

# Anisotropic Deformation Model for Hawkesbury Sandstone Incorporating Inherent Mobilised Shear Strength

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Received December 3, 2020; Revised July 26, 2021; Accepted August 9, 2021

## Cite This Paper in the following Citation Styles

(a): [1] Noorfaizah Hamzah, Nur Ain Mat Yusof, Mohd Jamaludin Md Noor, "Anisotropic Deformation Model for Hawkesbury Sandstone Incorporating Inherent Mobilised Shear Strength," *Civil Engineering and Architecture*, Vol. 9, No. 5A, pp. 93 - 100, 2021. DOI: 10.13189/cea.2021.091311.

(b): Noorfaizah Hamzah, Nur Ain Mat Yusof, Mohd Jamaludin Md Noor (2021). *Anisotropic Deformation Model for Hawkesbury Sandstone Incorporating Inherent Mobilised Shear Strength*. *Civil Engineering and Architecture*, 9(5A), 93 - 100. DOI: 10.13189/cea.2021.091311.

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**Abstract** An extensive study on modelling progressive Hawkesbury Sandstone degradation by anisotropic deformation subjected to monotonic loadings is presented and discussed in this study. Hawkesbury Sandstone was used due to its unique behaviour, which was initially assumed to be a uniform testing lithology with respect to grain size, compressive strength and stiffness. This study identified a theoretical approach to the anisotropic model of rock deformation. The model utilizes the stress-strain curve to derive the development of mobilised shear strength and applies it to the rock to simulate how it compresses in anisotropic. Monotonic loading tests were performed in triaxial conditions at variations of confining pressure, 4MPa and 8MPa. An increment of confining pressure was used to obtain elevation in the stress-strain curve. Progressive monotonic loading changed the mechanical characteristics of the rock; the level of the applied stress is compressed axially and then expanded laterally. During applied loading, the rock may experience damage or rock failure; the correlation between the magnitude of the mobilised shear strength and the axial stress associated with it is regarded as an intrinsic property in the rock mass. The stress-strain behaviour of rocks under anisotropic stress conditions can be predicted using this method. The mobilised minimum friction angle is used to determine the location of the mobilised shear strength

envelope. The results reveal that the mobilised intrinsic shear strength is developed if the rock is forced to compaction. This is evidenced by the envelope rotating upward to the shear force envelopes during failing. Consequently, it can be deduced that the cementation ( $c'$ ) of the rock increases.

**Keywords** Hawkesbury Sandstone, Mobilised Shear Strength, Monotonic Load, Stress-Strain, Shear Strength Envelope

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## 1. Introduction

Rocks can be a challenge as they included different types of inhomogeneities and discontinuities. In recent rock study, more emphasis is placed on rock behaviour that is subject to loading under varying loading conditions, in various stress-strain rates and pattern of behaviour that emerges after rock failure. The study of rock behaviour under different confining pressure is to demonstrate the application of loading condition during construction activity. Due to the fact that the intrinsic attributes of the rock are founded through loading, the rock may fail when it reaches the monotonic failure stress. Such mechanisms

may erode the stability of geomaterials and excavation boundaries, posing a significant risk of failure. Therefore, prediction of the peak strength is a certain step in order to avoid those hazards from happening. The deformation of evaporating sedimentary rock under cyclic loading needs further studies, which were based on their findings, during monotonic stress, the peak strengths of their rock samples decreased to varying degrees under cyclic loading. [1].

The damage or failure of faulted rock slope is a quite difficult problem in geotechnical engineering, and attention has been drawn from researchers for a long time. Under static loadings such as gravity or engineering building, the rock material within a certain zone behind the tip of a crack/fault will behave strain-softening property due to the action of stress erosion or chemical erosion.

Anisotropy is an essential characteristic that should be paid close attention to in rock engineering, regardless of the application. Mostly due to the presence of cleavage, foliation, bedding planes, schistosity, joints, and micro or macro fractures, the anisotropy of rock properties can be ascribed to anisotropy [2].

In this study, Hawkesbury sandstone was used, which

belongs to the catalogue sedimentary rock. The Hawkesbury sandstone is chosen as it was initially assumed to be a uniform testing lithology with respect to grain size, compressive strength, and stiffness. During uniaxial testing, the discrepancy between sample strength was observed due to the heterogeneity of Hawkesbury Sandstone. Therefore, the monotonic test is generally used with a condition, which, by assuming entire samples to be tested, are identical and have used average peak strength which were obtained from the monotonic test as the peak value for every Hawkesbury Sandstone specimen tested. The assessment of the Hawkesbury sandstone in terms of the stress-strain curve for primary loading testing has been studied by Taheri et al. [3].

Based on the previous researcher, the rock condition has been changed by the weathering process. Some unnatural characteristics of rock engineering in comparison with the fresh rock to residual soil are shown [4]. Physically, sandstone rock is considered as soft rock, and sandstone was formed when sand is buried under successive layers of sediment. The following Table 1 compares the physical characteristics of sandstone and shale [5].

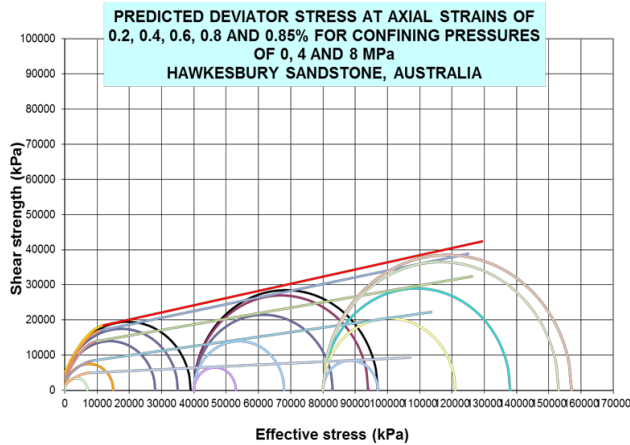
**Table 1.** Comparative of physical properties of weathered sandstone and shale of Kenny hill formation (Mohamed et al., 2007) [5]

Rock samples	Hardness (hand sample)	Rebound hardness	Surface texture and lamination	Dry density kN/m <sup>3</sup>	Slake
Slightly weathered Sandstone (BP2g, BP2s)	Very hard and intact	> 30	Smooth and intact	2.4 – 2.6	98
Moderately weathered Sandstone (BP3) BP4	Hard and slightly disintegrate	5-30	Slightly massive and gritty	2.2 – 2.7	94
Highly weathered Sandstone (BP5)	Breakable and easily disintegrate	10-15	Partially massive and gritty	2.0 – 2.7	-
Slightly weathered shale (S2)	Easily broken and disintegrate	dented	Very gritty and massive	1.5 – 1.9	46
Moderately to Highly weathered shale	Hard and slake	< 20	Soapy and intact	2.4 – 2.5	92
S5	Breakable and easily slake	10-20	Powdery and slightly massive	2.2 – 2.4	96
S5a	Breakable and slake	dented	Powdery, gritty and massive	2.5 – 2.6	92





width of the Mohr–Circle increases, the deviator stress increases. This is an equilibrium state, and the rock mass has generated a higher mobilised shear strength as a result. Different types of rock will exhibit a range of mobilised shear strength envelopes and shear strength envelopes at failure. As a result, it demonstrated the unique property of the rock.



**Figure 5.** Hawkesbury Sandstone axial stress envelopes with mobilised shear strength

To compare stress-strain curve predictions to experimental results, the normalised strain must be multiplied by the inverse factor calculated using Equation 2. Table 3 presents the complete prediction values. Figure 6 shows the projected stress-strain curve superimposed on the actual lab stress-strain curve of Hawkesbury Sandstone. The estimated stress-strain magnitudes for confining pressures of 0, 4, and 8 MPa are shown in Table 3 and plotted as stress-strain curves. In comparison to the charts obtained from the laboratory tests, a very closely predicted stress response had been achieved.

$$\text{Normalised inverse factor} = \frac{\text{Axial strain (\%)} \text{ at failure under consideration}}{\text{Maximum axial strain (\%)} \text{ at failure}} \quad (2)$$

**Table 3.** Mobilized stress strength envelope for different confining pressures

0 MPa			4 MPa			8 MPa	
Axial Strain (%)	Axial Strain (x inverse factor)	Deviator stress (MPa)	Axial Strain (%)	Axial Strain (x inverse factor)	Deviator stress (MPa)	Axial Strain (%)	Deviator stress (MPa)
0	0	0	0	0	0	0	0
0.2	0.12	7	0.2	0.17	13	0.2	17
0.4	0.23	15	0.4	0.34	28	0.4	41
0.6	0.35	28	0.6	0.51	43	0.6	58
0.8	0.46	35	0.8	0.68	54	0.8	73
0.85	0.49	39	0.85	0.72	57	0.85	77

The advantages that drive deformations and resisting variables are incorporated in the deformation framework. The framework is, moreover, derived from the actual behaviour of stress. Table 4 detailed the projection of Hawkesbury Sandstone stress-strain curves for the respective axial strain (0.2, 0.4, 0.6, 0.8, and 0.85 per cent). This process is repeated for all mobilised shear resistance envelopes and the connection between the deviating stress and the corresponding axial stress that may comprise the prediction of straining relationships for pressure-confining differences of Hawkesbury Sandstone as shown in Figure 6.

**Figure 6.** Predicted stress-strain curve superimposed on the Hawkesbury Sandstone's actual laboratory stress-strain curve

**Table 4.** Mobilized stress strength envelope for different confining pressures

Axial Strain (%)	Oo at 0MPa	Oo at 4MPa	Oo at 8MPa
0.2	7	13	17
0.4	15	28	41
0.6	28	43	58
0.8	35	54	73
0.85	39	59	77.5

### 2.3.1. Monitoring of Specimen, Vertical and Lateral Displacement during Shearing

The vertical compression of the specimen and the lateral dilation during shearing are monitored using LVDT's assembled, as shown in Figure 7. The latter substantiated that there is consideration of anisotropic condition. The graphically relationships between the stress and strain of each confined rock specimen in terms of the axial strain, lateral strain, LVDT, and volumetric strain would be exclusively highlighted in the research. The volumetric strain referred to the summations of the average of axial strain, and the doubled of average lateral strains recorded by the strain gauge. The volumetric strain of each specimen can be calculated using Equation 3, which is shown as follows:

$$\epsilon_{\text{volumetric}} = \epsilon_{\text{axial}} + 2 \epsilon_{\text{lateral}} \quad (3)$$

Where,

$\epsilon_{\text{volumetric}}$  = Volumetric strain

$\epsilon_{\text{axial}}$  = Average axial strain recorded from two axial

strain gauges

$\epsilon_{lateral}$  = Average lateral strain recorded from two lateral strain gauges

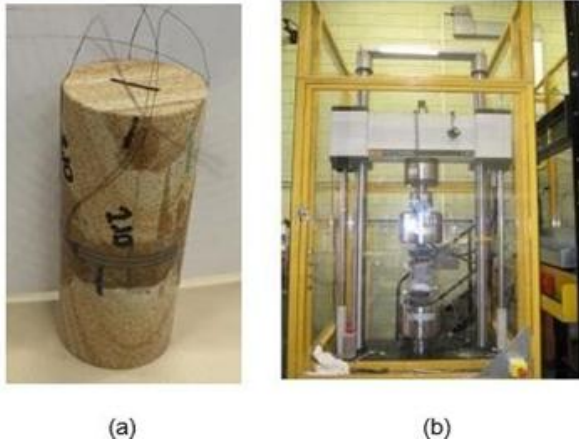


Figure 7. (a) Strain Gauge (b) Measurement of LVDT

In addition to that, the stress-strain graphs in this section show plots of the axial strain measured by the LVDT, the axial and lateral strain recorded by the gauges as well as the volumetric strain. Strain gauges are an accurate means of measuring the strain that each rock specimen is under during testing compared to the strain measured from the LVDT. Unfortunately, sometimes gauges are not always reliable as failure occurs commonly even after protection methods have been applied.

Figure 8 shows the relationship of volumetric strain with the axial and lateral strain of the specimen for this research. Figure 8(a) illustrates similar results for the uniaxial compressive loading test, while both Figure 8(b) and Figure 8(c) are demonstrating similar results for 4 MPa and 8 MPa confinements, respectively. In the beginning, the specimen exhibited volumetric compaction followed by volumetric dilation. The volumetric strain decrease with axial and lateral strain after the peak strength is reached. This could be explained by the fact that volumetric compaction induces the hardening of the specimen, thus incurring the crack damage.

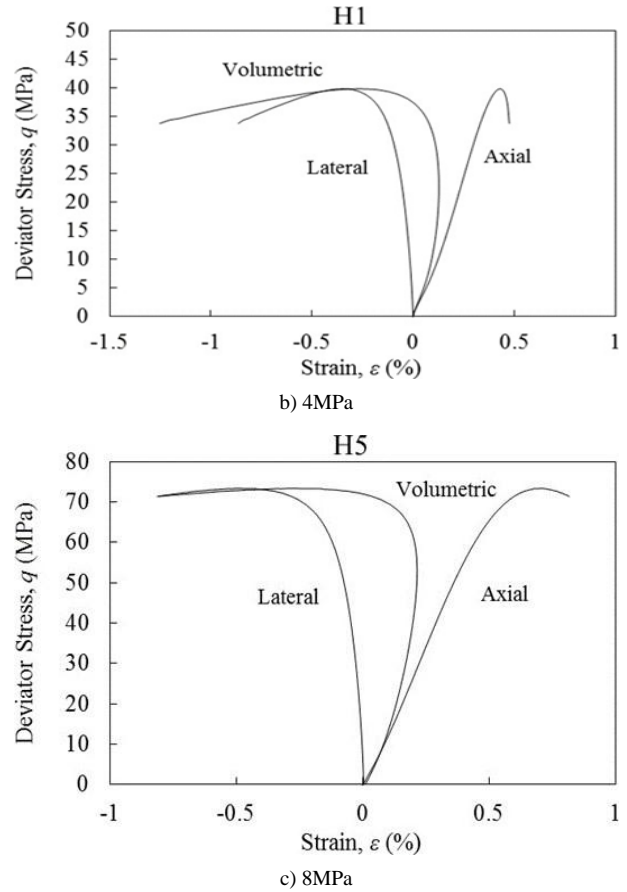
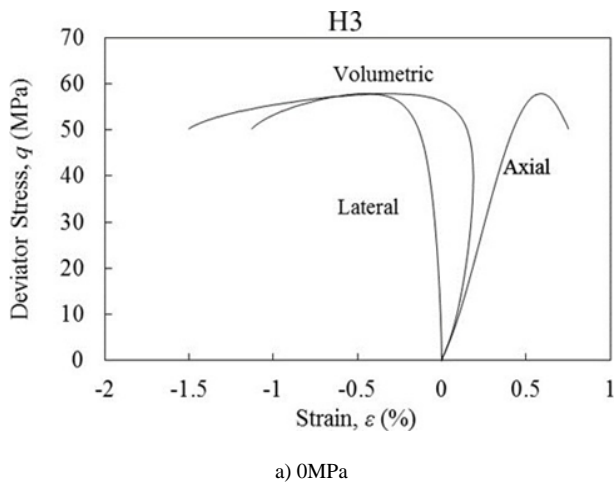


Figure 8. Volumetric strain relation with axial and lateral strain for a) 0 MPa, b) 4 MPa and c) 8 MPa

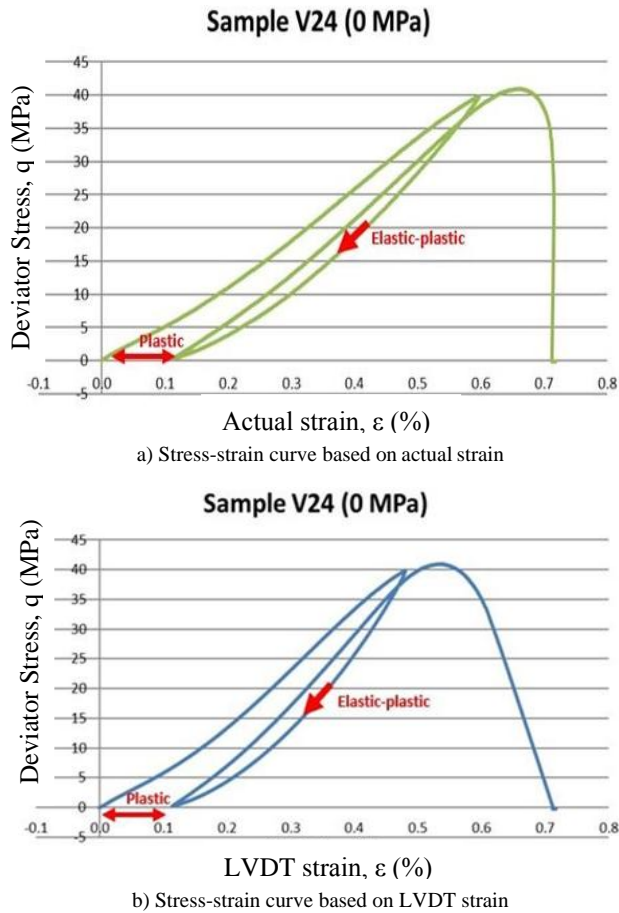
Md Noor and Jobli [10] stated a new theoretical approach for calculating anisotropic rock deformation models had been developed, and the model incorporates the stress-strain curves for rock in isotropic compression to derive the mobilisation of shear strength. The relationship between the position of the mobilised shear strength envelopes and the axial strain is regarded as an inherent property of the rock mass, and it is used to predict the rock stress-strain response under anisotropic stress conditions. In order to identify the location of the mobilised shear strength envelopes, a pilot run is performed, during which the mobilised minimum friction angle is recorded. When the rock is subject to anisotropic compression, this inherent mobilised shear strength essentially increases, as demonstrated by the envelope rotating upward toward the shear strength envelope at failure. This incorporates the mobilised shear force in the characterization of the rock deformation in terms of the applied stress.

Previous studies have confirmed laboratory testing of anisotropic rocks has been carried out in recent years by considering:

- 1) rocks which would be expected to behave as isotropic media and exhibit anisotropic properties (Class A)

- 2) rocks that are clearly anisotropic in nature and show directions of symmetry for their strength and deformation characteristics (Class B).

The anisotropic of rock can be an effect produced by microfractures and narrow cavities between adjacent particle boundaries in the rock. The stress-strain curve for the same rocks at low to moderate stress levels is nonlinear, which shows a concave upward curvature as shown in Figure 9. Thus, the nonlinear and anisotropic behaviour have the same origin [11].



**Figure 9.** Anisotropic volume change of rock based on a) Actual strain b) LVDT strain

Ge et al. [12] examined the three stages of fatigue behaviour: volumetric compaction, volumetric dilation with strain hardening behaviour, and volumetric dilation with strain-softening behaviour. It was found that the threshold for fatigue failure occurred at the transition point between volumetric compaction and volumetric dilation.

### 3. Conclusions

The mobilised shear strength was determined in this study using a stress-strain curve and an anisotropic deformation model in which the rock is compressed in an

anisotropic manner. The magnitude of the mobilised shear strength envelope in relation to the corresponding axial strain revealed the inherent property of the unique rock mass. The properties are utilised as guidelines for predicting rock stress-strength behaviour under anisotropic stress conditions. The research revealed that, based on mobilised minimum friction angle, the location of the mobilised shear strength envelope is considered. When the rock is compacted, however, the inherent mobilised shear strength of the rock increases due to the compaction process. As proven in this study, the envelope rotates upwards at the shear stress envelopes, demonstrating the phenomenon. Cementation ( $c'$ ) on the rock appears to increase, as can be seen in Table 5.

**Table 5.** Inherent properties of Hawkesbury Sandstone from mobilised shear strength envelopes Prediction of stress-strain behaviour of Hawkesbury Sandstone

Axial (%)	Cement (kPa)	Friction angle (deg)
0.2	1500	2.5
0.4	3000	7.5
0.6	3000	9.00
0.8	4000	10.85
0.85	5000	11.5

This research allows explaining the resultant deformation occurrence whereby at the point above the stress level where crack initiation occurs, the deformation in successive loads is cumulative. It is proposed that when the strain energy stored in the specimen exceeds a critical energy level, the failure occurs. Critical energy defined that it is equivalent to static loading (non-cyclic). Equation 1 given by Ge et. al. (2003) is an expression for decomposition where total strain = reversible strain + irreversible strain = reversible strain + plastic deformation + fatigue deformation. The monotonic loading causes the strain component by plastic deformation.

### Acknowledgments

The authors would like to express their gratitude to the School of Civil, Environmental and Mining Engineering, University of Adelaide Australia and School of Civil Engineering, College of Engineering, Universiti Teknologi MARA Malaysia for the laboratory expertise and contribution to this research.

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