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Some Properties of BP-Space

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Abstract Y. Imai, K. Iseki [4], and K. Iseki [5] presented types from summary algebras which are called BCK-algebras and BCI-algebras. It is known that the brand of BCK algebras is a suitable subtype from the type from BCI-algebras. The researchers Q. P. Hu [2] & X. Li [3] presented a width type from essence algebras: BCH-algebras. They have exhibited that the type of BCI-algebras is a suitable subtype of the type of BCH-algebras. Moreover, J. Neggers and H. S. K [9] presented the connotation from d - algebras that are else popularization from BCK-algebras, inspected kinsmen amidst d-algebras & BCK-algebras. They calculated diversified topologies to research from lattices but they did not discuss the experience of making the binary operation of d- algebra continuous. Topological set notions are famous and yet accurate by numerous mathematicians. Even global topographical algebraic structure is sought by several writers. We realize a Tb-algebra, get it several ownerships of such build, the generality significant flavors and arrive to realize a new gender of spaces designated BP- space, where we arrived the results. Let X be *a* B-space and $u \in X$ is periodic proportional. Then $\varphi(B, u)$ is a compact set in X and $\varphi(B, u) = \varphi(B, v)$, $\forall\ v \in \varphi(B, u)$. Also If $\Omega = \{G \subseteq X / G$ is an invariant under $B\}$, then $\bigcup_{G\in\Omega} G$, G^C and $G\text{-}Q$ are invariant under B for every Q in Ω if φ is also. If the function φ is closed (one to one) then $\bar{G}$,$(\bigcap_{G\in\Omega} G)$ is invariant under B and the set of interior points of G is invariant under B, if the function φ is open and $e \in B$.

Keywords Tb-Algebra, Periodic Point, B-Space, BP-Space

1. Introduction

We have begun the lesson of topological b-algebra in this paper, so we want some preliminary indispensable to amplify the paper and contain some of the outcrops of the basic knowledge of algebra b which is a prerequisite for lessoning this topic. We will learn on topological algebra and explore several universal facts of topological algebra all of this in Section two. In Section 3, we discuss the topological transformation (b-space) and its more serious characteristics. In Section 4, we present a definition of a novel sort from space termed BP-space and several characteristics of BP-space.

2. Topological b-Algebra

We tutored qualifier from Tb-algebra and examples concerning of object in this portion.

Definition 2.1: A non-empty set B with binary process $*$, $0 \in B$ to be b-algebra if next postulates comply with $\forall\ u, w, z \in B$ where 0 is called zero element:

1) $u * u = 0$
2) $u * 0 = u$
3) $(u * w) * z = u * (z * (0 * w))$.

Definition 2.2: Element e from B is named right-identity if $u*e = u$(left identity if $e*u=u$), for every $u \in B$ and $u \neq e$. If e is right(left) identity at that time e is namely identity. So $(B,*)$ is b-algebra containing identity.

Example 2.3: Put $B = \{0, ɑ, в, ɕ\}$ and the following table of $*$:

∗	0	ɑ	ϐ	ɕ
0	0	ɑ	ϐ	ɕ
ɑ	ɑ	0	ɕ	ϐ
ϐ	ϐ	ɕ	0	ɑ
ɕ	ɕ	ϐ	ɑ	0

Table above shows that the set B with binary operation* is B-Algebra.

From table, *B* is b-algebra and it contains identity element.

Definition 2.4. [3] A non-empty subset *S* of a b-algebra *B* is called a sub b-algebra of *B* if it is closed under the b-operation.

A non-empty subset *I* of a b-algebra *B* is called an ideal of *B* if

(B1) o∈ I

(B2) b ∈ I and a ∗ b ∈ I reveal a ∈ I. for all a, b∈ B.

Definition 2.5. [10] Let ϕ ≠ I⊆ B of a b-algebra $(B;*,0)$. We say that I is b-ideal if it holds (B1) and:

(B3) $(a*b)\in I$ and $(d*a)\in I$, Imply $(b*d)\in I$, for arbitrary a, b, d ∈B,

Definition 2.6. A b-ideal *I* of *B* is called a strongly b-ideal (sb - ideal) of *B* if, for arbitrary *a,b, u ∈B,*

(B4) a ∗ b ∈ I and b ∗ a ∈I imply (a ∗ u) ∗ (b ∗ u) ∈ I and (u∗ a) ∗ (u ∗ b) ∈ I.

Definition 2.7. If (τ, *B*) is topological space and (*B*,∗) is b-algebra. We called that (*B*,∗,τ) is a topological b-algebra (indicated that by Tb-algebra) if binary procedure ∗ is continuous. The continuity of the operation "∗" is equivalent to the following property:

(C): If *O* is an open set and *a, b ∈ B* ∋ *a ∗b ∈ O*, then ∃O_1, O_2 ∈τ such that *a ∈* O_1, *b ∈* O_2 and $O_1 * O_2 \subseteq O$.

Example 2.8: Let *B*={ *0,u,w,z*} and the following table of ∗ :

∗	0	u	w	z
0	0	u	w	z
u	u	0	z	w
w	w	z	0	u
z	z	w	u	0

Table above shows that the set B with binary operation∗ is B-Algebra.

Then (*B*,∗) is b-algebra and τ is discrete topology on *B*, then the triple (*B*,∗,τ) is a topological b-algebra.

ii) Let *R* be real numbers, the binary operation *, such that *a*∗*ϐ = a–ϐ*. Thus (*R*,∗,τ) is Tb-algebra where τ is usual topology.

Definition 2.7: Let B be a Tb-algebra, ϕ ≠ U ⊆ B and a ∈ B, then U_a= {x∈B/ xa∈U}[$_aU$={x∈B/ax∈U}]. So if K ⊆ B then $_KU=\bigcup_{a\in K}$ $_aU$ [U_K = $\bigcup_{a\in K}$ U_a].

Example 2.8: Let B={…, -3,-2,-1,0,1,2,3,…} , τ ={A/A ⊆ B} such that ɑ∗ϐ = ɑ-ϐ .So triple (B,∗,τ) is Tb-algebra and U = {x∈B / 0 ≤ x< 9} , K ={0,1,2} and a=2 then U_2={0,1,2,3,4,5,6}, $_2U$ = {0,1,2} and $_KU$ = {0,1,2}.

Proposition 2.9: If *B* is Tb-algebra then:

1) If A⊆ D then $_AK\subseteq {}_DK$.
2) If K⊆ W then $_AK\subseteq {}_AW$. Where A, D, W, K ⊆ B

Proof:

1) Let k∈ $_AK$ ⇒∃ a∈A, ∋ k∈$_aK$. Since A ⊆ D, so $_AK\subseteq {}_DK$.
2) Let k∈ $_AK$⇒∃ a∈A, ∋k∈$_aK$. So ak∈ K, since K⊆ W, then k∈$_aW$, so $_AK\subseteq {}_AW$

Proposition 2.10: Let B be Tb-algebra, ϕ ≠ O⊆ B, ϕ ≠ C ⊆ B, then:

i) Then $_aO$ and O_a are open sets ∀a∈B when O is an open set,
ii) Then $_aC$ and C_a are closed sets ∀ a∈B when C is a closed set

Proof:

i) Let a∈ B, U is an open set and x∈$_aO$. So ax ∈ O, since ∗ is eternal, consequently ∃ A, D of B Such that A, D are open sets and (a,x) ∈ A× D, ax∈ AD= ∗(A,D) ⊆ O, then aD ⊆ O. Thus x∈ D⊆ $_aO$ ⇒ $_aO$ is open in B. So the proof of O_a being open becomes clear.
ii) Let a∈B, C is a closed and x ∈$\overline{_aC}$, then ∃ $\{x_\alpha\}_{\alpha\in\Omega}$ in $_aC$ such that $x_\alpha\rightarrow$ x. Since B is Tb-algebra ⇒ $ax_\alpha\rightarrow$ ax and ax∈ C, ⇒ x∈$_aC$ and $_aC$ is a closed set and so the proof of C_a being closed becomes clear.

Corollary 2.11: Let B be Tb-algebra, ϕ ≠ A⊆ B and ϕ ≠ O ⊆ B, then:

i) If O is open, then sets $_AO$ and O_A are open.
ii) If O is closed and A={ a_1, a_2, … , a_n}. Then the sets $_AC$ and C_A are closed sets.

Proposition 2.12: Let B be T_2-compact Tb-algebra. Then $_aU[U_a]$ is a compact set, when U is compact in B for all a∈B.

Proof: It is clear

Proposition 2.13: Let J be sub b-algebra from Tb-algebra B. Consequently $\overline{J}$ is sub - b-algebra.

Proof: Put ɑ,ϐ∈ $\overline{J}$, consequently there dwell $\{a_\alpha\}_{\alpha\in\varphi}$,$\{b_\alpha\}_{\alpha\in\varphi}$ within J where $a_\alpha\rightarrow$ a & $b_\alpha\rightarrow$ b, since ∗ is persistent So a_α b_α →ab. Since $\overline{J}$ is closed, consequently ab∈ $\overline{J}$. Thus $\overline{J}$ is sub-b-algebra.

Proposition 2.14: The Tb-algebra B is discrete, if {o} is open in B.

Proof: Let a ∈B. Since a∗ a = 0, {0} is open and ∗ is continuity. Then dwell open sets N & M of X so that N ∗ M = {0}. Posit K = N ∩ M. Consequently K ∗ K = {0} and K = {a} with that, we get what we want.

Proposition 2.15: The Tb-algebra B is T_2 space if and only if {0} is closed.

Proof: ⇒) Let a,b $\in$ B such that a $\neq$ b. So a$*$b $\neq$ 0. Since B is a Tb-algebra. Thus $\exists$ N,M open sets of a and b respectively, such that

NM $\subseteq$ B / {0}

Thus N $\cap$ M = ϕ. Then B is T_2 Tb-algebra

⇐clear.

Proposition 2.16: Let *B* be a Tb – algebra. Then *J* is a closed set, if *J* is an open b-ideal in B.

Proof: Let $a \notin J$. Since $*$is a continuity of b-algebra. Then $\exists$ *N* open neighborhood so that $N*N\subseteq J$ (since $a*a = 0$). If for some a is contained in $N \cap J$, so $N \subseteq J$ (by definition of b-ideals). It is a contradiction. So $N \subseteq J^c$. Thus *J* is b-ideal.

3. B – Space

In that division, we study B-space, some expository examples, its issues and scores.

Definition 3.1: A topological transformation b-algebra is threefold (B,N,φ) so that *B* is a Tb-algebra, *N* is space and $\varphi : B\times N \to N$ is a persistent, ∍ $\varphi(b_1,\varphi(b_2, n)) = \varphi(b_1 * b_2, n)$ for all $b_1,b_2\in B$, $n\in N$, if $(B,*)$ is Tb – algebra containing identity *e*, Then (B,N,φ) is called topological transformation b-algebra together containing identity where $\varphi(e, n) = n$ $\forall n\in N$.

Example 3.2: If $(R,*,U)$ is Tb-algebra so that $r_1 * r_2 = r_1 - r_2$, $\forall$ $r_1,r_2 \in R$, (R, U) are usual space. Thus the triple (R,R,φ) is topological transformation b-algebra so that $\varphi(r_1,r_2) = r_2$, $\forall r_1,r_2\in R$.

Remark 3.3:

i) We say that the function φ is a b-action of *B* on *N*, (N, φ) is a *B*-space and pair (N, φ) contains identity, if $(B,*)$ is b-algebra containing identity.

ii) We will use *b.n* (*n.b)* instead $\varphi(b,n)$ ($\varphi(n,b)$) and $b_1.(b_2.n) = (b_1b_2).n$ for $\varphi(b_1,\varphi(b_2,n)) = \varphi(b_1b_2,n)$.

iii) If $H\subseteq B$ and $M \subseteq N$, we put $HM=\{h.m/ h\in H, m\in M\}$ for $\varphi(H,M)$.

iv) The function φ_b: $N\to N$ is continuous and defined by $\varphi_b(n) = \varphi(b, n) = b.n$ $\forall$ $b\in B$. Thus $\varphi_{b_1}\varphi_{b_2} = \varphi_{b_1b_2}$ and $\varphi_e = I_n$, where I_n identity function of *N*.

Proposition 3.4: If *N* is *B*-space, $M \subseteq N$, $H \subseteq B$ and $b\in B$ then:

i) $b\overline{M}\subseteq\overline{bM}$.

ii) $\overline{H\overline{M}} = \overline{\overline{H}M} = \overline{HM} = \overline{H}\overline{M}$.

iii) The set *HM* is compact in *N*, if *M*, *H* are compact in *N*, *B* respectively.

iv) If *G* is a neighborhood of *HM*. Then $\exists$ $M \subseteq V$, $H \subseteq W$ so that V, W are neighborhoods and $WV \subseteq G$.

Proof:

i) Since $b\overline{M} = \varphi(b, \overline{M}) = \varphi_b(\overline{M}) \subseteq \overline{\varphi_b(M)} = \overline{bM}$ and φ_b is continuous function.

ii) Since $\overline{H}\overline{M} = \varphi(\overline{H}\times\overline{M}) = \varphi(\overline{H\times M}) \subseteq \overline{\varphi(H\times M)} = \overline{HM} \Rightarrow HM\subseteq \overline{H}M \subseteq \overline{HM} \subseteq \overline{HM}$ (By containing φ). Then $\overline{\overline{H}M} \subseteq \overline{HM}$ and HM $\subseteq H\overline{M}\subseteq \overline{HM} \subseteq \overline{HM}$ then $\overline{H\overline{M}} \subseteq \overline{HM}$.

iii) Since $\varphi(H\times M) = HM$ and φ is continuous function. Thin *HM* is compact in *N*.

iv) By proposition (C).

Definition 3.5: Let (B,N,φ) be a topological transformation b-algebra and $n\in N$. Then we name that the set $B_n(\varphi)=\{b\in B/ \varphi(b, n) = n\}$ b-stabilizer of φ at *n* and the set $B(\varphi) = \bigcap_{n\in N}B_n(\varphi)$ is the stabilizer of φ.

Example 3.6: Let *Z* be the set of the integer number and $*$ be binary operation such that $a * b = a-b$ $\forall$ $a,b\in Z$. Then $(Z,*,\tau)$ is topological b-algebra where (Z, τ) is discrete topology. The triple (R,φ) is a B-space where (R,U) is usual topology and φ: $Z\times R\to R$ where $\varphi(z,r) = r$ $\forall z\in Z$, $r\in R$ then $Z_x(\varphi) =\{z\in Z / \varphi(z, x) = x\} = Z$ thus $Z(\varphi) = \bigcap_{r\in R} Z_r(\varphi) = Z$.

Definition 3.7: Let *N* be a B-space, φ is a minimal function if $\overline{\varphi(B,n)} =$ *N* for all $n\in N$.

Example 3.8: Let $B = \{0,1,2,3\}$ and $*$ is define by the table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Table above shows that the set B with binary operation$*$ is B-Algebra.

Thus $(B,*)$ is b-algebra and $(B,*,T)$ is Tb-algebra with the discrete topology *T* and (B,B,φ) is topological transformation b-algebra where (B, τ') is indiscrete topological space and $\varphi(a , b) = b$ so $\forall b\in B$, $\overline{\varphi(B,b)} = \overline{\{b\}} = B$. Thin φ is minimal function.

Definition 3.9: Let *N* be a B-space. We say that φ is faithful if $\forall$ $b_1, b_2\in B$ such that $b_1 \neq b_2$, $\exists$ $n\in N$ so that $\varphi(b_1,n) \neq \varphi(b_2,n)$.

Example 3.10: Let $B = \{0,a,b,c\}$ and $*$ be define by:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Table above shows that the set B with binary operation$*$ is B-Algebra.

So $(B,*,\tau)$ is Tb-algebra where τ is discrete topology and $\tau' = \{U_n / U_n = \{0, 1, 2, 3, m, m+1\}\cup \{\phi\}$ is a topology on $N^{\#}$. If $\varphi: B\times N^{\#}\to N^{\#}$ is defined by $\varphi(a, b) = a$, then the triple $(B, N^{\#},\varphi)$ is B-space and $\forall a , b\in B$ then there $n \in N^{\#}$ so that $a = \varphi(a, n) \neq \varphi(b,n) = b$. So φ is faithful.

Definition 3.11: Let B be a Tb-algebra, we say that the set I is right syndetic in B if there subset J from B so that it is a compact set in B, ${}_IJ=IJ=B$ and I is left syndetic in B if there subset J of B so that it is a compact set in B and $J_I=JI=B$.

Example 3.12: Let $A = \{a,b,c\}$, $(P(A), \cup, \tau)$ be Tb-algebra where τ discrete topology and $I=\{A, \{c\}, \{b\}, \{c\}\}$ and $J = \{\phi, \{a,b\},\{a,c\},\{b,c\}\}$ then ${}_JI= JI = P(A)$ and I are right syndetic in $P(A)$. If $I = \{\phi, A, \{a\}, \{a,b\}, \{a, c\}\}$, $H = \{\phi, \{a\}, \{b\}, \{c\}, \{b,c\}\}$ then $I_H=IH=P(A)$and $I$is left syndetic in $P(A)$.

Proposition 3.13: Let B be a Tb-algebra and $T \subseteq B$ then there $I, J \subseteq B$ such that I, J are compact and ${}_IT = B$ and $JT=B$ if and only if T is right syndetic in B.

Proof: $\Rightarrow$) If T is right syndetic then $\exists\ I \subseteq B$ is a compact set so that $B= {}_IT = IT$ by this completes the proof .

$\Leftarrow$) let I, J be a compact subset of B so that ${}_IT=B$ and $JT= B$.

Put $H= I \cup J$ then :

${}_HT = {}_{I\cup J}T = \bigcup_{h\in(I\cup J)} {}_hT = (\bigcup_{h\in I} {}_hT) \cup (\bigcup_{h\in J} {}_hT) = {}_IT \cup {}_JT$

$= B \cup {}_JT = B$ and $HT = (I \cup J)T = IT \cup JT = IT \cup B = B$

Then T is right syndetic in B

Proposition 3.14: If $N \subseteq B$ if N is a left (right) syndetic then $\overline{N}$ is left (right) syndetic.

Proof:- Since N is a left(right) syndetic from B, so $\exists I\subseteq B$ so that I is a compact in B such that $N_I = NI=B$ by Proposition (2.9) and $N\subseteq \overline{N}$ then $\overline{N}_I = \overline{N}I = B$ thus $\overline{N}$ is left(right) syndetic.

Definition 3.15: The point a in B-space X is named periodic proportional to φ if $B_x(\varphi)$ is right syndetic in B and φ is named periodic if $B(\varphi)$ is right syndetic.

Proposition 3.16: Let X be B-space then $\forall x \in X$ is periodic relative to φ if φ is periodic.

Proof: let φ be periodic. We get $B(\varphi)$ is right syndetic, by Definition (3,11). Then $\exists$ I $\subseteq$ B such that it is a compact and $IB(\varphi)= {}_IB(\varphi)=B$.

But $B(\varphi)= \cap_{x\in X} B_x(\varphi)$ and $\cap_{x\in X} B_x(\varphi) \subseteq B_x(\varphi)$. thus ${}_H(\cap_{n\in X} B_x(\varphi)) = H(\cap_{x\in X} B_X(\varphi)) = B$, then $I B_x(\varphi)= {}_IB_x(\varphi)=B$. Thus $\forall x\in X$ is periodic relative to φ.

Proposition 3.17: Let X be a B-space and $u\in X$ is periodic proportional. Then $\varphi(B,u)$ is a compact set in X and $\varphi(B, u) = \varphi(B, v)$, $\forall v\in\varphi(B, u)$.

Proof: let $u \in X$ and it is aperiodic. So $\exists I \subseteq B$ such that it is compact and $IB_x(\varphi)=B= {}_IB_x(\varphi)$. We will show that $\varphi(B, u) = \varphi(I, u)$. Let $v\in\varphi(I, u)$ then $\exists i\in H \ni v = \varphi(i,u)$. Since $I\subseteq B$, then $v = \varphi(i, u) \in \varphi(B, u)$. So $\varphi(I, u) \subseteq \varphi(B, u)$. Let $w\in\varphi(B, u)$. Then $\exists b_1\in B \ni w=\varphi(b_1,u)$. Since $IB_u(\varphi) = B$, then $\exists i_1\in I \ni b_1 = i_1b_2$ and $b_2\in B_u(\varphi)$, then $w=\varphi(b_1,u) =\varphi(i_1b_2,u) =\varphi(i_1,\varphi(b_2, u) =\varphi(i_1, u) \Rightarrow w\in\varphi(I, u)$, So $\varphi(B,u) \subseteq \varphi(I, u)$ thin $\varphi(B, u) =\varphi(I, u)$. Since $\varphi(I, u)$ is compact, thus $\varphi(B, u)$ is compact and let $v\in\varphi(B,u)$. We show that $\varphi(B,u) =\varphi(B,v)$. Let $w\in\varphi(B, u)$ then $\exists\ b_1, b_2\in B, \ni v = \varphi(b_1, u)$ and $w=\varphi(b_2, u)$. Since $B = {}_IB_u(\varphi)$ then $\exists i_3\in I$ and $b\in B$ so that $h_3 b\in B_u(\varphi)$[by defining ${}_IB_u(\varphi)$] then $\varphi(i_3 b, u) = u$. Then $w = \varphi(b_2,u)$, $\Rightarrow w= \varphi(b_2, \varphi(i_3 b,u)) =\varphi(b_2, \varphi(i_3, \varphi(b,u)) = \varphi(b_2i_3, \varphi(b,u)) =\varphi(b_3,\varphi(b,u))$ where $b_3= b_2i_3 \Rightarrow \varphi(b_3,v)\in\varphi(B,v)$. Then $\varphi(B,u) \subseteq \varphi(B,v)$. Let $z\in\varphi(B, u) \Rightarrow \exists b_4\in B \ni w=(b_4,v) \Rightarrow w= (b_4, \varphi(b,u) =\varphi(b_4b,u) = \varphi(b_5,u) \in\varphi(B,u) \Rightarrow\varphi(B,v) \subseteq \varphi(B,u)$ then $\varphi(B,u) = \varphi(B,v) \forall v \in\varphi(B,u)$.

Proposition 3.18: Let X be T_2-B-space and $x\in X$ is periodic relative. Then φ is minimal $\Leftrightarrow X = \varphi(B, x)$.

Proof: $\Rightarrow$) Since X is T_2-space $\Rightarrow \varphi(B, x)$ is closed (by Proposition (3.17)). Since φ is minimal, thin $\varphi(B, x) = X$.

$\Leftarrow$) let $X= \varphi(B, x)$, Then $\varphi(B, y)= \varphi(B, x) =X\ \exists\ y\in X$ (by Proposition (3.17)). So φ is minimal.

4. The Invariant, Minimal and Orbits in B-Space

In this part, we discussed the minimal sets, orbits of element, the invariant sets and discussed the relationship between these topics and some especial topics and clarification examples.

Note: Let X be a B-space, $K\subseteq B$ and $F\subseteq X$ then $K(F) = \varphi(K,F) =\{\varphi(k, f), k\in K, f\in F\}$

Definition 4.1: if $B(Y) = Y$ then we summonsed Y is invariant under B in B-space X.

Example 4.2: Let R be B-space so that $(R,*, U)$ is a Tb-algebra so that $a*b= a-b\ \forall\ a,b \in R$ and U is usual topology and φ is known by $\varphi(r_1,r_2)= r_2\ \forall r_1, r_2\in R$. Then $\forall\ Y \subseteq R$ is an invariant set.

Proposition 4.3: If X is B-space then:

1) If $\Omega = \{G\subseteq X/\ G$ be an invariant under $B\}$, then $\bigcup_{G\in\Omega} G$ is invariant under B and $\bigcap_{G\in\Omega} G$ if φ is one to one.
2) If G and Q are invariant under B then G^C and G-Q are invariant under B.
3) If the function φ is closed and G is invariant under B then $\bar{G}$ is invariant under B.
4) The set of interior points of G is invariant under B, if the function φ is open, $e\in B$ is identity and G is invariant under B then.

Proof:

1) Let G is an invariant under B, So $\varphi(B\times G)=G$, $\forall G\in\Omega$. $B(\bigcup_{G\in\Omega} G) = \varphi(B \times \bigcup_{G\in\Omega} G) = \varphi(\bigcup_{G\in\Omega} B \times G) = \bigcup_{G\in\Omega} G$. Thus $\bigcup_{G\in\Omega} G$ is invariant. Also $B(\bigcap_{G\in\Omega} G) = \varphi(B\times\bigcap_{G\in\Omega} G) = \varphi(\cap_{G\in\Omega} (B\times G))$ since φ is one to one so $\varphi(\cap_{G\in\Omega} (B\times G)) = \cap_{G\in\Omega} \varphi(B\times G) = \cap_{G\in\Omega} Y$ thus $\bigcap_{G\in\Omega} G)$ is invariant.
2) Let G and Q be invariant under B. So $\varphi(B\times G) = G$ then $(\varphi(B\times G))^c = G^c$. Since φ is one to one thus (φ

$(B\times G))^c = \varphi((B\times G)^c) = \varphi(B\times G^c)$. So $\varphi(B\times G^c) = G^c$. Thus G^c is invariant too. Also $\varphi(B\times(G\cap Q^c)) = G\cap Q^c$, since $G\text{-}Q = G\cap Q^c$. So $G\text{-}Q$ is invariant by (1).

3) Since $\bar{G} = \overline{\varphi(B\times G)} = \varphi(\overline{B\times G}) = \varphi(B\times\bar{G})$ thus $\bar{G}$ is invariant.

4) Since the identify element e in B so $G^{\circ}\subseteq\varphi(B\times G^{\circ})$. Since the function φ is open and $\varphi(B\times G) = G$ then $\varphi(B\times G^{\circ})\subseteq(\varphi(B\times G))^{\circ} = G^{\circ}$, then $\varphi(B\times G^{\circ}) = G^{\circ}$ so Y° is invariant.

Definition 4.4: In B-space X so that $x\in X$. The set $Bx=\{b\,x \,/\, b\in B\}$ is called orbit of x by B. and the set $\overline{Bx}$ is called by orbit closure to x by B.

Proposition 4.5: If X is B-space containing identity e then:

1) set Bx is a minimal invariant containing x.
2) The set $\overline{Bx}$ is a closed invariant minimal in X by B and contains x, when φ is closed.

Proof:

1) Suppose that $x\in X$. Since X is B-space and $(B,*)$ is b-algebra with identity so $B(Bx) = \varphi(B\times Bx) = \varphi(B\times\varphi(B,x)) = \varphi(*(B\times B),x) = \varphi(B,x) = Bx$. Then Bx is an invariant and contains x. Let $Y\subseteq X$ where Y is invariant, $x\in Y$ and $Y\subseteq Bx$. Since $B(Y) = Y$ then $B(Y)\subseteq B(x)$. But $B(x)\subseteq B(Y)$ (since $x\in Y$) so $Y = Bx$. Thin Bx is minimal.
2) Let $x\in X$, from (1) the set Bx is an invariant. So by Proposition (4.3). $\overline{Bx}$ is an invariant Let $Y\subseteq X$ where it is closed invariant under B and $x\in Y$. Thus $Bx\subseteq B(Y)$, then $\overline{Bx}\subseteq\overline{B(Y)} = \bar{Y}$ then $\overline{Bx} = \bar{Y}$(since Y is closed) then $\overline{Bx}$ is minimal.

Definition 4.6: Let X be B-space, $Y\subseteq X$ and $H\subseteq B$. Then Y is called minimal set by H if $Y = \overline{Hx}$ for some $x\in X$ and if $G\subseteq Y$ so that G is any orbit closure by H then $G = Y$. Y is a summonsed closure minimal orbit (CMO) if $H = B$

Remark 4.7: if X is B-space. Then X is CMO $\Leftrightarrow\overline{Bx} = X$, $\forall x\in X$

Proof⇒)If X is CMO then $\overline{Bx} = X$ for some $x\in X$. let $u\in X$ since $\overline{Bu}\subseteq X$. Then we have $\overline{Bu} = X$ by Definition (4.6).

Proposition 4.8: Let X be T_2-B-space containing identity e so that the action φ is closed. The following statements are equivalent:-

1) Y is CMO by B.
2) Y is not empty smaller closed and invariant set beneath B.
3) $Y = BZ$ for all not empty closed set Z from Y and Y is closed not empty set.

Proof: (1) → (2) let Y be CMO by B. So $\exists\, x\in X$ so that $Y = \overline{Bx}$, thus $Y\neq\phi$ and it is closed set. By proposition 3.5 then Y is invariant under B. Let $D\neq\phi$, it is invariant beneath B and closed so that $D\subseteq Y$. Since $Y = \overline{Bx}$ and $\overline{Bx}$ is junior invariant beneath B. Hence $Y = D$.

2)→(3) Let Z be closed set of Y and $Z\neq\phi$ then $BZ\subseteq BY = Y\Rightarrow BZ\subseteq Y$ and since $\overline{BZ} = \overline{\varphi(B\times Z)} = \varphi(\bar{B}\times\bar{Z}) = \varphi(B\times Z) = BZ\Rightarrow BZ$ is closed and since $B(BZ) = \varphi(B\times\varphi(B\times Z)) = \varphi(*(B,B), Z = \varphi(B,Z) = BZ$. Then BZ is an invariant and closed set by B. Since Y is junior and achieves this property, $Y\subseteq BZ$ the $Z = BZ$.

(3)→(1) Let $Y\neq\phi$, it is closed and $Y = BZ$ for all closed Z from Y so there dwells $z\in Z$. Since X is T_2 – space $\Rightarrow\{z\}$ is a closed set of Y, then $Y = B\{z\} = Bz\Rightarrow\overline{Bz} = \bar{Z} = Z$. Then Z is closure orbit. Put $y\in Y$ so that $\overline{By}\subseteq Y$. Then $By = Y\Rightarrow\overline{By} = Y$ (by $\{y\}$ is a closed set of Y) thus Y is CMO by B.

Remark 4.9: Let X be a B-space. If X is a compact space than $\overline{Bx}$ is compact.

Definition 4.10: Let X be a B-space such that B is a b-algebra with identity. If $\forall\, x\in X$ is periodic point, we called X is B-periodic space and denoted by BP-space.

Example 4.11: Let $(B,*,\lambda)$ be a Tb-algebra containing identity 0 so that $B=\{0, r, s, t\}$, λ is discrete topology on B and the function $*$ is defined by the following table:

*	0	r	s	t
0	0	r	s	t
R	r	0	t	s
S	s	t	0	r
T	t	s	r	0

Table above shows that the set B with binary operation* is B-Algebra.

And (R, U) is usual space where R is a real number then(B, R, φ) is a B-space, where $\varphi(b,\alpha) = \alpha\ \forall(b,\alpha)\in B\times R$ then $B_x = B$, $\forall x\in R$ and $0\in B$, where ${}_0B = 0B = B$. So (B, R,φ) is BP-space.

Proposition4.12: If X is BP-space with identity, then $\Omega = \{Bx \,/\, x\in X\}$ is partition for X.

Proof: Let's take $r,s\in X$, we proved $Br\cap Bs = \phi$ so that $Br\neq Bs$. Suppose that $Br\cap Bs\neq\phi,\exists\, t\in X$ so that $t\in Br$ and $t\in Bs$ then $Br = Bt$ and $Bs = Bt\Rightarrow Br = Bs = Bt$ by Proposition3;11. So $r\in Br\ \forall r\in X$(because $e\in B$), $Br\subseteq X$, thus $\cup Br = X$, thus Ω is partition for X.

Proposition 4.13: If X is BP-space containing identity e. Then relation $K=\{(u,v)\in X\times X/\ v\in B\,u\}$ is equivalent on X.

Proof:

1) $\varphi(e,u) = eu = u$ so $(u,u)\in K$.
2) If $(u,v)\in K$, thus $v\in Bu\Rightarrow Bv = Bu$ and by Proposition 2.11 we obtain $u\in Bv\Rightarrow(v,u)\in K$.
3) Put (u,v) and $(v,w)\in K$ so $v\in Bu$ and $w\in Bv$. Then $Bu = Bv = Bw$(by proposition 2.11) $\Rightarrow w\in Bu\Rightarrow(u, w)\in K$. Thus K is equivalent on X.

Proposition 4.14: Let X be BP-T_2-space then Bx is a minimal and closed set, $\forall x \in X$.

Proof: Put $x \in X$, thus x is periodic point. So $\varphi(B,x) = Bx$ is compact set by Proposition 3.11. Thus Bx is closed (since X is T_2), so $Bx = \overline{Bx}$, $\forall x \in X$. Then Bx is closure orbit, if $y \in Bx$ and since $\overline{By} = By = Bx \Rightarrow \overline{By} \subseteq Bx$. Thus Bx is a minimal set.

Proposition 4.15: Let X be a BP-T_2-space with identity e and $Y \subseteq X$. Then Y is an invariant set under $B \Leftrightarrow \overline{By} \subseteq Y$ $\forall y \subseteq Y$.

Proof:$\Rightarrow$) If $y \in Y$, then $By \subseteq B(Y) = Y$. Since y is a periodic point, thus By is closed by Proposition 4.14, so $By = \overline{By} \subseteq Y$.

$\Leftarrow$) Since $e \in B$, so $Y \subseteq B(Y)$ and since $\overline{By} \subseteq Y$, $\forall y \in Y$, thus $\cup_{y \in Y} \overline{By} \subseteq Y$ then $B(Y) = \cup_{y \in Y} \overline{By} = Y$. Hence $B(Y) = Y$, so Y is invariant.

REFERENCES

[1] Ahmed T.H., Habeeb. K. A., Haider J., A, "Uniformity in d-algebras", Earthline Journal of Mathematical Sciences, Vol. 2, No. 1, pp. 207-221, 2019, DOI: https://doi.org/10.34198/ejms.2119.207221

[2] Ahmed T.H, "The orbits and minimal sets in d – algebra", Journal of Physics: Conference Series, vol.1664, pp.1742-6596, 2020, DOI: 10.1088/1742-6596/1664/1/012048

[3] Sims B.T., "Fundamentals of Topology", Macmillan Publishing Co., Inc., New York, (1976).

[4] Yamini C., Kailasavalli S., "Fuzzy B – Ideals On B – algebra", International, Journal of Mathematical Archive, vol.5, no. 2, pp.227 – 233. 2014, www.ijma.info

[5] Yoon D. S., Kim H. S., "Uniform structures in BCL – algebras", Commun. Korean, Math. Soc., vol. 17, no.3, pp. 403407, 2002. DOI: http://dx.doi.org/10.4134/CKMS.2002.17.3.403

[6] Habeeb K., Haider J., and Ahmed T., "Limit Sets and Cartan D – space", Journal of AL-Qadisiyah for computer science and mathematics, vol.9, no.2, pp. 134 – 140, 2017, DOI: 10.29304 /jqcm. 2017.9.2.305

[7] Meng J. Y. Jun B., "BCK-algebras", Kyung Moon Sa, Seoul, Korea, pp. 293-377, 1994. DOI https://doi.org/10.1007/978-94-017-2422-7_6

[8] Neggers J., Hee S. K., "On B-algebras", Mate. Vesnik, vol.54, pp.21-29, 2002.

[9] Neggers J., Jun Y. B. and Hee. S. K, "On d-ideals in d-algebras", Math. Slovaca vol.49, no.3. pp.243-251, 1999.

[10] Young. B. J., Hee. S. K., "Uniform structures in positive implication algebras", Int. Math. J., vol.2, pp.215-2, 2002.