

Grey Relational Analysis Parameter-Based Predictive Modelling of Surface Roughness

Zvikomborero Hweju, Khaled Abou-El-Hossein *

Ultra-High Precision Engineering Research Unit, Department of Mechatronics, Nelson Mandela University,
Port Elizabeth, South Africa

Received May 19, 2021; Revised September 22, 2021; Accepted October 17, 2021

Cite This Paper in the following Citation Styles

(a): [1] Zvikomborero Hweju, Khaled Abou-El-Hossein , "Grey Relational Analysis Parameter-Based Predictive Modelling of Surface Roughness," *Universal Journal of Mechanical Engineering*, Vol. 9, No. 3, pp. 21 - 26, 2021. DOI: 10.13189/ujme.2021.090301.

(b): Zvikomborero Hweju, Khaled Abou-El-Hossein (2021). *Grey Relational Analysis Parameter-Based Predictive Modelling of Surface Roughness*. *Universal Journal of Mechanical Engineering*, 9(3), 21 - 26. DOI: 10.13189/ujme.2021.090301.

Copyright©2021 by authors, all rights reserved. Authors agree that this article remains permanently open access under the terms of the Creative Commons Attribution License 4.0 International License

Abstract Grey relational analysis is a widely used approach for the purposes of decision making, prediction and relational investigation. This study utilizes the grey relational analysis for modelling surface roughness during the single point diamond turning of RSA-443. The utilized parameter in this study is the grey relational grade together with cutting speed, feed, and depth of cut. The Taguchi L9 orthogonal array has been utilized for designing the experiment, with three extra experimental runs being carried out for the purposes of validating the developed model. The developed model indicates that the cutting parameters are insignificant as predictors of surface roughness. Grey relational grade is the only significant predictor of surface roughness. Acoustic emission signal root mean square has been used for determining the grey relational grade in the study. The grey relational analysis-based surface roughness values have been compared to experimentally obtained values by using the Mean Absolute Percentage Error (MAPE). The accuracy levels are an exhibition of high prediction power of the model. Pair t-test results indicate the lack of statistical significance in the difference between the experimentally measured and predicted surface roughness values.

Keywords Grey Relational Analysis, Surface Roughness, RSA-443, Predictive Modelling

1. Introduction

Single point diamond turning is a mechanical machining process that utilizes a cutting tool with a diamond tip to manufacture high quality aspheric and cylindrical components. Numerous techniques have been utilized to optimize and predict surface roughness in mechanical machining of components. One of the methods that have been employed is the grey relational analysis technique. Panda et al. utilized the technique to model and optimize cutting parameters in dry machining of AISI 52100 steel using a mixed ceramic insert tool [1]. In the study, first and second order models were developed for the purposes of response accuracy checking. There is a good correlation between the developed models and experimentally obtained surface roughness values. Hence, the grey relational analysis technique is efficient in modelling and predicting surface roughness.

In yet another study, Tosun [2] used the grey relational analysis technique to determine the optimum parameters for multi-performance characteristics in drilling of AISI 4140 steel. In the study, the following parameters have been used as drilling parameters: feed rate, cutting speed, drill and point angles. The experiment has been designed using the Taguchi orthogonal array. The study results indicate a significant enhancement of surface roughness and blur through using the grey relational analysis technique. In a similar study, Kao et al. [3] optimized the electrochemical polishing of 316L stainless steel using grey relational analysis. In the study, the control variables

are temperature, current density, and electrolyte composition, while the response variables are surface roughness and passivation strength. The results of the study successfully verified the effectiveness of the approach. Additionally, numerous studies have successfully used the same approach [4-6], albeit on materials other than RSA-443.

A study by Yang et al. [7] has utilized the grey relational analysis technique to optimize the machining parameters during the end milling of high purity graphite. The machining parameters in the study are cutting speed, depth of cut and feed rate, while the response variables are surface roughness and groove width.

The grey relational analysis approach was successfully used by Lu et al. [8] in conjunction with principal component analysis (PCA) to optimize process parameters during the in-feed centerless cylindrical grinding of EN52 steel. The use of the technique has proven to be efficient.

Based on the previous studies, it is evident that the Grey relational analysis technique is a powerful optimization tool that can be reliably used for machining processes such as milling, drilling, and turning. Hence, this paper is a presentation of grey relational analysis-based optimization of machining parameters in single point diamond turning of RSA 433.

2. Grey Relational Analysis

The grey relational analysis is a technique utilized to determine the influence and interrelationship among multiple parameters. It reduces multi response optimization problem into a single response relational grade. The stages of transformation are presented by the flow chat (Figure 1).

The first stage entails experimentally obtained data normalization. The normalization method employed is the mean-max method. This procedure performs a linear alteration on the original data. The data values are normalized in the range [0,1]. The normalization equation is presented by (1). In this study, data normalization has been performed to remove inconsistencies that can complicate data analysis. The inconsistencies may be due to any of the following: inserting more information than required, deleting data, or updating existing data.

$$Z = (\gamma - \text{mina}) / (\text{maxa} - \text{mina}) \quad (1)$$

Where Z= normalized value

γ = new value in the required range

mina= minimum value in the required range

maxa= maximum value in the required range

Immediately following the data normalization stage is the computation of the deviation sequence using (2).

$$\Delta_{oi}(k) = [X_0(k) - X_i(k)] \quad (2)$$

Where $\Delta_{oi}(k)$ = deviation sequence

$X_0(k)$ = reference sequence

$X_i(k)$ = comparability sequence

The grey relational coefficient (GRC) is calculated using (3).

$$\alpha_i(k) = (\Delta_{\min} + \alpha \Delta_{\max}) / (\Delta_{oi}(k) + \alpha \Delta_{\max}) \quad (3)$$

where $\alpha_i(k)$ = grey relational coefficient

Δ_{\min} = minimum absolute difference

Δ_{\max} = maximum absolute difference

Δ_{oi} = deviation sequence

Finally, a composite grey relational grade (GRG), is determined by averaging the GRC of each response variable as shown in (4).

$$\lambda_i = (1/n) \sum \alpha_i(k) \quad (4)$$

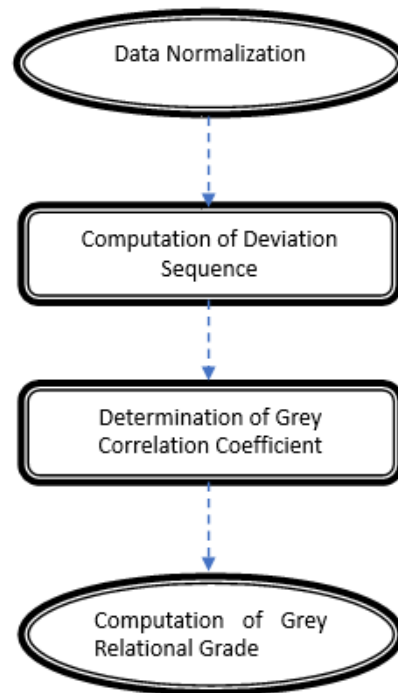


Figure 1. Grey Relational Analysis Process Flow

3. Materials and Methodology

In the present study, kerosene has been used as the coolant during the ultra-high precision diamond turning of RSA-443. The experiment has been designed using the Taguchi L9 orthogonal array and an extra three (3) experimental runs have been carried out for the purpose of testing the accuracy of the developed model. During the turning process, acoustic emission signals are captured, and the root mean square of the signal is employed for modelling purposes together with primary cutting parameters (cutting speed, feed rate and depth of cut). The stages followed in performing the experiment are presented by the flow diagram (Figure 2).

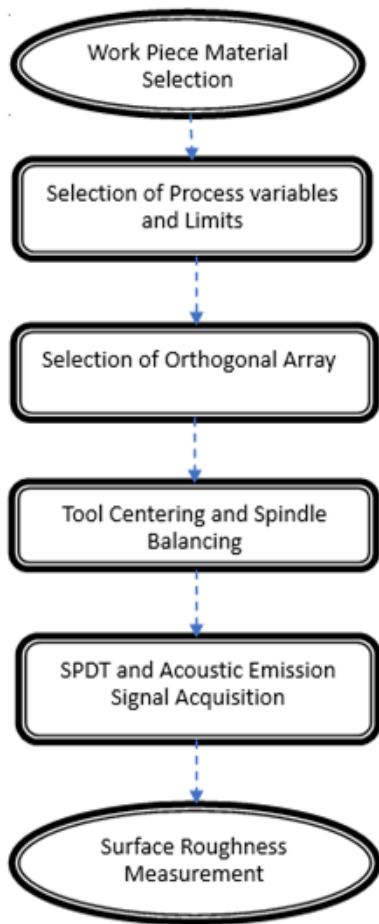


Figure 2. Experimentation Process Flow Chart

The first stage has been the selection of workpiece material. Use has been made of a 60 mm diameter specimen of RSA-443. RSA-443 contains 40% silicon by composition. Three levels for each of the three selected process parameters have been decided. Prior to performing the machining process, the single point diamond cutting tool has been centered, and the spindle balanced as a way of eliminating unwanted oscillations. In this study, spindle balancing has been carried out at 2 500 rpm, 2 000rpm and 1 000 rpm. The selected balancing values cover all chosen cutting speed levels. The diamond cutting tool centering image is presented in Figure 3.

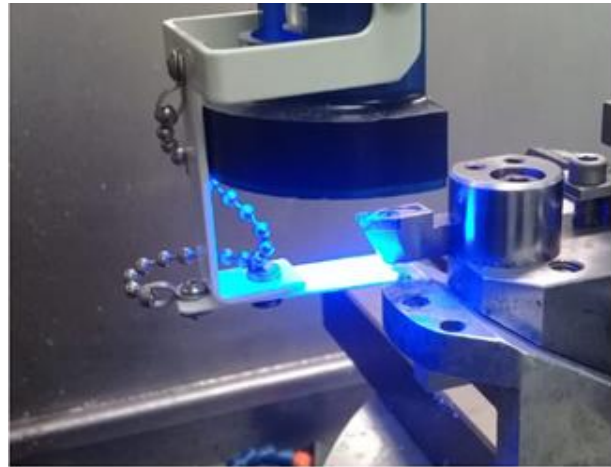


Figure 3. Diamond Cutting Tool Centering

4. Results and Analysis

The experimental results of the single point diamond turning process are presented in Table 1. The resultant surface roughness for each cutting parameter combination has been presented.

Table 1. Single Point Diamond Turning Results (Kerosene Coolant)

Run No.	Cutting Speed (rpm)	Feed (mm/min)	Depth of Cut (µm)	AERms (V)	R _a (nm)
1	1250	7.5	5	0.029679	21
2	1250	15	15	0.027504	33
3	1250	22.5	25	0.028245	38
4	1750	7.5	15	0.02781	17
5	1750	15	25	0.027856	23
6	1750	22.5	5	0.028368	26
7	2500	7.5	25	0.027855	12
8	2500	15	5	0.034323	18
9	2500	22.5	15	0.028314	19
10	1250	7.5	15	0.030895	24
11	1750	15	5	0.027612	23
12	2500	22.5	25	0.02721	22

Table 2. Normalized Data

Run No.	Cutting Speed (rpm)	Feed (mm/min)	Depth of Cut (μm)	AE _{rms} (V)	R _a (nm)
1	0.00	0.00	0.00	0.32	0.35
2	0.00	0.50	0.50	0.00	0.81
3	0.00	1.00	1.00	0.11	1.00
4	0.40	0.00	0.50	0.04	0.19
5	0.40	0.50	1.00	0.05	0.42
6	0.40	1.00	0.00	0.13	0.54
7	1.00	0.00	1.00	0.05	0.00
8	1.00	0.50	0.00	1.00	0.23
9	1.00	1.00	0.50	0.12	0.27
10	0.00	0.00	0.50	0.95	0.22
11	0.40	0.50	0.00	0.62	0.67
12	1.00	1.00	1.00	0.58	0.61

Table 3. Grey Relational Coefficient and Grey Relational Grade Values

Run	Evaluation of Δ_{oi}		Grey Relational Coefficient		GRG	Rank
	AE _{rms}	R _a	AE _{rms}	R _a		
1	0.68	0.65	0.424	0.435	0.429	5
2	1.00	0.19	0.333	0.725	0.529	7
3	0.89	0.00	0.360	1.000	0.680	11
4	0.96	0.81	0.342	0.382	0.362	2
5	0.95	0.58	0.345	0.463	0.404	4
6	0.87	0.46	0.365	0.521	0.443	6
7	0.95	1.00	0.345	0.333	0.339	1
8	0.00	0.77	1.000	0.394	0.697	12
9	0.88	0.73	0.362	0.407	0.384	3
10	0.05	0.78	0.909	0.391	0.650	10
11	0.38	0.33	0.568	0.602	0.585	9
12	0.68	0.65	0.424	0.435	0.429	8

Table 2 is a presentation of the normalized experimental data and results.

A presentation is made of the calculated values of the grey relational coefficient and grey relational grade values (Table 3). The computed rank is presented in the extreme right column of the table. Surface roughness values based on machining parameters and grey relational coefficient are presented in Table 4. The data is subsequently used to generate a model linking grey relational grade (GRG) and surface roughness. This model is essential as it adds to the already available surface roughness prediction tools during single point diamond turning of rapidly solidified aluminium (RSA-443).

Performing multiple linear regression to ascertain the correlation between grey relation grade and surface roughness yields (5).

$$Y = -8.967336 + 73.060687 \text{ GRG} \quad (5)$$

R square (R^2) equals 0.901298. It means that the predictors (X_i) explain 90.1% of the variance of Y . Adjusted R square equals 0.881557. The coefficient of multiple correlation (R) equals 0.949367. It means that there is a very strong direct relationship between the predicted data (\hat{y}) and the observed data (y). Overall regression: right-tailed, $F(1,5) = 45.657386$, p -value = 0.00107809. Since p -value $< \alpha$ (0.05), we reject the H_0 . The linear regression model (2) provides a better fit than the model without the independent variables resulting in, $Y = b_0$. The following independent variables are not significant as predictors for Y : X_1 , X_2 , and X_3 . Therefore, the calculator excluded these variables from the model.

$$Y = b_0 + b_1X_1 + \dots + b_pX_p \quad (6)$$

Table 4. Surface Roughness Based on Machining Parameters and Grey Relational Coefficient

C ₁ [rpm]	C ₂ [mm/min]	C ₃ [μm]	GRC		Measured Ra [nm]	Predicted Ra [nm]	Absolute Error (%)
			AERms [V]	Ra [nm]			
1250	7.5	5	0.424	0.435	21	22.375	6.54
1250	15.0	15	0.333	0.725	33	29.681	10.05
1250	22.5	25	0.360	1.000	38	40.713	7.13
1750	7.5	15	0.342	0.382	17	17.480	2.82
1750	15.0	25	0.345	0.463	23	20.549	10.65
1750	22.5	5	0.365	0.521	26	23.398	10.00
2500	7.5	25	0.345	0.333	12	15.800	31.66
2500	15.0	5	1.000	0.394	18	22.375	24.30
2500	22.5	15	0.362	0.407	19	19.087	0.45
1250	7.5	15	0.909	0.391	24	38.522	60.51
1750	15	5	0.568	0.602	23	33.773	46.83
2500	22.5	25	0.424	0.435	22	22.375	1.70

Table 5. Paired t-test Parameters

Parameter	Value
Average (\bar{X}_d)	2.51
Sample Size (n)	12
Sample SD (S_d)	5.37
Skewness	1.27
Normality p-value	0.06
Outliers _d	14.52

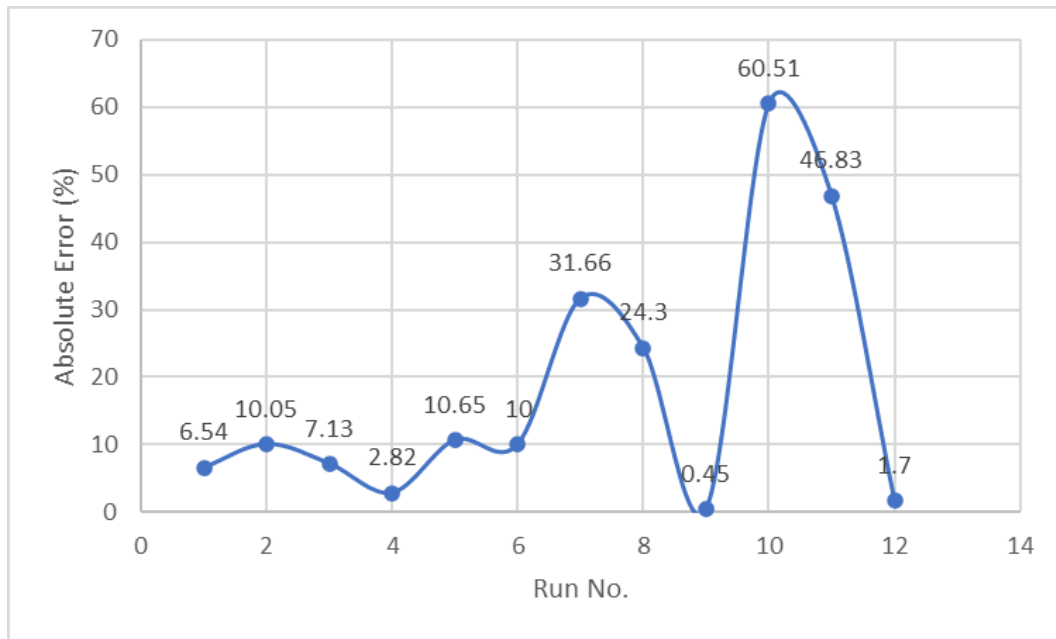


Figure 4. Plot of Absolute Error vs Run Number

The absolute percentage error values have been plotted against experiment run number to have a visual variation of the errors (Figure 4). The first eight (8) sets of absolute error values are obtained using training data, while the last four (4) values are obtained using testing data. Except for the two absolute error values at experiment run numbers

10 and 11 that have magnitudes of 60.51% and 46.83%, respectively, the rest of the values are below 30%. This is an indication of the high prediction accuracy of the formulated model. To assess the significance of the difference between the actual and predicted values of surface roughness, a paired t-test is conducted. The

parameters of the paired t-test are presented in Table 5. Since $p\text{-value} > \alpha$, H_0 cannot be rejected. The average of population of Predicted Value minus Actual Value is assumed to be equal to the μ_0 . In other words, the difference between the average of Predicted Value minus Actual Value and the μ_0 is not big enough to be statistically significant.

5. Conclusion

This study evaluated the effectiveness of using the grey relational grade parameter in modelling surface roughness during single point diamond turning of RSA-443. The modelling results reveal the insignificance of primary cutting parameters as predictors of surface roughness in the presence of grey relational grade. High R^2 value (0.9) of the model indicates a very strong direct relationship between the grey relational grade and the surface roughness. Predicted surface roughness values of the formulated model have been compared to the experimentally obtained values using the Mean Absolute Percentage Error (MAPE) method. The MAPE values indicate high prediction accuracy of the model. Furthermore, a paired t-test indicates the lack of statistical significance in the difference between the experimentally measured and predicted surface roughness values. Hence, the grey relational grade is a suitable and accurate predictor of surface roughness.

REFERENCES

- [1] Panda A., Sahoo A. K., Rout A. K., "Multi-attribute decision making parametric optimization and modeling in hard turning using ceramic insert through grey relational analysis: A case study," *Decision Science Letters*, vol. 5, no. 4, pp. 581–592, 2016. DOI: 10.5267/j.dsl.2016.3.001
- [2] Tosun N., "Determination of optimum parameters for multi-performance characteristics in drilling by using grey relational analysis," *International Journal Advanced Manufacturing Technology*, vol. 28, pp. 450–455, 2006. DOI 10.1007/s00170-011-3857-6
- [3] Kao P. S., Hocheng H., "Optimization of electrochemical polishing of stainless steel by grey relational analysis," *Journal of Materials Processing Technology*, vol. 140, no. 1-3, pp. 255–259, 2006. DOI: 10.1016/S0924-0136(03)00747-7
- [4] Lin J. L., Lin C. L., "The use of the orthogonal array with grey relational analysis to optimize the electrical discharge machining process with multiple performance characteristics," *International Journal of Machine Tools and Manufacture* vol. 42, no. 2, pp. 237-244, 2002. DOI:10.1016/S0890-6955(01)00107-9
- [5] Çaydaş U., Haşçalık A., "Use of the grey relational analysis to determine optimum laser cutting parameters with multi-performance characteristics," *Optics & Laser Technology*, vol. 40, no. 7, pp. 987-994, 2008. DOI: 10.1016/j.optlastec.2008.01.004
- [6] Huang J. T., Liao Y. S., "Optimization of machining parameters of Wire-EDM based on Grey relational and statistical analyses," *International Journal of Production Research*, vol. 41, no. 8, pp. 1707-1720, 2003.
- [7] Yang Y. Y., Shie J. R., Huang C. H., "Optimization of dry machining parameters for high purity graphite in end-milling process," *Materials and Manufacturing Processes*, vol. 21, no. 8, pp. 832 – 837, 2006. DOI: 10.1080/03602550600728141
- [8] Lu H. S., Chang C. K., Hwanga N. C., Chung C. T., "Grey relational analysis coupled with principal component analysis for optimization design of the cutting parameters in high-speed end milling," *Journal of Material Processes Technology*, 2008. DOI:10.1016/j.jmatprotec. 2008.08.030