

Modeling Euribor Rates Volatility: Application of the GARCH Model

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Abstract Euribor (Euro Interbank Offered Rate) is considered to be the most important base rate for all types of financial products like interest rate swaps, interest rate futures, saving accounts and mortgages. Euribor rates turned negative for the first time in January 2015 and have been negative ever since. In recent years, several European central banks have imposed negative interest rates on commercial banks, which are the only way to stimulate their nations' economies. Under these circumstances, the purpose of this study is to estimate the optimal equilibrium of the negative rates which are still increasing constantly. This fact raises doubts about the financial stability in many countries and the effect of monetary policy in stimulating economic growth in European countries. This study has analyzed the volatility of the Euribor rates related to the daily time series 2015-2021. Advanced volatility econometric methods are applied to GARCH models and volatility forecasting in the long-run equilibrium. The optimal model for the weekly and monthly maturity rates is identified; however, the larger the ARCH(p) and lag-variance(q) value we test, the poorer the performance of the obtained model is. Practical implications ought to be taken into consideration by the banking sector and other financial institutions.

Keywords Euribor, GARCH Volatility Modeling, Optimal Long-Run Equilibrium

Euro Interbank Offer Rate (Euribor) as a reference rate is constructed from the average interest rate at which Eurozone banks offer unsecured short-term lending on the inter-bank market. The maturities on loans used to calculate Euribor often range from one week to one year (which makes Euribor an index reference). These Euribor rates, which are updated daily, represent the average interest rate that Eurozone banks charge one another for uncollateralized loans. Euribor rates are an important benchmark for a range of euro-denominated financial products, including mortgages, savings accounts, car loans, and various derivatives securities [10,16,28].

The global interest rates have been declining for many years, even decades. This trend is related to many fundamental factors. There are two prevailing views:

- (1) Structural factors have pushed interest rates to record low levels. These structural factors include demographics and longer life expectancy, which affects individuals' propensity to save and invest.
- (2) The lower interest rates are a reaction to the high financial leverage levels, which contributed to the global financial crisis. According to this view, lower interest rates are necessary to facilitate the deleveraging process, thereby they are expected to return to normal in the future.

However, Euribor does not return to equilibrium, as it has been performing at negative rates for years. The *figure 1 A and B* below give the trend of the Euribor rates with maturity from one week to one year:

1. Introduction

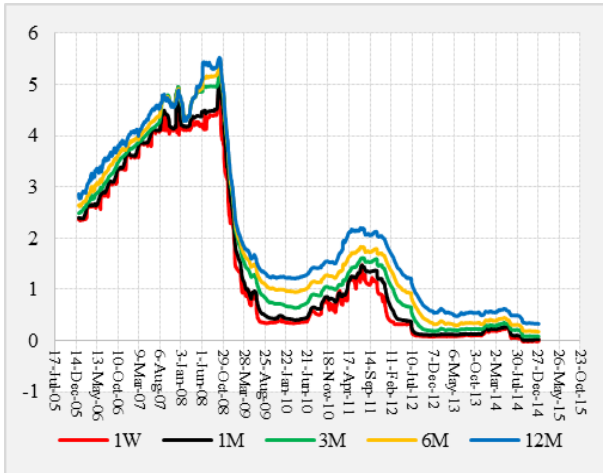


Figure 1 A. Euribor rates with positive values (in %), 2006-2014

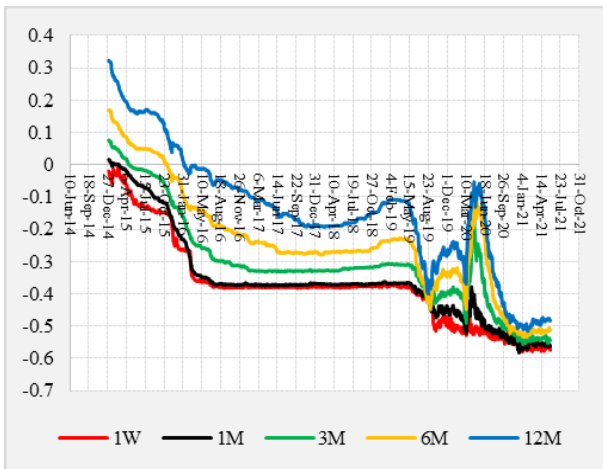


Figure 1 B. Euribor rates with negative values (in %), 2015-2021

For the first time in January 2014 – February 2016 these rates have turned negative and have continued with a negative trend until today, *figure 1 B*. Weekly and monthly maturity Euribor rates have performed with

negative value since 2014. Whereas quarterly and semi-annual maturity Euribor rates have performed with negative value since 2015 and annual Euribor rate has performed with negative value since 2016. The performance at negative rates presents new challenges to the Eurozone, as they have exceeded the medium term.

As you can see from *figure 1 B*, the values of Euribor rates have plunged as a result of Covid-19 pandemic (during this period European banks have injected liquidity into the economy). Consequently, the rates declined sharply. The monetary policy has been constrained by low interest rates in many major economies caused by covid-19 pandemic [2]. Furthermore, during the covid-19 pandemic, the liquidity in the Euribor benchmark started to decrease but it continued to react smoothly to monetary decisions for the eurozone [7,25]. Whether they are part of the Eurozone or not, the economies of many European countries are significantly affected by the Euribor. There are several challenges, two of them being more crucial:

- (1) They complicate the estimation of the lower bound monetary policy rate in these economies.
- (2) There is a situation that alters the relative incentives toward domestic and foreign currency denominated assets and liabilities when currency is different from euro.

Related to the system of the required reserves, the lower return on foreign exchange reserve assets, and at times, negative, makes the scope for currency diversification more limited.

The specific monetary effects and the effects on the general economy that Euribor has, are closely related to the economic euroization. According to Veyrune [24], there are four main phases in the cycle of euroization, with small deviations in some countries, as shown in *figure 2*.

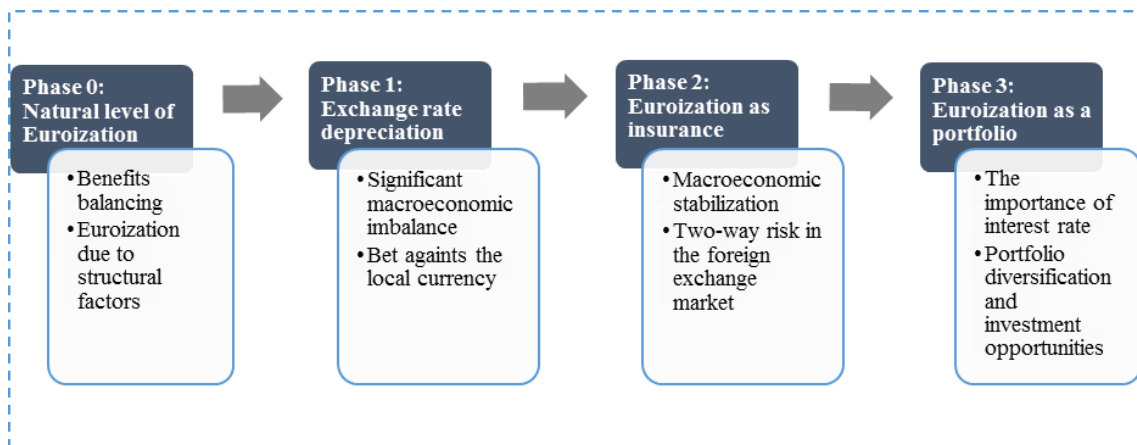


Figure 2. The stages of the euroization cycle

In the Southeastern European economies, the financial euroization contributes by encouraging banks deposits rather than hoarding foreign currency cash, the integration in the international capital markets, and the access to financially attractive foreign funding sources. A large problem is related to the effectiveness of the monetary policy. Euroization directly reduces the scope of financial intermediation that the central bank can influence. Sometimes the financial euroization impairs the monetary policy transmission, as the exchange rate transmission channel negatively interacts with the interest rate channel.

This study has analyzed the volatility of the Euribor rates (also known as index rates because they serve as a benchmark for many interest rates in the financial system) based on daily data for the period 2015 – 2021 (the period in which Euribor rates turned negative, until now). Advanced volatility econometric methods have been applied, more specifically GARCH volatility models to forecasting the long-run equilibrium. The aim of this research is to measure the financial stability in different countries and the effect of euro monetary policy.

2. Literature Review

Euribor performance analysis and forecasting must be estimated in three dimensions of time. There are three types of classification methods based on time, which are short-term forecasting, medium-term forecasting, and long-term forecasting [6]. According to researchers, short-term forecasting is used in forecasting on a daily, weekly, and monthly basis, such as the market model [27], whose typical volatility is forecasted by using GARCH (1,1) model [4,17].

Modeling autocorrelation in daily and weekly frequency data in the last decades are being used for ARCH/GARCH models [5], but more recent research has improved the volatility estimations and jump estimates by using these models. Researchers also found it useful to incorporate information about periodic volatility patterns and macroeconomic announcements in their calculations.

In financial investment field, there is a correlation between volatility in the capital market and high uncertainty of return, known as “Risk and Return Tradeoff”-phenomenon. Consequently, for the low volatility share prices, investors must hold the share as a long-term investment in order to receive capital gain. When the daily volatility of a share price is high, meaning that the price fluctuates, it provides trading opportunities for investors who can gain from the differences in the opening and closing share prices, also known as “High Risk High Return” [14]. Volatility is also considered fundamental to asset pricing and important information for investment [33]. According to Blaskowitz and Herwartz [20] in the benchmark models, like Euribor, the adaptive approach offers additional forecast accuracy in

terms of directional accuracy and directional forecast value.

According to a study by the European Central Bank conducted by researchers Ivanova and Gutiérrez [31], it was analyzed that the option-implied interest rate forecasts and the development of risk premium and state prices are correlated with the Euribor futures options market. They found out that the real-world option-implied distributions can be used to forecast the futures rate, while the forecasting ability of the risk-neutral distributions is rejected. Also, a negative market price of interest rate risk is documented, which generates a positive premium for the futures contract.

Other authors like Pelizzon and Sartore [18] concluded that the Euribor rates cannot be used anymore as a benchmark for all market rates except credit risk indicators. They studied that based on credit risk and liquidity tensions in the short-term securities market, the Euribor rates are dynamically and largely unrelated to the Central Banks target rates. In accordance with the importance of Euribor volatility, Alfred [32] found out that the Euribor-Overnight Indexed Swap (credit risk is not a major factor in determining the OIS rate) spread incorporates rich information regarding future FX market uncertainty. He expressed “this result supports the view that adverse information flow over the sample period is transitory, suggesting that market participants are mainly concerned about currency jumps during periods surrounding the crises, and prior crises jump are generally ignored.”

The importance of Euribor volatility estimation is widely viewed as a risk indicator of financial distress associated with insolvency within the interbank lending market [15]. Therefore, this study motivates the estimation of the Euribor volatility, as for many years it has been performing with negative rates, causing the reduction of the monetary policy efficiency.

3. Research Methodology

Compounded rate r_t (as a continuous estimation) evaluates the return from time $t - 1$ to time t . The volatility of a variable is its standard deviation. Performing the annual standard deviation of the compounded returns, two main assumptions need to be emphasised:

The first assumption is that interest rates are not correlated over time or that the weak form of efficient market hypothesis approximately holds, i.e., the interest rates are not predictable from past interest rates. In this case, the variance over n days equals n times variance of one day. In particular, if we want to compute variance for one year we get *annualized variance*= $252 \cdot \text{daily variance}$. Equivalently we get the square root of the variance to get the standard deviation or volatility of an asset.

The second assumption is that the expected value of the interest rates equals zero. We make this assumption of zero mean return in the calculation of standard deviation for a short period of time.

$$\sigma^2 = \frac{\sum_{t=1}^n (r_t - E(r))^2}{n} \quad \text{where } E(r) = \frac{\sum_{t=1}^n r_t}{n} \rightarrow 0$$

where we get,

$$\sigma^2 = \frac{\sum_{t=1}^n (r_t)^2}{n} \tag{1}$$

3.1. Efficient Market Hypothesis (EMH)

The weak form of efficient market hypothesis (EMH) indicates that interest rates are almost unpredictable from their past history [21]. There are many tests of EMH in the academic literature and below we will perform one simple test for correlation between interest rates at time t and past interest rates at times $t - 1, t - 2, \dots, t - k$. Correlation for a variable with its lags is called autocorrelation. The null hypothesis: $H_0: \rho = 0$ (no autocorrelation up to order k , lags = k), which means the market is efficient. We are going to perform Ljung Box Q test for this purpose. The Q statistics are based on the normalized sum of squared autocorrelations, which has chi-squared distribution. Note that the underlying assumption of the Q test under the null hypothesis is the independent identical distribution (iid) for the interest rates:

$$Q = n(n+2) \sum_{j=1}^k \frac{\rho_j^2}{n-j} \tag{2}$$

Where $\rho_j = \text{correl}(r_t, r_{t-j})$ with n data sample.

3.2. GARCH Volatility Model

When performing volatility calculations σ_t at time t , the simplest method is to apply equal weight for each term $r_{t-1}^2, r_{t-2}^2, \dots, r_{t-k}^2$. The equation would be:

$$\sigma_t^2 = \sum_{i=1}^k \alpha_i r_{t-i}^2$$

The variable α_i is the given weight at time i . It is understandable that α is positive. If we choose $\alpha_i < \alpha_j$ when $i > j$, less weight is applied to earlier observations. Whatever method is used to estimate the weights, their overall sum should be: $\alpha_1 + \alpha_2 + \dots + \alpha_k = 1$. Assuming an average long-run variance rate σ^2 exists, we get:

$$\sigma_t^2 = \gamma \sigma^2 + \sum_{i=1}^k \alpha_i r_{t-i}^2$$

Where γ is the run variance parameter. Consequently,

we arrive at the equation: $\gamma + \sum_{i=1}^k \alpha_i = 1$.

The variance estimation is based on long-term variance and p variables that are time lags. This model was first introduced as the ARCH (p) model. One of the most widely used models in risk management is the GARCH model, but this model is understood if we first know the ARCH model. ARCH (autoregressive conditional heteroscedasticity model) makes the prediction of time variance based on information obtained from daily squared interest rates (in our case is Euribor rates). The ARCH(p) model is:

$$\sigma_t^2 = w + \sum_{i=1}^p \alpha_i r_{t-i}^2 \tag{3}$$

Where w is the constant parameter of the variance according to this model. The model includes an autoregressive structure in the form of a regression based on past square return observations. This model is conditioned by past information and by the variance of returns which varies with respect to time. So, the variance of the dependent variable is a function of the retrospective values of the dependent variable, or exogenous variables. The generalized ARCH form is the GARCH model (generalized ARCH). Forecasting variance at time t is the weighted average of the long-term variances, i.e., from the forecasting variances and information on the squared interest rates.

Preposition 1. The general form of the GARCH(p,q) model estimates that the parameter " p " is ARCH(p), while the parameter " q " shows that we have lag = q of the variances:

$$\sigma_t^2 = w + \sum_{i=1}^p \alpha_i r_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \tag{4}$$

Where β shows the coefficient of variables (according to the model are lag-variances) in the change of current variance. The main properties of GARCH(p,q) coefficients to satisfy a good model are:

- (1) Positivity $w > 0$; $\alpha_i \geq 0$, for $i = 1, 2, \dots, p$; and $\beta_j \geq 0$ for $j = 1, 2, \dots, q$.
- (2) Stability or mean reversion:

$$\sum_{i,j=1}^{\max\{p,q\}} (\alpha_i + \beta_j) < 1$$

If model coefficients are good (respectively with the two conditions above), the equilibrium level of variance is:

$$\sigma^2 = \frac{w}{1 - \sum_{i,j=1}^{\max\{p,q\}} (\alpha_i + \beta_j)} \tag{5}$$

In general, a GARCH(1,1) model forecasting will be sufficient to capture the volatility clustering in the daily

data, and rarely is any higher-order model estimated or even entertained in the academic finance literature [3,8].

Lemma 1. The GARCH(p,q) model is an extension of the ARCH(p) and EWMA (Exponentially weighted moving average) models.

Proof: The simplest form of the GARCH(p,q) model is GARCH(1,1):

$$\sigma_t^2 = w + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Based on stability of the coefficients, the variance σ_t^2 is calculated with the long-run variance σ^2 :

$$\sigma_t^2 = \gamma \sigma^2 + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

Where the parameters α , β and γ are also the weights of variation respectively according to the variances in the model, $\alpha + \beta + \gamma = 1$. If $\gamma = 0$, $\alpha = 1 - \lambda$ and $\beta = \lambda$ (λ is exponential smoothing parameter, so the weight λ to previous day variance and $(1 - \lambda)$ to previous daily squared return). Note $w = \gamma \sigma^2$ the model stand in the initial form GARCH(1,1). Using GARCH(1,1) model, we can perform $\sigma_{t-1}^2 = w + \alpha r_{t-2}^2 + \beta \sigma_{t-2}^2$ and we replace this expression in the initial model:

$$\sigma_t^2 = w + \alpha r_{t-1}^2 + \beta (w + \alpha r_{t-2}^2 + \beta \sigma_{t-2}^2)$$

or,

$$\sigma_t^2 = w + \beta w + \alpha r_{t-1}^2 + \alpha \beta r_{t-2}^2 + \beta^2 \sigma_{t-2}^2$$

In the same way, we can perform σ_{t-2}^2 and we replace this in the expression above:

$$\sigma_t^2 = w + \beta w + \beta^2 w + \alpha r_{t-1}^2 + \alpha \beta r_{t-2}^2 + \alpha \beta^2 r_{t-3}^2 + \beta^3 \sigma_{t-3}^2$$

Continuing in this way, we see that the weights applied to r_{t-i}^2 is $\alpha \beta^{i-1}$. This means that the weight value decreases exponentially with parameter β . This process is similar to the λ parameter in the exponentially weighted moving average model. Doing these successive substitutions we conclude that the general form of the GARCH(p,q) model has ARCH(p) and lag = q variances. GARCH(p,q) model can include other independent variables in this form:

$$\sigma_t^2 = w + \sum_{i=1}^p \alpha_i r_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \pi Z_t \tag{6}$$

Where Z_t is a vector of exogenous independent variables that could be included in the model [12].

3.3. GARCH Model Testing

GARCH(p,q) model testing is performed by maximum likelihood method.

Proposition 2. When we have k observations of interest rates r such that $r_{t-1}, r_{t-2}, \dots, r_{t-k}$ with expected value $\mu = 0$ and variance σ^2 the likelihood of r_{t-i} is density function:

$$f(r_{t-i}, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\left(\frac{-r_{t-i}^2}{2\sigma^2}\right)} \tag{7}$$

Lemma 2: The efficient estimation of GARCH(p,q) model is the maximum log-likelihood method.

Proof: According to proposition 2, it can be generalized, so the likelihood of k observations is:

$$\prod_{i=1}^k \left[\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\left(\frac{-r_{t-i}^2}{2\sigma^2}\right)} \right]$$

Maximum likelihood method is used. The best estimation of the variance is the maximum of expression. Maximizing this expression is equivalent to maximizing its logarithm:

$$\sum_{i=1}^k \left[-\ln(\sigma^2) - \frac{r_{t-i}^2}{\sigma^2} \right] \text{ or } k \cdot \ln(\sigma^2) - \sum_{i=1}^k \frac{r_{t-i}^2}{\sigma^2}$$

Deriving this expression in relation to the variance and finding the point that the derivative equals to zero:

$$\sigma_t^2 = \frac{1}{k} \sum_{i=1}^k r_{t-i}^2$$

This expression shows the variance of the k returns or the best quantitative assessment of volatility like the assumption 2, (1). Lemma 2 is true for any random interest rate Y which is a linear combination of the random interest rate X with normal distribution.

Lemma 3. If X is a random variable with normal distribution with parameters $N(\mu, \sigma^2)$, then $Y = aX + b$ has normal distribution as well, with parameters $N(a\mu + b, a^2\sigma^2)$.

Proof: Assume that $a > 0$ (in the same way we prove the function for the inverse form) and let F_Y be the probability accumulation function of Y , then:

$$F_Y(x) = P(Y \leq x) = P(aX + b \leq x) = P\left(X \leq \frac{x-b}{a}\right) = F_X\left(\frac{x-b}{a}\right)$$

Deriving both sides of this equation, we have:

$$f_Y(x) = \frac{1}{a} f_X\left(\frac{x-b}{a}\right) = \frac{1}{a\sigma\sqrt{2\pi}} e^{\left[\frac{-\left(\frac{x-b}{a}-\mu\right)^2}{2\sigma^2}\right]}$$

or normal distribution:

$$f_Y(x) = \frac{1}{a\sigma\sqrt{2\pi}} e^{\left[\frac{-(x-(a\mu+b))^2}{2(a\sigma)^2}\right]}$$

showing that Y is of normal distribution with the parameters $N(a\mu + b, a^2\sigma^2)$.

In many statistical software GARCH(p,q) model testing is performed by maximum log-likelihood method. In

order to find optimal number of lags for fitting the model to data, information criteria are commonly used. In addition to the estimates of coefficients and other statistics the AIC (Akaike Information Criterion) and SIC (Schwarz Information Criterion) are reported. When GARCH models are estimated the method is based on maximizing the log-likelihood function for observing actual returns in the data:

$$\log-Likelihood(w, \alpha, \beta) = -\frac{1}{2} \sum_{t=1}^T \left(\log(\sigma_t^2) + \frac{r_t^2}{\sigma_t^2} \right) \quad (8)$$

Thus, models which achieve the highest value of the log-likelihood are preferred. But if models have different numbers of parameters there needs to be some adjustment or penalty for a number of estimated parameters in order to avoid overfitting. The AIC and SIC information criteria are computed based on the negative log-likelihood with an added penalty for the number of parameters included in the model. The smaller the information criterion, the better the model. Moreover, the SIC criterion imposes a higher penalty for extra parameters ; and based on this criterion, a simpler (more parsimonious) model may be selected compared to other information criteria [23].

3.4. Forecasting with GARCH Model

Forecasting out-of-sample for volatility for 2-day based on information up to time t is based on the following formula:

$$\begin{aligned} \sigma_{t+2|t}^2 &= w + \alpha E(r_{t+1}^2) + \beta \sigma_{t+1}^2 + w + (\alpha + \beta) \sigma_{t+1}^2 = \\ &= \sigma^2 + (\alpha + \beta)(\sigma_{t+1}^2 - \sigma^2) \end{aligned}$$

Here $E(r_{t+1}^2)$ is conditional forecast (expectation) of r_{t+1}^2 based on information at time t. This is the same as conditional variance forecast σ_{t+1}^2 . Instead of w we plug in its expression through the long-run unconditional variance $w = \sigma^2 - (\alpha + \beta)\alpha^2$. Using similar re-arrangement, we can derive a formula for k-day forecast conditional on time t:

$$\sigma_{t+k|t}^2 = \sigma^2 + (\alpha + \beta)^{k-1}(\sigma_{t+1}^2 - \sigma^2) \quad (9)$$

When k is large the forecast converges to long run unconditional variance σ^2 if the model is stationary and $(\alpha + \beta) < 1$.

4. Empirical Analysis and Findings

The Euribor data series is taken from official Thomson Reuters daily publications [34], (from January 2015 to May 2021), the time when Euribor rates started performing on

negative values). The data is processed in R programming and final results are presented in the following tables.

Table 1 summarizes the descriptive statistics where we can notice the Euribor rates - time series for all five maturities which have a normal distribution.

Table 1. Description statistics and normal distribution test

(in %)	Weekly	Monthly	Quarterly	Semi-annual	Annual
Min.	-0.579	-0.582	-0.556	-0.534	-0.515
1st Qu.	-0.484	-0.443	-0.375	-0.323	-0.242
Median	-0.379	-0.371	-0.323	-0.257	-0.148
Mean	-0.373	-0.355	-0.302	-0.236	-0.141
3rd Qu.	-0.373	-0.366	-0.279	-0.170	-0.050
Max.	-0.004	0.016	0.076	0.169	0.323
J-Bera Test	Normal	Normal	Normal	Normal	Normal

While table 2 shows the correlation matrix of Euribor rates, by statistical testing results that each pair-correlation is statistically significant with the statistical significance p < 1%. This also shows a correlation trend where all rates converging together.

Table 2. Correlation matrix of Euribor rates by maturity

	1W	1M	3M	6M	12M
1W	1.0000				
1M	0.9982	1.0000			
3M	0.9949	0.9981	1.0000		
6M	0.9913	0.9949	0.9986	1.0000	
12M	0.9854	0.9893	0.9944	0.9983	1.0000

Note: Maturity 1W, 1M, 3M, 6M, 12M respectively weekly, monthly, quarterly, semi-annual, annual.

Ljung-Box test is applied as shown in table 3. None of the interest rates satisfy the weak form of the efficient market hypothesis, even though statistical control is done with lag = 0, 1, 2, ..., 30.

Table 3. Summary of the efficient market hypothesis (EMH)

Euribor rate maturity	Ljung Box test	Result
Weekly	Q > 0	No efficient
Monthly	Q > 0	No efficient
Quarterly	Q > 0	No efficient
Semi-annual	Q > 0	No efficient
Annual	Q > 0	No efficient

Note: All Euribor rates are not market efficient, checked by lag = 30.

According to the statistical estimation of the GARCH(p,q) model for p = 1 and 2 as well as q = 1 and 2, the results are given for the five Euribor rates, in tables 4, 5, 6, 7, and 8 respectively.

Table 4. GARCH(p,q) for Euribor rate with weekly maturity

	ARCH(1)		ARCH(2)	
	GARCH(1,1) model		GARCH(2,1) model	
Variance lag(1)	w	1.3834	w	1.38*10 ⁻⁷
	α ₁	0.5276*	α ₁	0.5279*
	α ₂	---	α ₂	1*10 ⁻⁸
	β ₁	0.4734*	β ₁	0.4734
	β ₂	---	β ₂	---
	AIC	-5.4373	AIC	-5.4359
Variance lag(2)	GARCH(1,2) model		GARCH(2,2) model	
	w	1.65*10 ⁻⁷	w	1.65*10 ⁻⁷
	α ₁	0.6211*	α ₁	0.6211*
	α ₂	---	α ₂	1*10 ⁻⁸
	β ₁	0.2016*	β ₁	0.2016
	β ₂	0.1753*	β ₂	0.1753
AIC	-5.4392	AIC	-5.4380	

Note: * is statistical significance less than 1% and ** is statistical significance less than 5%.

According to the estimation in table 4, all models satisfy the positivity condition, while GARCH(1,2) and GARCH(2,2) also satisfy the stability condition, so the best fitting of these two models is the one with minimum AIC i.e. GARCH(1, 2). It is also highlighted by its statistical significance. The general form of the optimal GARCH(1,2) model, in this case, is:

$$\sigma_t^2 = 0.000000165 + 0.6211r_{t-1}^2 + 0.2016\sigma_{t-1}^2 + 0.1753\sigma_{t-2}^2$$

Table 5. GARCH(p,q) for Euribor rate with monthly maturity

	ARCH(1)		ARCH(2)	
	GARCH(1,1) model		GARCH(2,1) model	
Variance lag(1)	w	3.55*10 ⁻⁷	w	3.55*10 ⁻⁷
	α ₁	0.6182*	α ₁	0.6185*
	α ₂	---	α ₂	1*10 ⁻⁸
	β ₁	0.3649**	β ₁	0.3649
	β ₂	---	β ₂	---
	AIC	-5.2189	AIC	-5.2176
Variance lag(2)	GARCH(1,2) model		GARCH(2,2) model	
	w	4.16*10 ⁻⁷	w	4.16*10 ⁻⁷
	α ₁	0.7121*	α ₁	0.7121*
	α ₂	---	α ₂	1*10 ⁻⁸
	β ₁	0.1328**	β ₁	0.1328
	β ₂	0.1336**	β ₂	0.1336
AIC	-5.2201	AIC	-5.2189	

Note: * is statistical significance less than 1% and ** is statistical significance less than 5%.

According to the estimation in table 5, all models satisfy the positivity condition and the stability condition, so the best fitting among these models is the one with minimum AIC i.e. GARCH(1, 2). The Other models have statistically insignificant variables and the more alpha (or ARCH (p)) or beta (or variance lag (q)) increases the less significant they become. The general form of the optimal GARCH(1,2) model, in this case, is:

$$\sigma_t^2 = 0.000000416 + 0.7121r_{t-1}^2 + 0.1328\sigma_{t-1}^2 + 0.1336\sigma_{t-2}^2$$

Table 6. GARCH(p,q) for Euribor rate with quarterly maturity

		ARCH(1)	ARCH(2)
		GARCH(1,1) model	GARCH(2,1) model
Variance lag(1)	w	2.06*10 ⁻⁷	w 2.06*10 ⁻⁷
	α ₁	0.7628*	α ₁ 0.7634*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	0.2405*	β ₁ 0.2403*
	β ₂	---	β ₂ ---
	AIC	-4.3037	AIC -4.3022
		GARCH(1,2) model	GARCH(2,2) model
Variance lag(2)	w	2.26*10 ⁻⁷	w 2.26*10 ⁻⁷
	α ₁	0.8275*	α ₁ 0.8275*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	0.1180*	β ₁ 0.1180*
	β ₂	0.5611	β ₂ 0.5611
	AIC	-4.3043	AIC -4.3031

Note: * is statistical significance less than 1% and ** is statistical significance less than 5%.

According to the estimation in table 6, all models satisfy the positivity condition but not the stability condition, so none of them is an optimal model. We get the same conclusion from table 7 and table 8.

Table 7. GARCH(p,q) for Euribor rate with semi-annual maturity

		ARCH(1)	ARCH(2)
		GARCH(1,1) model	GARCH(2,1) model
Variance lag(1)	w	4.52*10 ⁻⁷	w 4.51*10 ⁻⁷
	α ₁	0.7199*	α ₁ 0.7204*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	0.2931*	β ₁ 0.2930*
	β ₂	---	β ₂ ---
	AIC	-3.3635	AIC -3.3621
		GARCH(1,2) model	GARCH(2,2) model
Variance lag(2)	w	4.74*10 ⁻⁷	w 4.74*10 ⁻⁷
	α ₁	0.7532*	α ₁ 0.7532*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	0.1838**	β ₁ 0.1838
	β ₂	0.0739	β ₂ 0.7396
	AIC	-3.3630	AIC -3.3618

Note: * is statistical significance less than 1% and ** is statistical significance less than 5%.

Table 8. GARCH(p,q) for Euribor rate with annual maturity

		ARCH(1)	ARCH(2)
		GARCH(1,1) model	GARCH(2,1) model
Variance lag(1)	w	1.08*10 ⁻⁶	w 1.08*10 ⁻⁶
	α ₁	0.8848*	α ₁ 0.8858*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	0.1323**	β ₁ 0.1322*
	β ₂	---	β ₂ ---
	AIC	-2.2438	AIC -2.2421
		GARCH(1,2) model	GARCH(2,2) model
Variance lag(2)	w	1.23*10 ⁻⁶	w 2.12*10 ⁻⁷
	α ₁	0.9529*	α ₁ 0.8419*
	α ₂	---	α ₂ 1*10 ⁻⁸
	β ₁	1*10 ⁻⁸	β ₁ 1*10 ⁻⁸
	β ₂	0.0634	β ₂ 0.1805*
	AIC	-2.2438	AIC -2.2458

Note: * is statistical significance less than 1% and ** is statistical significance less than 5%.

Based on the research methodology, we expressed that the GARCH(p,q) model can get values of p and q greater than 2. The reason why we were satisfied with this length $p = 2$ and $q = 2$, is the positivity condition and stability condition (preposition 1) which defines when a model is called good fitting (or optimal). Consequently, if a model is not optimal in small values of p and q it will not be optimal even in its higher-order (larger values). So, it is obviously based on the stability condition (if it is rejected, it will never change with increasing p and q). Moreover, during the statistical tests with the increasing length of p and q , we figured out that the statistical significance of the model parameters decreased. It is the same as the financial literature argues.

Table 9. Summary results for the long-run equilibrium forecasting

Euribor rate maturity	The best model	Long-run
Weekly	GARCH(1,2)	14.4%
Monthly	GARCH(1,2)	7.0%
Quarterly	No exist	No exist
Semi-annual	No exist	No exist
Annual	No exist	No exist

Note: Long-run is the annualized variance.

Table 9 shows the information, which are the optimal models used for forecasting Euribor rates (according to the respective maturity) and the level of long-run equilibrium. What we found out is that by the increasing maturity (over a month) the interest rate becomes more fluctuating and unstable. The matter of fact is that many financial instruments indexed at these Euribor rates have an increased risk during the time. In addition, more worrying is the fact that the monetary policy of the European Bank is losing its function more and more without being able to identify an optimal threshold of the rate reductions.

5. Conclusion

From January 2015 until now, Euribor rates have been performing at negative rates. Lately, as a consequence of Covid-19 pandemic, the rates declined sharply, by presenting new challenges for the Eurozone in the monetary policies related to the lower rate bound in these economies; and relative incentives toward domestic and foreign currency denominated assets and liabilities for currencies other than euro.

This study has analyzed the volatility of the Euribor rates related to efficient market hypothesis and modeling volatility forecasting in the long-run equilibrium, by relying on daily data from 2015 to 2021. Ljung-Box discovered out that none of the interest rates (weekly, monthly, quarterly, semi-annual and annual) satisfy the weak form of the efficient market hypothesis, even though statistical control will be with lag = 0, 1, 2, ..., 30, so the

autocorrelation of these series is a phenomenon that tends them.

The best estimator of volatility is GARCH(p,q) model which is the statistical technique to figure out if long-run equilibrium exists and its value. The most efficient estimation (optimal model) among the GARCH(p,q) models is the one which uses the maximum log-likelihood method.

When testing the GARCH(p,q) model, we discovered that two optimal GARCH(1,2) models exist for the weekly and monthly maturity rates, with an annualized volatility of 14.4% and 7% respectively. We can forecast the optimal value with these two Euribor rates, but what we figured out is that the largest weight of the parameter in the optimal model is related to the rate squared α not the volatility of the previous day.

All in all, our optimal GARCH(p,q) model could be used for forecasting rates, but when forecasting out-of-sample for volatility is longer than the following day, based on information at time t , the variances of the values get larger and larger. Also, during the statistical tests with the increasing length of p and q , we figured out that the statistical significance of the model parameters decreased.

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