}T^3-\frac{hr\theta}{24}T^4]\\
&+I_cC\lambda\frac{T}{2}+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3-m+\frac{m^2}{2T}-\frac{\alpha mT}{2}+\frac{\alpha m^3}{6T}\\
&-\frac{m\theta}{6}T^2+\frac{\theta m^3}{6}+\frac{\theta m^4}{12T}-\frac{r}{4}T^2-\frac{ra}{6}T^3-\frac{r\theta}{24}T^4+\frac{rmT}{2}\\
&-\frac{rm^2}{4}+\frac{rm\alpha T^2}{4}-\frac{r\alpha m^3}{12}+\frac{rm\theta}{12}T^3-\frac{r\theta m^3}{12}T-\frac{r\theta m^4}{24}]\\
&-\frac{Ar}{2}-\frac{I_dP\lambda}{2T}(m^2(m\alpha+1))
\end{aligned}$$

$$+\frac{I_dP\lambda r}{2\alpha}(m^2(m\alpha+1))](H(1+\frac{rH}{2})) \quad (27)$$

Now defuzzifing the total cost $\widetilde{Z_3(T)}$ by using Signed distance method, we have

$$\begin{aligned}
&\widetilde{Z_{3(sd)}}(T)=\frac{1}{4}\{\widetilde{Z_{3(sd_1)}}(T)+2\widetilde{Z_{3(sd_2)}}(T)+\widetilde{Z_{3(sd_3)}}(T)\}\\
&=\{\frac{1}{4}[\frac{a_1}{T}+C_1\lambda_1[1+\frac{\alpha_1T}{2}+\frac{\theta_1}{6}T^2+\frac{h_1T}{2}+\frac{h_1\alpha_1}{3}T^2\\
&+\frac{h_1}{12}T^3-\frac{rT}{2}-\frac{r\alpha_1}{4}T^2-\frac{r\theta_1}{12}T^3-\frac{h_1rT^2}{4}-\frac{h_1r\alpha_1}{6}T^3\\
&-\frac{h_1r\theta_1}{24}T^4]+I_cC_1\lambda_1[\frac{T}{2}+\frac{\alpha_1}{3}T^2+\frac{\theta_1}{12}T^3-m_1+\frac{{m_1}^2}{2T}\\
&-\frac{\alpha_1m_1T}{2}+\frac{\alpha_1{m_1}^3}{6T}-\frac{\theta_1}{6}T^2+\frac{\theta_1{m_1}^3}{6}+\frac{\theta_1{m_1}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_1}{6}T^3-\frac{r\theta_1}{24}T^4+\frac{rm_1T}{2}-\frac{r{m_1}^2}{4}+\frac{rm_1}{4}-\frac{r\alpha_1{m_1}^3}{12}\\
&+\frac{rm_1\theta_1}{12}T^3-\frac{r\theta_1{m_1}^3}{12}T-\frac{r\theta_1{m_1}^4}{24}]-\frac{\alpha_1r}{2}\\
&-\frac{I_dP\lambda_1}{2T}({m_1}^2(m_1\alpha_1+1))+\frac{I_dP\lambda_1r}{2a_1}({m_1}^2(m_1+1))]\}\\
&+\{\frac{1}{2}[\frac{a_2}{T}+C_2\lambda_2[1+\frac{\alpha_2T}{2}+\frac{\theta_2}{6}T^2+\frac{h_2T}{2}+\frac{h_2\alpha_2}{3}T^2\\
&+\frac{h_2}{12}T^3-\frac{rT}{2}-\frac{r\alpha_2}{4}T^2-\frac{r\theta_2}{12}T^3-\frac{h_2rT^2}{4}-\frac{h_2r\alpha_2}{6}T^3\\
&-\frac{h_2r\theta_2}{24}T^4]+I_cC_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2+\frac{\theta_2}{12}T^3-m_2+\frac{{m_2}^2}{2T}\\
&-\frac{\alpha_2m_2T}{2}+\frac{\alpha_2{m_2}^3}{6T}-\frac{\theta_2}{6}T^2+\frac{\theta_2{m_2}^3}{6}+\frac{\theta_2{m_2}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_2}{6}T^3-\frac{r\theta_2}{24}T^4+\frac{rm_2T}{2}-\frac{r{m_2}^2}{4}+\frac{rm_2}{4}-\frac{r\alpha_2{m_2}^3}{12}\\
&+\frac{rm_2\theta_2}{12}T^3-\frac{r\theta_2{m_2}^3}{12}T-\frac{r\theta_2{m_2}^4}{24}]-\frac{\alpha_2r}{2}\\
&-\frac{I_dP\lambda_2}{2T}({m_2}^2(m_2\alpha_2+1))+\frac{I_dP\lambda_2r}{2\alpha_2}({m_2}^2(m_2+1))]\}\\
&+\{\frac{1}{4}[\frac{a_3}{T}+C_3\lambda_3[1+\frac{\alpha_3T}{2}+\frac{\theta_3}{6}T^2+\frac{h_3T}{2}+\frac{h_3a_3}{3}T^2\\
&+\frac{h_3}{12}T^3-\frac{rT}{2}-\frac{r\alpha_3}{4}T^2-\frac{r\theta_3}{12}T^3-\frac{h_3rT^2}{4}-\frac{h_3r\alpha_3}{6}T^3\\
&-\frac{h_3r\theta_3}{24}T^4]+I_cC_3\lambda_3[\frac{T}{2}+\frac{a_3}{3}T^2+\frac{\theta_3}{12}T^3-m_3+\frac{{m_3}^2}{2T}\\
&-\frac{\alpha_3m_3T}{2}+\frac{\alpha_3{m_3}^3}{6T}-\frac{\theta_3}{6}T^2+\frac{\theta_3{m_3}^3}{6}+\frac{\theta_3{m_3}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_3}{6}T^3-\frac{r\theta_3}{24}T^4+\frac{rm_3T}{2}-\frac{r{m_3}^2}{4}+\frac{rm_3}{4}-\frac{r\alpha_3{m_3}^3}{12}\\
&+\frac{rm_3\theta_3}{12}T^3-\frac{r\theta_3{m_3}^3}{12}T-\frac{r\theta_3{m_3}^4}{24}]-\frac{\alpha_3r}{2}\\
&-\frac{I_dP\lambda_3}{2T}\,({m_3}^2(m_3\alpha_3+1))\\
&+\frac{I_dP\lambda_3r}{2\alpha_3}({m_3}^2(m_3+1))]\}(H(1+\frac{rH}{2})) \quad (28)
\end{aligned}$$

$$\begin{aligned}
&\{\{\frac{1}{4}[\frac{-a_1}{T^2}+C_1\lambda_1[\frac{\alpha_1}{2}+\frac{\theta_1}{3}T-\frac{r}{2}-\frac{r\alpha_1}{2}T-\frac{r\theta_1}{4}T^2]\\
&+h_1C_1\lambda_1[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3]\\
&+C_1\lambda_1I_c[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{{m_1}^2}{2T^2}-\frac{\alpha_1m_1}{2}-\frac{\alpha_1{m_1}^3}{6T^2}\\
&-\frac{\theta_1m_1T}{3}-\frac{\theta_1{m_1}^4}{12T^2}-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3+\frac{mr}{2}\\
&+\frac{m_1r\alpha_1}{2}T+\frac{m_1r\theta_1}{4}T^2+\frac{r\theta_1{m_1}^3}{12}]+\frac{I_dP\lambda_1}{2T^2}({m_1}^2(m_1\alpha_1\\
&+1))]\}+\{\frac{2}{3}[\frac{-a_2}{T^2}+C_2\lambda_2[\frac{\alpha_2}{2}+\frac{\theta_2}{3}T-\frac{r}{2}-\frac{r\alpha_2}{2}T\\
&-\frac{r\theta_2}{4}T^2]+h_2C_2\lambda_2[\frac{1}{2}+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{rT}{2}-\frac{r\alpha_2}{2}T^2\\
&-\frac{r\theta_2}{6}T^3]+C_2\lambda_2I_c[\frac{1}{2}+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{{m_2}^2}{2T^2}-\frac{\alpha_2m_2}{2}\\
&-\frac{\alpha_2{m_2}^3}{6T^2}-\frac{\theta_2m_2T}{3}-\frac{\theta_2{m_2}^4}{12T^2}-\frac{rT}{2}-\frac{r\alpha_2}{2}T^2-\frac{r\theta_2}{6}T^3\\
&+\frac{mr}{2}+\frac{m_2r\alpha_2}{2}T+\frac{m_2r\theta_2}{4}T^2+\frac{r\theta_2{m_2}^3}{12}]\\
&+\frac{I_dP\lambda_2}{2T^2}\,({m_1}^2(m_1\alpha_2+1))]\}+\{\frac{1}{4}[\frac{-a_3}{T^2}+C_3\lambda_3[\frac{\alpha_3}{2}\\
&+\frac{\theta_3}{3}T-\frac{r}{2}-\frac{r\alpha_3}{2}T-\frac{r\theta_3}{4}T^2]+h_3C_3\lambda_3[\frac{1}{2}+\frac{2\alpha_3}{3}T\\
&+\frac{\theta_3}{4}T^2-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\theta_3}{6}T^3]+C_3\lambda_3I_c[\frac{1}{2}+\frac{2\alpha_3}{3}T\\
&+\frac{\theta_3}{4}T^2-\frac{{m_3}^2}{2T^2}-\frac{\alpha_3m_3}{2}-\frac{\alpha_3{m_3}^3}{6T^2}-\frac{\theta_3m_3T}{3}-\frac{\theta_3{m_3}^4}{12T^2}\\
&-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\theta_3}{6}T^3+\frac{mr}{2}+\frac{m_3r\alpha_3}{2}T+\frac{m_3r\theta_3}{4}T^2\\
&+\frac{r\theta_3{m_3}^3}{12}]+\frac{I_dP\lambda_3}{2T^2}({m_3}^2(m_3\alpha_3+1))]\}\}(H(1+\frac{rH}{2}))=0
\end{aligned} \quad (29)$$

Now defuzzifing the total cost $\widetilde{Z_2(T)}$ by using Graded mean integration method, we have

$$\begin{aligned}
&\widetilde{Z_{3(gm)}}(T)=\frac{1}{6}\{\widetilde{Z_{3(gm_1)}}(T)+4\widetilde{Z_{3(gm_2)}}(T)+\widetilde{Z_{3(gm_3)}}(T)\}\\
&=\{\frac{1}{6}[\frac{a_1}{T}+C_1\lambda_1[1+\frac{\alpha_1T}{2}+\frac{\theta_1}{6}T^2+\frac{h_1T}{2}+\frac{h_1\alpha_1}{3}T^2\\
&+\frac{h_1}{12}T^3-\frac{rT}{2}-\frac{r\alpha_1}{4}T^2-\frac{r\theta_1}{12}T^3-\frac{h_1rT^2}{4}-\frac{h_1r\alpha_1}{6}T^3\\
&-\frac{h_1r\theta_1}{24}T^4]+I_cC_1\lambda_1[\frac{T}{2}+\frac{\alpha_1}{3}T^2+\frac{\theta_1}{12}T^3-m_1+\frac{{m_1}^2}{2T}\\
&-\frac{\alpha_1m_1T}{2}+\frac{\alpha_1{m_1}^3}{6T}-\frac{\theta_1}{6}T^2+\frac{\theta_1{m_1}^3}{6}+\frac{\theta_1{m_1}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_1}{6}T^3-\frac{r\theta_1}{24}T^4+\frac{rm_1T}{2}-\frac{r{m_1}^2}{4}+\frac{rm_1}{4}-\frac{r\alpha_1{m_1}^3}{12}\\
&+\frac{rm_1\theta_1}{12}T^3-\frac{r\theta_1{m_1}^3}{12}T-\frac{r\theta_1{m_1}^4}{24}]-\frac{\alpha_1r}{2}\\
&-\frac{I_dP\lambda_1}{2T}({m_1}^2(m_1\alpha_1+1))+\frac{I_dP\lambda_1r}{2\alpha_1}({m_1}^2(m_1+1))]\}\\
&+\{\frac{2}{3}[\frac{a_2}{T}+C_2\lambda_2[1+\frac{\alpha_2T}{2}+\frac{\theta_2}{6}T^2+\frac{h_2T}{2}+\frac{h_2\alpha_2}{3}T^2\\
&+\frac{h_2}{12}T^3-\frac{rT}{2}-\frac{r\alpha_2}{4}T^2-\frac{r\theta_2}{12}T^3-\frac{h_2rT^2}{4}-\frac{h_2r\alpha_2}{6}T^3\\
&-\frac{h_2r\theta_2}{24}T^4]+I_cC_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2\frac{\theta_2}{12}T^3-m_2+\frac{{m_2}^2}{2T}
\end{aligned}$$

$$\begin{aligned}
&-\frac{\alpha_2 m_2 T}{2}+\frac{\alpha_2 m_2^3}{6T}-\frac{\theta_2}{6}T^2+\frac{\theta_2 m_2^3}{6}+\frac{\theta_2 m_2^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_2}{6}T^3-\frac{r\theta_2}{24}T^4+\frac{rm_2T}{2}-\frac{rm_2^2}{4}+\frac{rm_2}{4}-\frac{r\alpha_2 m_2^3}{12}\\
&+\frac{rm_2\theta_2}{12}T^3-\frac{r\theta_2 m_2^3}{12}T-\frac{r\theta_2 m_2^4}{24}]-\frac{\alpha_2 r}{2}\\
&-\frac{I_dP\lambda_2}{2T}(m_2^2(m_2\alpha_2+1))+\frac{I_dP\lambda_2 r}{2\alpha_2}(m_2^2(m_2+1))]\}\\
&+\{\frac{1}{6}[\frac{a_3}{T}+C_3\lambda_3[1+\frac{\alpha_3T}{2}+\frac{\theta_3}{6}T^2+\frac{h_3T}{2}+\frac{h_3\alpha_3}{3}T^2\\
&+\frac{h_3}{12}T^3-\frac{rT}{2}-\frac{r\alpha_3}{4}T^2-\frac{r\theta_3}{12}T^3-\frac{h_3rT^2}{4}-\frac{h_3r\alpha_3}{6}T^3\\
&-\frac{h_3r\theta_3}{24}T^4]+I_cC_3\lambda_3[\frac{T}{2}+\frac{\alpha_3}{3}T^2+\frac{\theta_3}{12}T^3-m_3+\frac{m_3^2}{2T}\\
&-\frac{\alpha_3 m_3 T}{2}+\frac{\alpha_3 m_3^3}{6T}-\frac{\theta_3}{6}T^2+\frac{\theta_3 m_3^3}{6}+\frac{\theta_3 m_3^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_3}{6}T^3-\frac{r\theta_3}{24}T^4+\frac{rm_3T}{2}-\frac{rm_3^2}{4}+\frac{rm_3}{4}-\frac{r\alpha_3 m_3^3}{12}\\
&+\frac{rm_3\theta_3}{12}T^3-\frac{r\theta_3 m_3^3}{12}T-\frac{r\theta_3 m_3^4}{24}]-\frac{\alpha_3 r}{2}\\
&-\frac{I_dP\lambda_3}{2T}(m_3^2(m_3\alpha_3+1))\\
&+\frac{I_dP\lambda_3 r}{2\alpha_3}(m_3^2(m_3+1))]\}(H(1+\frac{rH}{2})) \qquad (30)
\end{aligned}$$

The necessary condition for minimizing the total cost is $\frac{\partial Z_{3(\widetilde{gm})}(T)}{\partial T}=0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 Z_{3(\widetilde{gm})}(T)}{\partial T^2}>0$

$$\begin{aligned}
&\{\{\frac{1}{4}[\frac{-a_1}{T^2}+C_1\lambda_1[\frac{\alpha_1}{2}+\frac{\theta_1}{3}T-\frac{r}{2}-\frac{ra_1}{2}T-\frac{r\theta_1}{4}T^2]\\
&+h_1C_1\lambda_1[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3]\\
&+C_1\lambda_1I_c[\frac{1}{2}+\frac{2\alpha_1}{3}T+\frac{\theta_1}{4}T^2-\frac{m_1^2}{2T^2}-\frac{\alpha_1m_1}{2}-\frac{\alpha_1m_1^3}{6T^2}\\
&-\frac{\theta_1m_1T}{3}-\frac{\theta_1m_1^4}{12T^2}-\frac{rT}{2}-\frac{r\alpha_1}{2}T^2-\frac{r\theta_1}{6}T^3+\frac{mr}{2}\\
&+\frac{m_1r\alpha_1}{2}T+\frac{m_1r\theta_1}{4}T^2+\frac{r\theta_1m_1^3}{12}]\\
&+\frac{I_dP\lambda_1}{2T^2}(m_1^2(m_1\alpha_1+1))]\}+\{\frac{2}{3}[\frac{-a_2}{T^2}+C_2\lambda_2[\frac{\alpha_2}{2}+\frac{\theta_2}{3}T\\
&-\frac{r}{2}-\frac{r\alpha_2}{2}T-\frac{r\theta_2}{4}T^2]+h_2C_2\lambda_2[\frac{1}{2}+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{rT}{2}\\
&-\frac{r\alpha_2}{2}T^2-\frac{r\theta_2}{6}T^3]+C_2\lambda_2I_c[\frac{1}{2}+\frac{2\alpha_2}{3}T+\frac{\theta_2}{4}T^2-\frac{m_2^2}{2T^2}\\
&-\frac{\alpha_2m_2}{2}-\frac{\alpha_2m_2^3}{6T^2}-\frac{\theta_2m_2T}{3}-\frac{\theta_2m_2^4}{12T^2}-\frac{rT}{2}-\frac{r\alpha_2}{2}T^2\\
&-\frac{r\theta_2}{6}T^3+\frac{mr}{2}+\frac{m_2r\alpha_2}{2}T+\frac{m_2r\theta_2}{4}T^2+\frac{r\theta_2m_2^3}{12}]\\
&+\frac{I_dP\lambda_2}{2T^2}(m_2^2(m_2\alpha_2+1))]\}+\{\frac{1}{4}[\frac{-a_3}{T^2}+C_3\lambda_3[\frac{\alpha_3}{2}\\
&+\frac{\theta_3}{3}T-\frac{r}{2}-\frac{r\alpha_3}{2}T-\frac{r\theta_3}{4}T^2]+h_3C_3\lambda_3[\frac{1}{2}+\frac{2\alpha_3}{3}T\\
&+\frac{\theta_3}{4}T^2-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\theta_3}{6}T^3]+C_3\lambda_3I_c[\frac{1}{2}+\frac{2\alpha_3}{3}T\\
&+\frac{\theta_3}{4}T^2-\frac{m_3^2}{2T^2}-\frac{\alpha_3m_3}{2}-\frac{\alpha_3m_3^3}{6T^2}-\frac{\theta_3m_3T}{3}-\frac{\theta_3m_3^4}{12T^2}\\
&-\frac{rT}{2}-\frac{r\alpha_3}{2}T^2-\frac{r\theta_3}{6}T^3+\frac{mr}{2}+\frac{m_3r\alpha_3}{2}T+\frac{m_3r\theta_3}{4}T^2\\
&+\frac{r\theta_3m_3^3}{12}]+\frac{I_dP\lambda_3}{2T^2}(m_3^2(m_3\alpha_3+1))]\}\}\\
&\times(H(1+\frac{rH}{2}))=0 \qquad (31)
\end{aligned}$$

Case (iv) $m\le T_d\le T$

$$\begin{aligned}
&\widetilde{Z_4(T)}=[\frac{A}{T}+C\lambda[1+\frac{\alpha T}{2}+\frac{\theta}{6}T^2+\frac{hT}{2}+\frac{h\alpha}{3}T^2\\
&+\frac{h\theta}{12}T^3-\frac{rT}{2}-\frac{r\alpha}{4}T^2-\frac{r\theta}{12}T^3-\frac{hrT^2}{4}-\frac{hr\alpha}{6}T^3\\
&-\frac{hr\theta}{24}T^4]+I_cC\lambda[\frac{T}{2}+\frac{\alpha}{3}T^2+\frac{\theta}{12}T^3-m+\frac{m^2}{2T}\\
&-\frac{\alpha mT}{2}+\frac{\alpha m^3}{6T}-\frac{m\theta}{6}T^2+\frac{\theta m^3}{6}+\frac{\theta m^4}{12T}-\frac{r}{4}T^2-\frac{r\alpha}{6}T^3\\
&-\frac{r\theta}{24}T^4+\frac{rmT}{2}-\frac{rm^2}{4}+\frac{rm\alpha T^2}{4}-\frac{r\alpha m^3}{12}+\frac{rm\theta}{12}T^3\\
&-\frac{r\theta m^3}{12}T-\frac{r\theta m^4}{24}]-\frac{Ar}{2}-\frac{I_dP\lambda}{2T}(m^2(m\alpha+1))\\
&+\frac{I_dP\lambda r}{2\alpha}(m^2(m\alpha+1))](H(1+\frac{rH}{2})) \qquad (32)
\end{aligned}$$

Now defuzzifing the total cost $\widetilde{Z_4(T)}$ by using Signed distance method, we have

$$\begin{aligned}
&Z_{4(\widetilde{sd})}(T)=\frac{1}{4}\{Z_{4(\widetilde{sd_1})}(T)+2Z_{4(\widetilde{sd_2})}(T)+Z_{4(\widetilde{sd_3})}(T)\}\\
&=\{\frac{1}{4}[\frac{a_1}{T}+C_1\lambda_1[1+\frac{\alpha_1T}{2}+\frac{\theta_1}{6}T^2+\frac{h_1T}{2}+\frac{h_1\alpha_1}{3}T^2\\
&+\frac{h_1}{12}T^3-\frac{rT}{2}-\frac{r\alpha_1}{4}T^2-\frac{r\theta_1}{12}T^3-\frac{h_1rT^2}{4}-\frac{h_1r\alpha_1}{6}T^3\\
&-\frac{h_1r\theta_1}{24}T^4]+I_cC_1\lambda_1[\frac{T}{2}+\frac{\alpha_1}{3}T^2+\frac{\theta_1}{12}T^3-m_1+\frac{m_1^2}{2T}\\
&-\frac{\alpha_1m_1T}{2}+\frac{\alpha_1m_1^3}{6T}-\frac{\theta_1}{6}T^2+\frac{\theta_1m_1^3}{6}+\frac{\theta_1m_1^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_1}{6}T^3-\frac{r\theta_1}{24}T^4+\frac{rm_1T}{2}-\frac{rm_1^2}{4}+\frac{rm_1}{4}-\frac{r\alpha_1m_1^3}{12}\\
&+\frac{rm_1\theta_1}{12}T^3-\frac{r\theta_1m_1^3}{12}T-\frac{r\theta_1m_1^4}{24}]-\frac{\alpha_1r}{2}\\
&-\frac{I_dP\lambda_1}{2T}(m_1^2(m_1\alpha_1+1))+\frac{I_dP\lambda_1r}{2\alpha_1}(m_1^2(m_1+1))]\}\\
&+\{\frac{1}{2}[\frac{a_2}{T}+C_2\lambda_2[1+\frac{\alpha_2T}{2}+\frac{\theta_2}{6}T^2+\frac{h_2T}{2}+\frac{h_2\alpha_2}{3}T^2\\
&+\frac{h_2}{12}T^3-\frac{rT}{2}-\frac{r\alpha_2}{4}T^2-\frac{r\theta_2}{12}T^3-\frac{h_2rT^2}{4}-\frac{h_2r\alpha_2}{6}T^3\\
&-\frac{h_2r\theta_2}{24}T^4]+I_cC_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2+\frac{\theta_2}{12}T^3-m_2+\frac{m_2^2}{2T}\\
&-\frac{\alpha_2m_2T}{2}+\frac{\alpha_2m_2^3}{6T}-\frac{\theta_2}{6}T^2+\frac{\theta_2m_2^3}{6}+\frac{\theta_2m_2^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_2}{6}T^3-\frac{r\theta_2}{24}T^4+\frac{rm_2T}{2}-\frac{rm_2^2}{4}+\frac{rm_2}{4}-\frac{r\alpha_2m_2^3}{12}\\
&+\frac{rm_2\theta_2}{12}T^3-\frac{r\theta_2m_2^3}{12}T-\frac{r\theta_2m_2^4}{24}]-\frac{\alpha_2r}{2}
\end{aligned}$$

$$
\begin{aligned}
&- \frac{I_dP\lambda_2}{2T}(m_2{}^2(m_2\alpha_2+1)) + \frac{I_dP\lambda_2 r}{2\alpha_2}(m_2{}^2(m_2+1))]\}\\
&+\{\frac{1}{4}[\frac{a_3}{T} + C_3\lambda_3[1+\frac{\alpha_3 T}{2}+\frac{\theta_3}{6}T^2+\frac{h_3T}{2}+\frac{h_3\alpha_3}{3}T^2\\
&+\frac{h_3}{12}T^3 - \frac{rT}{2} - \frac{r\alpha_3}{4}T^2 - \frac{r\theta_3}{12}T^3 - \frac{h_3rT^2}{4} - \frac{h_3r\alpha_3}{6}T^3\\
&-\frac{h_3r\theta_3}{24}T^4] + I_cC_3\lambda_3[\frac{T}{2}+\frac{\alpha_3}{3}T^2+\frac{\theta_3}{12}T^3 - m_3 + \frac{m_3{}^2}{2T}\\
&-\frac{\alpha_3m_3T}{2}+\frac{\alpha_3m_3{}^3}{6T} - \frac{\theta_3}{6}T^2+\frac{\theta_3m_3{}^3}{6}+\frac{\theta_3m_3{}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_3}{6}T^3-\frac{r\theta_3}{24}T^4+\frac{rm_3T}{2}-\frac{rm_3{}^2}{4}+\frac{rm_3}{4}-\frac{r\alpha_3m_3{}^3}{12}\\
&+\frac{rm_3\theta_3}{12}T^3-\frac{r\theta_3m_3{}^3}{12}T-\frac{r\theta_3m_3{}^4}{24}]-\frac{\alpha_3r}{2}\\
&-\frac{I_dP\lambda_3}{2T}(m_3{}^2(m_3\alpha_3+1))\\
&+\frac{I_dP\lambda_3r}{2\alpha_3}(m_3{}^2(m_3+1))]\}(H(1+\frac{rH}{2}))
\end{aligned} \tag{33}
$$

The necessary condition for minimizing the total cost is $\frac{\partial \widetilde{Z_{4(sd)}(T)}}{\partial T} = 0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 \widetilde{Z_{4(sd)}(T)}}{\partial T^2} > 0$

Now defuzzifing the total cost $\widetilde{Z_2(T)}$ by using Graded mean integration method ,we have

$$
\begin{aligned}
&\widetilde{Z_{4(gm)}}(T) = \frac{1}{6}\{\widetilde{Z_{4(gm_1)}}(T) + 4\widetilde{Z_{4(gm_2)}}(T) + \widetilde{Z_{4(gm_3)}}(T)\}\\
&=\{\frac{1}{6}[\frac{a_1}{T} + C_1\lambda_1[1+\frac{\alpha_1 T}{2}+\frac{\theta_1}{6}T^2+\frac{h_1T}{2}+\frac{h_1\alpha_1}{3}T^2\\
&+\frac{h_1}{12}T^3 - \frac{rT}{2} - \frac{r\alpha_1}{4}T^2 - \frac{r\theta_1}{12}T^3 - \frac{h_1rT^2}{4} - \frac{h_1r\alpha_1}{6}T^3\\
&-\frac{h_1r\theta_1}{24}T^4] + I_cC_1\lambda_1[\frac{T}{2}+\frac{\alpha_1}{3}T^2+\frac{\theta_1}{12}T^3 - m_1 + \frac{m_1{}^2}{2T}\\
&-\frac{\alpha_1m_1T}{2}+\frac{\alpha_1m_1{}^3}{6T} - \frac{\theta_1}{6}T^2+\frac{\theta_1m_1{}^3}{6}+\frac{\theta_1m_1{}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_1}{6}T^3-\frac{r\theta_1}{24}T^4+\frac{rm_1T}{2}-\frac{rm_1{}^2}{4}+\frac{rm_1}{4}-\frac{r\alpha_1m_1{}^3}{12}\\
&+\frac{rm_1\theta_1}{12}T^3-\frac{r\theta_1m_1{}^3}{12}T-\frac{r\theta_1m_1{}^4}{24}]-\frac{\alpha_1r}{2}\\
&-\frac{I_dP\lambda_1}{2T}(m_1{}^2(m_1\alpha_1+1)) + \frac{I_dP\lambda_1r}{2\alpha_1}(m_1{}^2(m_1+1))]\}\\
&+\{\frac{2}{3}[\frac{a_2}{T} + C_2\lambda_2[1+\frac{\alpha_2 T}{2}+\frac{\theta_2}{6}T^2+\frac{h_2T}{2}+\frac{h_2a_2}{3}T^2\\
&+\frac{h_2}{12}T^3 - \frac{rT}{2} - \frac{r\alpha_2}{4}T^2 - \frac{r\theta_2}{12}T^3 - \frac{h_2rT^2}{4} - \frac{h_2r\alpha_2}{6}T^3\\
&-\frac{h_2r\theta_2}{24}T^4] + I_cC_2\lambda_2[\frac{T}{2}+\frac{\alpha_2}{3}T^2+\frac{\theta_2}{12}T^3 - m_2 + \frac{m_2{}^2}{2T}\\
&-\frac{\alpha_2m_2T}{2}+\frac{\alpha_2m_2{}^3}{6T} - \frac{\theta_2}{6}T^2+\frac{\theta_2m_2{}^3}{6}+\frac{\theta_2m_2{}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_2}{6}T^3-\frac{r2}{24}T^4+\frac{rm_2T}{2}-\frac{rm_2{}^2}{4}+\frac{rm_2}{4}-\frac{r\alpha_2m_2{}^3}{12}\\
&+\frac{rm_2\theta_2}{12}T^3-\frac{r\theta_2m_2{}^3}{12}T-\frac{r\theta_2m_2{}^4}{24}]-\frac{\alpha_2r}{2}\\
&-\frac{I_dP\lambda_2}{2T}(m_2{}^2(m_2\alpha_2+1)) + \frac{I_dP\lambda_2r}{2\alpha_2}(m_2{}^2(m_2+1))]\}\\
&+\{\frac{1}{6}[\frac{a_3}{T} + C_3\lambda_3[1+\frac{\alpha_3 T}{2}+\frac{\theta_3}{6}T^2+\frac{h_3T}{2}+\frac{h_3a_3}{3}T^2\\
&+\frac{h_3}{12}T^3 - \frac{rT}{2} - \frac{r\alpha_3}{4}T^2 - \frac{r\theta_3}{12}T^3 - \frac{h_3rT^2}{4} - \frac{h_3r\alpha_3}{6}T^3\\
&-\frac{h_3r\theta_3}{24}T^4] + I_cC_3\lambda_3[\frac{T}{2}+\frac{\alpha_3}{3}T^2+\frac{\theta_3}{12}T^3 - m_3 + \frac{m_3{}^2}{2T}\\
&-\frac{\alpha_3m_3T}{2}+\frac{\alpha_3m_3{}^3}{6T} - \frac{\theta_3}{6}T^2+\frac{\theta_3m_3{}^3}{6}+\frac{\theta_3m_3{}^4}{12T}-\frac{r}{4}T^2\\
&-\frac{r\alpha_3}{6}T^3-\frac{r3}{24}T^4+\frac{rm_3T}{2}-\frac{rm_3{}^2}{4}+\frac{rm_3}{4}-\frac{r\alpha_3m_3{}^3}{12}\\
&+\frac{rm_3\theta_3}{12}T^3-\frac{r\theta_3m_3{}^3}{12}T-\frac{r\theta_3m_3{}^4}{24}]-\frac{\alpha_3r}{2}\\
&-\frac{I_dP\lambda_3}{2T}(m_3{}^2(m_3\alpha_3+1))\\
&+\frac{I_dP\lambda_3r}{2\alpha_3}(m_3{}^2(m_3+1))]\}(H(1+\frac{rH}{2})) = 0
\end{aligned} \tag{34}
$$

The necessary condition for minimizing the total cost is $\frac{\partial \widetilde{Z_{4(gm)}(T)}}{\partial T} = 0$ provided that they satisfy the sufficient conditions $\frac{\partial^2 \widetilde{Z_{4(gm)}(T)}}{\partial T^2} > 0$
case (iv) is same as case (iii)

5 Numerical Examples

Consider an inventory system with the parametric values in proper units

Example : Case(i) $0 < T < T_d$

Suppose $\tilde{A} = (100, 200, 300)$, $\tilde{C} = (15, 25, 35)$, $\tilde{h} = (2, 5, 8)$, $\tilde{\alpha} = (0.25, 0.5, 0.75)$, $\tilde{\theta} = (0.1, 0.2, 0.3)$,$\tilde{\lambda} = (100, 200, 300)$, $I_c = 0.10$, $I_d = 0.05$, $r = 0.05$, $H = 1$, by signed distance method and graded mean integration we get

$T = 0.0902$, $\widetilde{Z_{1(sd)}}(T) = 9715.1$ and $T = 0.0870$, $\widetilde{Z_{1(gm)}}(T) = 9403.4$

Case(ii) $T_d \leq m < T$

Suppose $\tilde{A} = (80, 160, 240)$, $\tilde{C} = (10, 20, 30)$, $\tilde{h} = (2, 5, 8)$, $\tilde{\alpha} = (0.2, 0.4, 0.6)$, $\tilde{\theta} = (0.1, 0.2, 0.3)$, $\tilde{\lambda} = (50, 100, 150)$, $\tilde{m} = (5, 10, 15)$, $r = 0.05$, $n = 40$, $I_d = 0.01$, $H = 1$, by signed distance method and graded mean integration we get

$T = 0.1375$, $\widetilde{Z_{1(sd)}}(T) = 4143.2$ and $T = 0.1462$, $\widetilde{Z_{1(gm)}}(T) = 3951.1$

Case(iii) $T_d \leq m \leq T$

Suppose $\tilde{A} = (20, 40, 60)$, $\tilde{C} = (20, 30, 40)$, $\tilde{h} = (2, 5, 8)$, $\tilde{\alpha} = (0.2, 0.4, 0.6)$, $\tilde{\theta} = (0.1, 0.2, 0.3)$, $\tilde{\lambda} = (50, 75, 100)$, $\tilde{m} = (5, 10, 15)$, $r = 0.05$, $n = 40$, $I_c = 0.01$, $I_d = 0.01$, $H = 1$, by signed distance method and graded mean integration we get

$T = 0.6803$, $\widetilde{Z_{1(sd)}}(T) =$ and $T = 0.5674$, $\widetilde{Z_{1(gm)}}(T) = 48775$

Case (iv) $m \leq T_d \leq T$

Case (iv) is same as case(iii).

6 Sensitivity Analysis

Case(i) $0 < T < T_d$

Table 1. Effects of different parameters using signed distance method

Parameter		T	TC_{GM}(T)
A	(150,250,350)	0.1005	10205
	(200,300,400)	0.1097	10650
	(250,350,450)	0.1181	11061
	(300,400,500)	0.1258	11445
θ	(0.2, 0.3 ,0.4)	0.0902	9716.1
	(0.4, 0.5 ,0.6)	0.0901	9718.5
	(0.6, 0.7 ,0.8)	0.0900	9720.9
	(0.8, 0.9 ,1.0)	0.0899	9723.3
C	(20, 30 ,40)	0.0837	11063
	(25, 35 ,45)	0.0784	12389
	(30, 40 ,50)	0.0740	13697
	(35, 45 ,55)	0.0702	14990
h	(3, 6 ,9)	0.0844	9996.7
	(4,7,10)	0.0796	10261
	(5,8,11)	0.0755	10511
	(6,9,12)	0.0721	10748
α	(0.3,0.55,0.80)	0.0897	9734.8
	(0.35,0.6,0.85)	0.0892	9754.4
	(0.40,0.65,0.9)	0.0887	9773.9
	(0.45,0.7,0.95)	0.0882	9793.3
λ	(125,225,325)	0.0942	10536
	(150,250,350)	0.0902	11366
	(175,275,375)	0.0868	12189
	(200,300,400)	0.0837	13005

Table 2. Effects of different parameters using graded mean integration method

Parameter		T	TC_{GM}(T)
A	(150,250,350)	0.0969	9876.8
	(200,300,400)	0.1058	10306
	(250,350,450)	0.1139	10703
	(300,400,500)	0.1214	11073
θ	(0.2, 0.3 ,0.4)	0.0869	9405.1
	(0.4, 0.5 ,0.6)	0.0868	9407.6
	(0.6, 0.7 ,0.8)	0.0867	9410.2
	(0.8, 0.9 ,1.0)	0.0866	9412.7
C	(20, 30 ,40)	0.0803	10758
	(25, 35 ,45)	0.0750	12087
	(30, 40 ,50)	0.0706	13398
	(35, 45 ,55)	0.0669	14692
h	(3, 6 ,9)	0.0810	9694.0
	(4,7,10)	0.0761	9964.9
	(5,8,11)	0.0720	10220
	(6,9,12)	0.0685	10462
α	(0.3,0.55,0.80)	0.0864	9425.1
	(0.35,0.6,0.85)	0.0859	9445.1
	(0.40,0.65,0.9)	0.0855	9464.2
	(0.45,0.7,0.95)	0.0850	9484.1
λ	(125,225,325)	0.0984	10182
	(150,250,350)	0.0941	11014
	(175,275,375)	0.0903	11838
	(200,300,400)	0.0870	12654

Case(ii) $T_d \leq m < T$

Table 3. Effects of different parameters using signed distance method

Parameter		T	TC_{GM}(T)
A	(90,170,250)	0.1415	4215.3
	(100,180,260)	0.1454	4285.5
	(110,190,270)	0.1492	4353.8
	(120,200,280)	0.1529	4420.4
θ	(0.2, 0.3 ,0.4)	0.1374	4144.2
	(0.4, 0.5 ,0.6)	0.1371	4146.4
	(0.6, 0.7 ,0.8)	0.1368	4148.5
	(0.8, 0.9 ,1.0)	0.1366	4150.6
C	(15, 25 ,35)	0.1260	4874.2
	(20, 30 ,40)	0.1171	5587.4
	(25, 35 ,45)	0.1098	6286.6
	(30, 40 ,50)	0.1037	6974.4
h	(3, 6 ,9)	0.1094	5784.3
	(4,7,10)	0.1031	5968.6
	(5,8,11)	0.0978	6142.4
	(6,9,12)	0.0932	6307.3
α	(0.25,0.45,0.65)	0.1163	6405.2
	(0.30,0.50,0.70)	0.1155	6415.5
	(0.35,0.55,0.75)	0.1147	6425.6
	(0.40,0.60,0.80)	0.1140	6436.3
λ	(60,110,160)	0.1127	6809.2
	(70,120,170)	0.1088	7218.2
	(80 ,130,180)	0.1053	7622.2
	(90,140,190)	0.1021	8021.4
m	(6,11,16)	0.1170	6352.0
	(7,12,17)	0.1170	6310.1
	(8,13,18)	0.1170	6268.1
	(9,14,19)	0.1169	6225.5

Table 4. Effects of different parameters using graded mean integration method

Parameter		T	TC_{GM}(T)
A	(90,170,250)	0.1505	4025.5
	(100,180,260)	0.1546	4092.0
	(110,190,270)	0.1586	4156.8
	(120,200,280)	0.1626	4220.0
θ	(0.2, 0.3 ,0.4)	0.1460	3958.2
	(0.4, 0.5 ,0.6)	0.1457	3960.5
	(0.6, 0.7 ,0.8)	0.1454	3962.8
	(0.8, 0.9 ,1.0)	0.1451	3965.0
C	(15, 25 ,35)	0.1331	4689.2
	(20, 30 ,40)	0.1231	5402.7
	(25, 35 ,45)	0.1151	6105.1
	(30, 40 ,50)	0.1084	6788.4
h	(3, 6 ,9)	0.1362	4121.0
	(4,7,10)	0.1281	4274.0
	(5,8,11)	0.1212	4418.1
	(6,9,12)	0.1154	4554.6
α	(0.25,0.45,0.65)	0.1452	4643.2
	(0.30,0.50,0.70)	0.1443	4653.0
	(0.35,0.55,0.75)	0.1433	4662.1
	(0.40,0.60,0.80)	0.1424	4671.5
λ	(60,110,160)	0.1406	4978.9

	(70,120,170)	0.1356	5262.0
	(80 ,130,180)	0.1310	5539.9
	(90,140,190)	0.1270	5815.0
m	(6,11,16)	0.1462	5920.7
	(7,12,17)	0.1463	5861.9
	(8,13,18)	0.1464	5803.1
	(9,14,19)	0.1465	5744.3

Case(iii) $T_d \leq m \leq T$

Table 5. Effects of different parameters using signed distance method

Parameter		T	TC_{GM}(T)
A	(30,50,70)	0.6811	19579
	(40,60,80)	0.6816	19593
	(50,70,90)	0.6822	19622
	(60,80,100)	0.6827	19636
θ	(0.12, 0.22 ,0.32)	0.7321	21183
	(0.14, 0.24 ,0.34)	0.7781	22695
	(0.16, 0.26,0.36)	0.8198	24133
	(0.18, 0.28,0.38)	0.8578	25512
C	(30, 40 ,50)	0.8201	29968
	(40, 50 ,60)	0.8926	39864
	(50, 60 ,70)	0.9379	49577
	(60, 70 ,80)	0.9689	59198
h	(3, 6 ,9)	0.6403	20568
	(4,7,10)	0.6067	21493
	(5,8,11)	0.5783	22363
	(6,9,12)	0.5537	23188
α	(0.25,0.45,0.65)	0.6236	18370
	(0.30,0.50,0.70)	0.5625	17012
	(0.35,0.55,0.75)	0.4951	15457
	(0.40,0.60,0.80)	0.4166	13619
λ	(60,85,110)	0.6751	16426
	(70,95,120)	0.6707	17953
	(80 ,105,130)	0.6670	19478
	(90,115,140)	0.6638	21001
m	(6,11,16)	0.7925	18475
	(6.5,11.5,16.5)	0.8500	20435
	(7,12,17)	0.9086	22517
	(7.5,12.5,17.5)	0.9681	24722

Table 6. Effects of different parameters using graded mean integration method

Parameter		T	TC_{GM}(T)
A	(30,50,70)	0.5682	48781
	(40,60,80)	0.5690	48787
	(50,70,90)	0.5697	48796
	(60,80,100)	0.5705	48802
θ	(0.12, 0.22 ,0.32)	0.6251	52073
	(0.14, 0.24 ,0.34)	0.6755	55199
	(0.16, 0.26 ,0.36)	0.7203	58248
	(0.18, 0.28 ,38)	0.7608	61219
C	(30, 40 ,50)	0.7309	75523
	(40, 50 ,60)	0.8104	97541
	(50, 60 ,70)	0.8587	119440
	(60, 70 ,80)	0.8913	141260
h	(3, 6 ,9)	0.5311	182090
	(3.5, 6.5 ,9.5)	0.5155	186910
	(4,7,10)	0.5012	191590
	(4.5,7.5,10.5)	0.4881	196130
α	(0.25,0.45,0.65)	0.4977	185170
	(0.30,0.50,0.70)	0.4172	207490
	(0.35,0.55,0.75)	0.3147	255830
	(0.40,0.60,0.80)	0.1341	542940
λ	(60,85,110)	0.5606	185090
	(70,95,120)	0.5550	203740
	(80 ,105,130)	0.5503	222410
	(90,115,140)	0.5463	241090
m	(6,11,16)	0.6767	265700
	(7 ,12,17)	0.7910	299520
	(8,13,18)	0.9098	375450
	(8.5,13.5,18.5)	0.9707	357970

All of the preceding points can be summarised as follows:

According to Table 1, increasing the ordering cost 'A' leads to an increase in the optimal cycle time T and total relevant cost, thus increasing the $\tilde{C}$, $\tilde{h}$, $\tilde{\alpha}$, $\tilde{\theta}$, $\tilde{\lambda}$ leads to a decrease in the optimal cycle time T and an increase in the total relevant cost.

Table 2 shows that increasing the ordering cost 'A' leads to an increase in the optimal cycle time T and total relevant cost, while increasing the $\tilde{C}$, $\tilde{h}$, $\tilde{\alpha}$, $\tilde{\theta}$, $\tilde{\lambda}$ leads to a decrease in the optimal cycle time T and an increase in the total relevant cost.

Table 3 shows that increasing the ordering cost 'A' leads to an increase in the optimal cycle time T and total relevant cost, thus increasing $\tilde{C}$, $\tilde{h}$, $\tilde{\alpha}$, $\tilde{\theta}$, $\tilde{\lambda}$ leads to a decrease in the optimal cycle time T and total relevant cost.

Table 4 shows that increasing the ordering cost 'A' leads to an increase in the optimal cycle time T and overall related cost, as well as an increase in $\tilde{C}$, $\tilde{h}$, $\tilde{\alpha}$, $\tilde{\theta}$, $\tilde{\lambda}$ increase in $\tilde{m}$ and decrease in optimal cycle time T and overall related cost decrease in overall relevant cost and increase in optimal cycle time T.

According to Table 5, raising the ordering cost 'A', $\tilde{C}$, $\tilde{m}$, $\tilde{\theta}$ results in an increase in the optimal cycle time T and total related cost, as well as an increase in $\tilde{\alpha}$ decrease in optimal cycle time T and overall related cost and increase in $\tilde{h}$, $\tilde{\lambda}$ decrease in optimal cycle time T and increase the total relevant cost.

According to Table 6, increasing the ordering cost 'A', $\tilde{C}$, $\tilde{m}$, $\tilde{\theta}$ results in an increase in the optimal cycle time T and total relevant cost, thus increasing the in $\tilde{\alpha}$, $\tilde{h}$, $\tilde{\lambda}$ results in a decrease in the optimal cycle time T and an increase in the total relevant cost.

7 Conclusion

When the supplier enables a payment delay connected to order quantity ,in this study we use an fuzzy inventory model for decreasing products to determine the best ordering strategy under inflation . The theoretical repercussion in each case are explained through numerical examples.The variance of different parameters is revealed through sensitivity analysis. A greater buying cost leads to a shorter cycle time and a higher total relevant cost, whereas higher holding cost leads to a shorter cycle time and a higher total relevant cost is obtained. Finally, a higher decay rate leads to a shorter optimal cycle time and a higher total relevant cost. The suggested work can be used to predict demand as a function of stock level time, linear and quadratic demand , selling price, and other factors.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest

---

REFERENCES

[1] D.Dutta, Pavan Kumar, Fuzzy inventory model without shortages using trapezoidal fuzzy number with sensitivity analysis, IOSR Journal of Mathematics, Vol.4, 32- 37, 2015.

[2] Dutta, Pavan Kumar, Optimal policy for an inventory model without shortages consider fuzziness in demand, holding cost and ordering cost, International Journal of Advanced Innovation and Research, Vol.2, No.3, 320-325, 2013.

[3] Goyal,S.K, EOQ under conditions of permissible delay in payments,Journal of Operational Research Society, Vol.36, 335-338, 1985.

[4] Goyal,S.K, Giri,B.C, Recent trends in modelling of deteriorating inventory, European Journal of Operational Research, Vol.134, 1-16, 2001.

[5] Harish Nagar, Priyanka Surana, Fuzzy inventory model for deteriorating items with fluctuating demand and using inventory parameters as Pentagonal Fuzzy numbers, Journal of computer and Mathematical Science, Vol.6, No.2, 55-65, 2015.

[6] Huey-Ming Lee, Jing-Shing Yao, Economic order quantity in Fuzzy sense for Inventory without Back order Fuzzy sets,Journal of Information and Engineering, Vol.21, 673-694, 2005.

[7] S.K.Indirajit Sinha, P.N.Samantha, U.K.Mishra, Fuzzy Inventory Model with Shortages under Full Backlogged Using Signed Distance Method,International Journal for research in Applied Science and Engineering Technology, Vol.4, No.1, 197-203, 2016.

[8] Jaggi C.K, Pareek S, Khanna A, Nidhi, Optimal Replenishment policy for fuzzy Inventory model with Deteriorating items and under allowable shortages under Inflationary conditions, Yugoslav Journal of Operations Research, Vol.10, 2298/YJOR150202002Y, 2016.

[9] Jershan Chiang-Shing S Yao, huey M Lee, Economic reorder point for fuzzy backorder quantity, European Journal of Operation Research, Vol.109, 183-202, 1998.

[10] Khanna,S, Ghosh, S.K, and Chaudhuri, K.S, An EOQ model for a deteriorating item with time-dependent quadratic demand under permissible delay in payment. Applied Mathematics and Computation, Vol.218, 1-9, 2011.

[11] Kumar,S, Rajput ,U.S, Fuzzy Inventory model for Deteriorating items with Time dependent demand and partial backlogging, Applied Mathematics, Vol.6, 496- 509, 2015.

[12] G.Michael Rosario, R.M.Rajalakshmi, A Fuzzy Inventory model with allowable shortage using different fuzzy numbers, International Journal of computational and Applied Mathematics, Vol.12, No.1, 227-240, 2017.

[13] Sana, S.S, Chaudhuri, K.S, A deterministic EOQ model with delays in payments and price discount offers. European Journal of Operations Research, Vol.184, 509-533, 2008.

[14] Shinn, S.W., Hwang, H.P., Sung, S.,Joint price and lot size determination under condition of permissible delay in payments and quantity discounts for freight cost, European Journal of Operational Research, Vol.91, 528-542, 1996.

[15] Taheri-Tolgari, J, Mirzazadeh, A, Jolai, F., An inventory model for imperfect items under inflationary conditions with considering inspection errors, Computers and Mathematics with Applications, Vol.63, 1007-1019, 2012.

[16] Teng, J.T. Yang, H.L, Chern, M.S, An inventory model for increasing demand under two levels of trade credit linked to order quantity. Applied Mathematical Modelling, Vol.37, 2624-2632, 2013.

[17] Tripathi, R.P, Inventory model with time-dependent demand rate under inflation when supplier credit linked to order quantity. International Journal of Business and Information Technology, Vol.1, No.3, 174-183, 2011.

[18] Tripathi, R.P., Dinesh Singh, Mishra.T, An EOQ model for deteriorating Items with Exponential Time- Dependent Demand Rate under Inflation when supplier credit is linked to order quantity. International Journal of Supply and Operations Management, Vol.1, No.1, 20-37, 2014.

[19] Tripathi, R.P, Pandey, H.S, An EOQ model for deteriorating Items with Weibull Time- Dependent Demand Rate under Trade Credits. International Journal of Information and Management Sciences, Vol.24, No.4, 329-347, 2013.

[20] Ouyang, L.Y, Chang, C.T, Optimal production lot with imperfect production process under permissible delay in payment and complete backlogging. International Journal of Production Economics, Vol.144, 610-617, 2013.