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An Approach to Solve Multi Attribute Decision-making Problem Based on the New Possibility Measure of Picture Fuzzy Numbers

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Abstract A picture fuzzy set is a more powerful tool to deal with uncertainties in the given informa- tion as compared to fuzzy set and intuitionistic fuzzy set and has energetic applications in decision-making. The aim of this study is to develop a new possibility measure for ranking picture fuzzy numbers and then some of its basic properties are proved. The proposed method provides the same ranking order as the score function in the literature. Moreover, the new possibility measure can provide additional information for the relative comparison of the picture fuzzy numbers. A picture fuzzy multi attribute decision-making problem is solved based on the possibility matrix generated by the proposed method after being aggregated using picture fuzzy Einstein weighted averaging aggregation operator. To verify the importance of the proposed method, an picture fuzzy multi attribute decision-making strategy is presented along with an application for selecting suitable alternative. The superiority of the proposed method and limitations of the existing methods are discussed with the help of a comparative study. Finally, a numerical example and comparative analysis are provided to illustrate the practicality and feasibility of the proposed method.

Keywords Picture Fuzzy Number, Aggregation Operator, New Possibility Measure, Multi Attribute Decision-making

1 Introduction

Multi attribute decision-making (MADM) problems are an important part of decision theory in which the optimal alternative is chosen from the different alternatives. The fundamental concept of fuzzy set (FS) theory was initially developed by Zadeh [1] in 1965. In 1986, Atanassov [2] implemented the intuitionistic fuzzy set (IFS) as an extension of the FS. Picture fuzzy set (PFS) is a direct extension of IFS, as proposed by Cuong [3,4] who also described some operations on PFS. Phong [5] investigated some properties of compositions of picture fuzzy (PF) relations. Singh [6] proposed a correlation coefficient for PFSs and used it for clustering analysis in the PF environment. Two correlation coefficients of PFSs, as well as some of their properties were suggested by Ganie [7]. Wei [8] discussed a MADM problem based on the PF weighted cross-entropy as an extension of the cross-entropy of PFSs, which is used to rank the different alternatives. Wang [9] developed some geometric aggregation operators under the PFSs. Dutta [10] introduced the equivalence of PF relation based on $(\alpha, \beta, \gamma) -$ cuts. Shovan Dogra [11] The notions of PF matrix, restricted PF matrix and special restricted PF matrix are established. Amalendu Si [12] proposed a decision-making approach for preferable medicine selection using PFS, Dempster–Shafer theory of evidence and grey relational analysis. Wei [13] introduced arithmetic and geometric operations to develop some PF aggregation operators and also studied their application in MADM problem. Garg [14] proposed some aggregation operators under PF environment, their desirable properties and their application in MADM problem. Wei [15] developed some picture fuzzy aggregation operators on the basic of the traditional arithmetic, geometric operations and Hamacher operations and utilized these operators to develop some approaches to solve the picture fuzzy MADM. Khan [16] examined the MADM problem based on the arithmetic aggregation operators under PF environment with Einstein t-norms and t-conorms. Qiyas [17] generated the Concept of Yager operators with the picture fuzzy set environment and its application to emergency program selection. Cuong [18,19] investigated the classification of representable picture t-norms, picture t-conorms operators and PF logic operators.

It is very important for PF decision-making to rate PFS. Thus, a comprehensive literature review survey of many research studies on the PF environment. The present research study aims to improve the New possibility measure for picture fuzzy numbers (PFNs). The possibility theory is a mathematical concept of dealing with several kinds of ambiguities and is a superior alternative to the probability theory that uses the possibility measure between 0 and 1. Zadeh [20] discussed the theory of possibility with the concept of possibility and probability in order to understand the role of human ability to interpret possibilities to imprecisely defined problems in a fuzzy environment. Wenxiu [21] proposed the extension principle through the extended fuzzy operators with t-norms and t-conorms for a new ordering and also introduced sigma-algebra to define fuzzy truth-valued possibility. Wang [22] developed the estimation of utility intervals with aggregation method associated with linear programming and simple additive weighting method to generate an aggregation ranking, which demonstrated the validity and potential applications of the proposed method. Van Dinh et al. [23] presented his ideas on difference and dissimilarity between two PFSs; the authors also studied, the application of dissimilarity measure in MADM. Garg [24] developed a possibility degree measure for ranking IFNs and also presented, some shortcomings of the existing possibility degree method and score function of intuitionistic fuzzy numbers.

In this study, a new possibility measure is proposed for ranking PFNs by extending the possibility measure of intuitionistic fuzzy (IF) environment [24-27]. The present study demonstrates that the same ranking can be obtained by both the new possibility measure and current approaches. Meanwhile, ranking result based on the new possibility measure will indicate additional information for the relative comparison of the PFNs.

The motivations of this article are outlined as follows:

(i) A new possibility measure for ranking PFNs is proposed by extending the possibility measure of the IF environment [24-27].

(ii) The proposed new possibility measure deal with the relative comparison of the PFNs.

(iii) The proposed method give more accurate and exact choice values in decision results when applied to MADM problem

(iv) The relative comparisons are more realistic in terms of knowing how good or bad an alternative is when compared to the rest of the alternatives on the multi attribute evaluation.

The contributions of this article are outlined as follows:

(i) The new possibility measure is proposed.

(ii) An algorithm for MADM problem using the proposed method is described and an application is presented to show the method's applicability in the real world.

(iii) The proposed method is demonstrated by a comparative study with other existing approaches.

The present research article is structured into seven comprehensive sections. Some of the basic definitions of PFSs and picture fuzzy Einstein weighted averaging (PFEWA) operator are outline in Section 2. The newly proposed possibility measure is covered under Section 3. Section 4 discusses a method of decision-making based on the proposed new possibility measure to solve the problem of MADM in the PF environment. A numerical example is given in Section 5 to establish the validity of this study. In Section 6, a comparative analysis between the existing methods and the proposed method is reviewed. In the final Section, the conclusions drawn from the present study are detailed.

2 Preliminaries

In this section, the basic concept of picture fuzzy set has been reviewed.

Definition 2.1: [3] A picture fuzzy set A on universal set X is defined by

$$\mathrm{A} = \{(x,\mu_A(x),\eta_A(x),\nu_A(x))\,|x \in X\} \tag{1}$$

where $\mu_A(x) \in [0,1]\,,\eta_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ are the degree of membership, the degree of neutral membership and the degree of non-membership of $x \in A$ such that $0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1 \forall x \in X$.

The degree of refusal membership is given by for $x \in X, \pi_A(x) = 1 - \mu_A(x) - \eta_A(x) - \nu_A(x)$.

Definition 2.2: [14] For convenience, let us denote $\alpha = (\mu_\alpha,\, \eta_\alpha, \nu_\alpha)$ as a picture fuzzy number (PFN), and its score function $S(\alpha)$ is defined by

$$S(\alpha) = \mu_\alpha - \eta_\alpha - \nu_\alpha;\; S(\alpha) \in [-1,1]. \tag{2}$$

Definition 2.3: [14] Let $\alpha = (\mu_\alpha,\, \eta_\alpha, \nu_\alpha)$be a PFN and its accuracy function $H(\alpha)$is defined by

$$H(\alpha) = \mu_\alpha +\, \eta_\alpha + \nu_\alpha; H(\alpha) \in [0,1]\,. \tag{3}$$

Definition 2.4: [14] Let $\alpha_1 = (\mu_{\alpha_1},\, \eta_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2},\, \eta_{\alpha_2}, \nu_{\alpha_2})$be two PFNs. then the following comparison rules can be used.

Table 1. Comparative analysis between some existing methods and the proposed approach

Reference Article	Weight information	Aggregation operators	Ranking method	Relative comparison
Wang (2017)	Known	Algebraic	Score function	×
Garg (2017)	Known	Archimedean	Score function	×
Wei (2017)	Known	Algebraic	Score function	×
Wei (2018)	Known	Hamacher	Score function	×
Khan (2019)	Known	Einstein Averaging	Score function	×
Qiyas (2020)	Known	Yager operators	Score function	×
Proposed Method	Known	Einstein Averaging	New possibility measure	✓

(i) If $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 > \alpha_2$;

(ii) If $S(\alpha_1) = S(\alpha_2)$, then

(a) If $H(\alpha_1) > H(\alpha_2)$, then $\alpha_1 > \alpha_2$;

(b) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$.

Definition 2.5: [14] Let $\alpha_i = \langle \mu_{\alpha_i}, \eta_{\alpha_i}, \nu_{\alpha_i} \rangle, (i = 1, 2, ..., n)$ be family of PFNs an picture fuzzy Einstein weighted arithmetic aggregation operator is defined as,

$$PFEWA(\alpha_1, \alpha_2, ..., \alpha_n) = w_1\alpha_1 \oplus w_2\alpha_2 \oplus ... \oplus w_n\alpha_n$$

$$= \left\langle \begin{array}{c} \frac{\prod_{i=1}^{n}(1+\mu_{\alpha_i})^{w_i} - \prod_{i=1}^{n}(1-\mu_{\alpha_i})^{w_i}}{\prod_{i=1}^{n}(1+\mu_{\alpha_i})^{w_i} + \prod_{i=1}^{n}(1-\mu_{\alpha_i})^{w_i}}, \\ \frac{2\prod_{i=1}^{n}(\eta_{\alpha_i})^{w_i}}{\prod_{i=1}^{n}(2-\eta_{\alpha_i})^{w_i} + \prod_{i=1}^{n}(\eta_{\alpha_i})^{w_i}}, \\ \frac{2\prod_{i=1}^{n}(\nu_{\alpha_i})^{w_i}}{\prod_{i=1}^{n}(2-\nu_{\alpha_i})^{w_i} + \prod_{i=1}^{n}(\nu_{\alpha_i})^{w_i}} \end{array} \right\rangle \tag{4}$$

where $w = (w_1, w_2, ..., w_n)^T$ be the weighing vector" of $\alpha_i\,(i = 1, 2, 3, ..., n)$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

3 New Possibility measure for Picture fuzzy numbers

A new possibility measure is proposed in this section to the rank of PFNs $\alpha_i = \langle \mu_{\alpha_i}, \eta_{\alpha_i}, \nu_{\alpha_i} \rangle, (i = 1, 2, ..., n)$.

Definition 3.1: Let $\alpha_1 = \langle \mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2} \rangle$ be two PFNs, then the new possibility measure $p(\alpha_1 \succeq \alpha_2)$ for $\alpha_1 \succeq \alpha_2$ is proposed as,

$$p(\alpha_1 \succeq \alpha_2) = \min\left(\max\left(\frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_2})+\pi_{\alpha_2}}{3(\pi_{\alpha_1}+\pi_{\alpha_2})}, 0\right), 1\right) \tag{5}$$

Where, either $\pi_{\alpha_1} \neq 0$ or $\pi_{\alpha_2} \neq 0$. On the other hand, if $\pi_{\alpha_1} = \pi_{\alpha_2} = 0$, then

$$p(\alpha_1 \succeq \alpha_2) = \begin{cases} 1; & \mu_{\alpha_1} > \mu_{\alpha_2} \\ 0; & \mu_{\alpha_1} < \mu_{\alpha_2} \\ 0.5; & \mu_{\alpha_1} = \mu_{\alpha_2}. \end{cases} \tag{6}$$

Theorem 3.1: Let $\alpha_1$and $\alpha_2$be two PFNs, then

(i) $0 \leq p(\alpha_1 \succeq \alpha_2) \leq 1$;

(ii) $p(\alpha_1 \succeq \alpha_2) = 0.5$ if $\alpha_1 = \alpha_2$;

(iii) $p(\alpha_1 \succeq \alpha_2) + p(\alpha_2 \succeq \alpha_1) = 1$;

Proof:

(i) $p(\alpha_1 \succeq \alpha_2) \leq 0$ is evidently, so we need to prove only $p(\alpha_1 \succeq \alpha_2) \leq 1$. for it, we take

$$a = \frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_2})+\pi_{\alpha_2}}{3(\pi_{\alpha_1}+\pi_{\alpha_2})}$$

Then, the following cases are discussed:

(a) If $a \geq 1$,then
$p(\alpha_1 \succeq \alpha_2) = \min(\max(a, 0), 1)$
$= 1$

(b) If $0 < a < 1$,then
$p(\alpha_1 \succeq \alpha_2) = \min(\max(a, 0), 1)$
$= a$

(c) If $a \leq 0$,then
$p(\alpha_1 \succeq \alpha_2) = \min(\max(a, 0), 1)$
$= 0$

Hence, in all cases,
we have $0 \leq p(\alpha_1 \succeq \alpha_2) \leq 1$.

(ii) Let $\alpha_1 = \langle \mu_{\alpha_1}, \eta_{\alpha_1}, \nu_{\alpha_1} \rangle$ and $\alpha_2 = \langle \mu_{\alpha_2}, \eta_{\alpha_2}, \nu_{\alpha_2} \rangle$ be two PFNs. If $\alpha_1 = \alpha_2$, which implies that $\mu_{\alpha_1} = \mu_{\alpha_2}, \eta_{\alpha_1} = \eta_{\alpha_2}$and $\nu_{\alpha_1} = \nu_{\alpha_2}$. then Equation (2) become

$p(\alpha_1 \succeq \alpha_2)$

$= \min\left(\max\left(\frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_2})+\pi_{\alpha_2}}{3(\pi_{\alpha_1}+\pi_{\alpha_2})},0\right),1\right)$

$= \min\left(\max\left(\frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_1})+\pi_{\alpha_1}}{3(\pi_{\alpha_1}+\pi_{\alpha_1})},0\right),1\right)$

$= \min\left(\max\left(\frac{3\pi_{\alpha_1}}{6\pi_{\alpha_1}},0\right),1\right)$

$= \min\left(\max\left(0.5,0\right),1\right)$

$= 0.5$

(iii) For two PFNs $\alpha_1 = \langle\mu_{\alpha_1},\eta_{\alpha_1},\nu_{\alpha_1}\rangle$ and $\alpha_2 = \langle\mu_{\alpha_2},\eta_{\alpha_2},\nu_{\alpha_2}\rangle$ we take

$a = \frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_2})+\pi_{\alpha_2}}{3(\pi_{\alpha_1}+\pi_{\alpha_2})}$

$b = \frac{2((\mu_{\alpha_2}+\pi_{\alpha_2})-\mu_{\alpha_1})+\pi_{\alpha_1}}{3(\pi_{\alpha_2}+\pi_{\alpha_1})}$

Such that

$a+b$

$= \frac{2((\mu_{\alpha_1}+\pi_{\alpha_1})-\mu_{\alpha_2})+\pi_{\alpha_2}+2((\mu_{\alpha_2}+\pi_{\alpha_2})-\mu_{\alpha_1})+\pi_{\alpha_1}}{3(\pi_{\alpha_2}+\pi_{\alpha_1})}$

$= \frac{2\mu_{\alpha_1}+2\pi_{\alpha_1}-2\mu_{\alpha_2}+\pi_{\alpha_2}+2\mu_{\alpha_2}+2\pi_{\alpha_2}-2\mu_{\alpha_1}+\pi_{\alpha_1}}{3(\pi_{\alpha_2}+\pi_{\alpha_1})}$.

$= \frac{3(\pi_{\alpha_2}+\pi_{\alpha_1})}{3(\pi_{\alpha_2}+\pi_{\alpha_1})}$

$= 1.$

Then, for the following cases:

(a) If $a \le 0$ and $b \le 0$ then
$p(\alpha_1 \succeq \alpha_2) + p(\alpha_2 \succeq \alpha_1)$
$= \min\left(\max\left(a,0\right),1\right) + \min\left(\max\left(b,0\right),1\right)$
$= 0+1$
$= 1.$

(b) If $0 < a, b < 1$ then
$p(\alpha_1 \succeq \alpha_2) + p(\alpha_2 \succeq \alpha_1)$
$= \min\left(\max\left(a,0\right),1\right) + \min\left(\max\left(b,0\right),1\right)$
$= a+b$
$= 1.$

(c) If $a \ge 1$ and then
$p(\alpha_1 \succeq \alpha_2) + p(\alpha_2 \succeq \alpha_1)$
$= \min\left(\max\left(a,0\right),1\right) + \min\left(\max\left(b,0\right),1\right)$
$= 1+0$
$= 1.$

We have from all the cases $p(\alpha_1 \succeq \alpha_2) + p(\alpha_2 \succeq \alpha_1) = 1$.

Example 3.1: Let $\alpha_1 = \langle 0.2, 0.5, 0.1\rangle$ and $\alpha_2 = \langle 0.3, 0.1, 0.3\rangle$ be two PFNs then

$$\begin{aligned} p(\alpha_1 \succeq \alpha_2) &= \min\left(\max\left(\tfrac{2((0.2+0.2)-0.3)+0.3}{3(0.2+0.3)},0\right),1\right) \\ &= \min\left(\max\left(0.333,0\right),1\right) \\ &= 0.333. \end{aligned}$$

In addition, $p = (p_{kj})_{n\times n}$ is the possibility matrix, where $p_{kj} = p(\alpha_k \succeq \alpha_j); k, j \in (1, 2, ..., n)$. Thus,

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}$$

Determine the alternatives ranking order, according to the decreasing order of $\alpha_k\, ; k \in (1, 2, ..., n)$ [27] defined as

$$r_k = \frac{1}{n(n-1)}\left(\sum\nolimits_{j=1}^{n} p_{kj} + \frac{n}{2} - 1\right). \tag{7}$$

Then, the values of α_k are in degreasing order with the ranking order of all $\alpha_k; k \in (1, 2, ..., n)$ alternatives.

Example 3.2: Consider $\alpha_1 = \langle 0.5, 0.1, 0.3\rangle$ and $\alpha_2 = \langle 0.4, 0.3, 0.2\rangle$ which are two PFNs that are equal in score value. The new possibility measure is applied to this numbers,

$$\begin{aligned} p(\alpha_1 \succeq \alpha_2) &= \min\left(\max\left(\tfrac{2((0.5+0.1)-0.4)+0.1}{3(0.1+0.1)},0\right),1\right) \\ &= \min\left(\max\left(0.833,0\right),1\right) \\ &= 0.833. \end{aligned}$$

And $p(\alpha_2 \succeq \alpha_1) = 0.167$

Therefore, using Equation (5), the possibility matrix is formulated as,

$$P = \begin{pmatrix} 0.500 & 0.833 \\ 0.167 & 0.500 \end{pmatrix}$$

The ranking value for the PFNs $\alpha_k(k = 1, 2)$is thus determined using Equation (7), Here, $r_1 = 0.665$ and $r_2 = 0.335$. since $r_1 > r_2$, thus we get $\alpha_1 \overset{0.833}{\succ} \alpha_2$.

4 Application of new possibility measure to the multi-attribute decision making

In this section, we present a MADM problem based on new possibility measure under PF environment. The decision-making problem consists of $'m'$ various alternatives $S_1, S_2, ..., S_m$ which are evaluated by a decision maker under the set of $'n'$ various attributes $T_1, T_2, ..., T_n$ which are represented by PFNs. The decision matrix of PFNs is represented by $\alpha_{kt} = \langle \mu_{\alpha_{kt}}, \eta_{\alpha_{kt}}, \nu_{\alpha_{kt}} \rangle_{m\times n}$ where $\mu_{\alpha_{kt}}, \eta_{\alpha_{kt}}$ and $\nu_{\alpha_{kt}}$ indicates the degrees that the alternative S_i satisfies, neutral and does not satisfy the attribute T_j, respectively. Such that, $0 \leq \mu_{\alpha_{kt}} + \eta_{\alpha_{kt}} + \nu_{\alpha_{kt}} \leq 1, \mu_{\alpha_{kt}}, \eta_{\alpha_{kt}}, \nu_{\alpha_{kt}} \in [0,1]$, $k = 1,2,...,m$ and $t = 1,2,...,n$ with weighing vector $w = (w_1, w_2, ..., w_n)^T$ such that $w_i \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

In the following, we develop a method based on the proposed new po ssibility measure to solve the decision-making problem with PF environment, which involves the following steps.

Step 1: The decision matrix is normalized, if applicable. The benefit type is transformed into the cost type, by using Equation (8).

$$\alpha_{kt} = \begin{cases} \langle \mu_{\alpha_{kt}}, \eta_{\alpha_{kt}}, \nu_{\alpha_{kt}} \rangle; & \text{for cost type attribute} \\ \langle \nu_{\alpha_{kt}}, \eta_{\alpha_{kt}}, \mu_{\alpha_{kt}} \rangle; & \text{for benefit type attribute} \end{cases} \tag{8}$$

Step 2: The PFEWA operator is used to aggregate the comprehensive evaluation of each alternative.

$$PFEWA(\alpha_{k1}, \alpha_{k2}, ..., \alpha_{kn}) = \left\langle \begin{array}{c} \frac{\prod_{i=1}^{n}(1+\mu_{\alpha_{kt}})^{w_i} - \prod_{i=1}^{n}(1-\mu_{\alpha_{kt}})^{w_i}}{\prod_{i=1}^{n}(1+\mu_{\alpha_{kt}})^{w_i} + \prod_{i=1}^{n}(1-\mu_{\alpha_{kt}})^{w_i}}, \\ \frac{2\prod_{i=1}^{n}(\eta_{\alpha_{kt}})^{w_i}}{\prod_{i=1}^{n}(2-\eta_{\alpha_{kt}})^{w_i} + \prod_{i=1}^{n}(\eta_{\alpha_{kt}})^{w_i}}, \\ \frac{2\prod_{i=1}^{n}(\nu_{\alpha_{kt}})^{w_i}}{\prod_{i=1}^{n}(2-\nu_{\alpha_{kt}})^{w_i} + \prod_{i=1}^{n}(\nu_{\alpha_{kt}})^{w_i}} \end{array} \right\rangle \tag{9}$$

Step 3: Compute the possibility matrix $P = (p_{kj})_{m\times m}$ as

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}$$

Where $p_{kj} = p(\alpha_k \succ \alpha_j)(k, j = 1,2,...,n)$ is defined" as

(i) If either $\pi_{\alpha_k} \neq 0$ or $\pi_{\alpha_j} \neq 0$ then

$$p(\alpha_k \succeq \alpha_j) = \min\left(\max\left(\frac{2\left((\mu_{\alpha_k}+\pi_{\alpha_k})-\mu_{\alpha_j}\right)+\pi_{\alpha_j}}{3\left(\pi_{\alpha_k}+\pi_{\alpha_j}\right)}, 0\right), 1\right) \tag{10}$$

(ii) If $\pi_{\alpha_k} = \pi_{\alpha_j} = 0$, then

$$p(\alpha_1 \succeq \alpha_2) = \begin{cases} 0; & \mu_{\alpha_k} > \mu_{\alpha_j} \\ 1; & \mu_{\alpha_k} < \mu_{\alpha_j} \\ 0.5; & \mu_{\alpha_k} = \mu_{\alpha_j}. \end{cases} \tag{11}$$

Further, it is clearly observed $0 \leq p_{kj} \leq 1$ and $p_{kj} + p_{jk} = 1; (k, j = 1,2,...,,n)$ which denotes that the possibility matrix, P.

Step 4: The ranking value for alternative $S_k (k = 1,2,...,n)$ is determined using,

$$r_k = \frac{1}{n(n-1)}\left(\sum_{j=1}^{n} p_{kj} + \frac{n}{2} - 1\right) \tag{12}$$

The order in which the values of $r_k (k = 1,2,...,n)$are decreasing is found with the ranking order of all alternatives on the basis of these membership values, and then the best alternative is selected.

5 Numerical example

A numerical example is taken [12] from literature with the PF decision matrix below and weighing vector $w = (0.2, 0.3, 0.1, 0.4)^T$ is considered in Table 2.

Table 2. PF Decision matrix

	T_1	T_2	T_3	T_4
S_1	$\langle 0.2, 0.1, 0.6 \rangle$	$\langle 0.5, 0.3, 0.1 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0.4, 0.3, 0.2 \rangle$
S_2	$\langle 0.1, 0.4, 0.4 \rangle$	$\langle 0.6, 0.3, 0.1 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$	$\langle 0.2, 0.1, 0.7 \rangle$
S_3	$\langle 0.3, 0.2, 0.2 \rangle$	$\langle 0.6, 0.2, 0.1 \rangle$	$\langle 0.4, 0.1, 0.3 \rangle$	$\langle 0.3, 0.3, 0.4 \rangle$
S_4	$\langle 0.3, 0.1, 0.6 \rangle$	$\langle 0.1, 0.2, 0.6 \rangle$	$\langle 0.1, 0.3, 0.5 \rangle$	$\langle 0.2, 0.3, 0.2 \rangle$

Step 1: The parameters here are cost type attributes, T_2 and T_3 while all the other attributes are benefit type attributes.

Using Equation (8), the PF decision matrix is converted into the following PF normalized decision matrix as follow in Table 3.

Table 3. PF normalized decision matrix

	T_1	T_2	T_3	T_4
S_1	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.5, 0.3, 0.1 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0.2, 0.3, 0.4 \rangle$
S_2	$\langle 0.4, 0.4, 0.1 \rangle$	$\langle 0.6, 0.3, 0.1 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$	$\langle 0.7, 0.1, 0.2 \rangle$
S_3	$\langle 0.2, 0.2, 0.3 \rangle$	$\langle 0.6, 0.2, 0.1 \rangle$	$\langle 0.4, 0.1, 0.3 \rangle$	$\langle 0.4, 0.3, 0.3 \rangle$
S_4	$\langle 0.6, 0.1, 0.3 \rangle$	$\langle 0.1, 0.2, 0.6 \rangle$	$\langle 0.1, 0.3, 0.5 \rangle$	$\langle 0.2, 0.3, 0.2 \rangle$

Step 2: Using PFEWA Equation (9), the following is obtained.

$$S_1 = \langle 0.413, 0.218, 0.227 \rangle,$$
$$S_2 = \langle 0.600, 0.200, 0.142 \rangle,$$
$$S_3 = \langle 0.430, 0.220, 0.218 \rangle,$$
$$S_4 = \langle 0.254, 0.215, 0.338 \rangle.$$

Step 3: The possibility matrix obtained by pairwise comparison of these PFNs using Equation (10), is given by

$$P = \begin{pmatrix} 0.50 & 0.00 & 0.46 & 0.79 \\ 1.00 & 0.50 & 1.00 & 1.00 \\ 0.54 & 0.00 & 0.50 & 0.83 \\ 0.21 & 0.00 & 0.17 & 0.50 \end{pmatrix}$$

Step 4: $S_k (k = 1, 2, 3, 4)$ is computed by Equation (12) and is found that $r_1 = 0.229, r_2 = 0.375, r_3 = 0.239$ and $r_4 = 0.156$. Consequently, the ranking of the alternatives is as follows, $S_2 \overset{1.00}{\succ} S_3 \overset{0.54}{\succ} S_1 \overset{0.79}{\succ} S_4$.

6 Comparative analysis

This section consists of the comparative analysis of several existing methods with proposed method are shown in the Table 4.

Table 4. Comparison with existing approaches

Existing operators	Ranking
PFWG [13]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFOWG [13]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFHA [13]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFHG [13]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFWA [14]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFOWA [14]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFHA [14]	$S_2 \succ S_1 \succ S_3 \succ S_4$
PFHWA [15]	$S_2 \succ S_1 \succ S_3 \succ S_4$
PFHWG [15]	$S_2 \succ S_1 \succ S_3 \succ S_4$
PFEWA [16]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFYWAA [17]	$S_2 \succ S_3 \succ S_1 \succ S_4$
PFYWGA [17]	$S_2 \succ S_3 \succ S_1 \succ S_4$
Proposed Method	$S_2 \overset{1.00}{\succ} S_3 \overset{0.54}{\succ} S_1 \overset{0.79}{\succ} S_4$

According to Table 4, the rank is same by using our new possibility measure and the existing approaches. While the proposed method provides additional information for the relative comparison to the decision makers, where S_2 alternative is more superior to S_3 than S_1 superior to S_4.

7 Conclusions

In this research study, a new possibility measure for ranking PFNs is proposed. This method brings the same ranking order of PFNs that is derived by the score function in the literature. Furthermore, the proposed new possibility measure can provide relative comparison of the PFNs. The proposed new possibility measure generates a possibility matrix, which is used to build a basic method for solving picture fuzzy multi attribute decision-making problems. After aggregating a decision matrix with the picture fuzzy Einstein weighted averaging aggregation operator, the proposed method is applied to it. An illustrative example is used to demonstrate the developed approach, and the result is compared with some of the current ranking approaches in the PF environment. In future work, we shall continue working in the extension and application of the possibility measure to the other domain [28-30].

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