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Market Efficiency Dynamics and Chaotic Behavior of Dhaka Stock Exchange: Evidence from Mutual Information and Lyapunov Exponents Models

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Abstract This study investigates the evidence of market efficiency dynamics and chaotic behavior of the Dhaka Stock Exchange benchmark index (DSEX) over the 2000-2020 period. We employed the newly developed model of mutual informational and global correlation coefficient in addition to the traditional linear and nonlinear techniques. Results suggest there is evidence of serial dependence in the DSEX returns. We attempted the Lyapunov exponent model to evaluate the possibility of chaos and nonlinear dynamics in the market. The results conspicuously represent the existence of chaotic behavior-a nonlinearity-based profitability pattern revealed in the DSEX return series in its short run behavior. By applying two technical trading indicators, we justify the predicting trend of the Bangladesh stock market and conclude that investors active in the Dhaka Stock Exchange can earn abnormal returns. Findings have practical implications for general investors and professional fund managers to exploit the profitable opportunities and reshuffle the investment decisions. Results also convey the message to the regulatory body to initiate the strategies for intervening in the operating mechanisms to reduce the market inefficiency.

Keywords Market Efficiency, Chaotic Behavior, Mutual Information, Global Correlation, Lyapunov Exponents

1. Introduction

Stock market efficiency is treated as a puzzling issue for academic research in corporate finance to validate the random walk model in equity returns behavior. It has become imperative for academicians to develop theories and models that explain how stock market prices behave. This theory became popular in early 1970s with the understanding that equity price movements can be modeled [1]. Random walk theory enables to develop the pattern of shocks that drive stock prices to predict successive price changes. If a random walk process follows, there should not be any mean-reversion tendency of equity price changes and, hence, market will gradually lead to the equilibrium in the long-run. But if price changes follow random walk, it is possible to predict the future security movements based on past prices. According to efficient market hypothesis (EMH), investors in the market act rationally and security prices will instantly reflect all the information about the particular investment. Therefore, random walk theory justifies that market movement cannot be predicted capitalizing any historical information. Stock market efficiency represents important implications for the equity market participants to make effective investment decisions. If equity market efficiency prevails, then any attempt made by the investors to generate abnormal returns will be meaningless. Thus, detecting undervalued or overvalued securities to make above-average potential returns remains effortless works for the market participants. Although the equity market efficiency is widely tested empirically, still understanding the market behavior has been a crucial area for all kinds of investors to design their investment strategies commensurate with risk-return tradeoff. This framework will also lead to the efficient allocation of resources in the market and contribute the overall economic growth.

1.1. Motivation and Objective of the Research

If equity price changes follow random walk process, it manifests that a particular equity market is not weak-form efficient. This has been a challenge for the efficient market hypothesis (EMH), a mainstream theory describing the risk-return relationship in all financial decision-making process. Weak-form efficiency has extensively been studied for all the equity markets: developed, developing, and emerging. In all previous studies conducted on weak-form efficiency of Bangladesh equity market, the methodologies regarding linear dependency are considered. Bangladesh equity market is not matured market and it is expected that nonlinear dependency in equity return would be a common phenomenon for this emerging equity market. Recognizing and modeling the nonlinear structure in the equity returns might have practical implications for both the individual and portfolio investors as well as for other participants to understand the directions of the future market movements. This is the first study attempts to investigate the market efficiency and chaotic behavior applying the nonlinear econometric techniques like mutual information and lyapunov exponents models. The application of nonlinear and chaos theory to any financial time series evidenced the presence of nonlinearity structure [2]. Any deviation to test the nonlinearity may violate the efficient market hypothesis.

By incorporating the importance of nonlinear dependency on equity returns, the study investigates whether Dhaka Stock Exchange returns display any nonlinear dynamics and chaotic behavior inferring a possible (in) efficiency of the stock market in its weak-form.

Our paper has been developed in the following ways: Section 2 states the theoretical framework and empirical evidence on the efficient market hypothesis. Section 3 presents the data and methodologies; Section 4 represents the empirical analysis and its results; Section 5 demonstrates predictable pattern, and in last part, section 6 makes the conclusion.

2. Theoretical Framework

There is a growing concern among researchers to examine how the equity markets actually move. The two evocative theories that characterize the market behaviors are described below:

2.1. The Efficient Market Hypothesis (EMH)

The efficient market hypothesis is still represented a dominating theory in corporate finance decisions [1]. The theory defends that when any new information arrives in the market, that is instantly incorporated in the current prices of the securities. Therfore, investors cannot exploit the market to take advantage of that information to generate any above-average potential market returns [3]. [4] makes a clear and representative definition of weak form efficiency asserts that current security prices reflect all past price information. Therefore, it is almost impossible to beat the market, and technical analysis fails to predict future prices. Semi-strong efficiency states that current security prices capture all public information, and thus fundamental and technical analysis may not be useful to make abnormal profits. Strong form becomes the strongest form of market efficiency**,** claiming that current security prices incorporate private and public information; even insider information may not bring an advantage for investors.

2.2. Random Walk Theory

The random walk theory or random walk hypothesis originated by Economist Burton Malkiel in 1973. The theory presumes that equity price changes are independent of each other and should follow the random walk. It defines there should be no apparent trend, and equity prices change randomly. Therefore, any effort on equity market participants to predict future market movement using any trading strategy remains worthless. Thus both technical and fundamental analysis are undependable to generate above-average returns in the long run.

The basic random walk model looks like:

$$P_{t+1} = P_t + \varepsilon_{t+1} \tag{1}$$

Where, P_{t+1} is the value of stock at the end of period t+1, and P_t is the current value, and ε_{t+1} is the unpredictable component of the price.

2.3. Empirical Evidences on Efficient Market Hypothesis

A few studies documented on equity market efficiency in developing markets compared to the empirical works done on developed markets. The empirical findings on the efficient market hypothesis in emerging equity markets are disputed. Some report the presence of weak-form efficiency [5];[6]and some studies against it[7]; [8]; [9]; [10] A very few studies are done in case of Dhaka Stock Exchange.

The pioneering work by [11] examined the U.S. security prices based on the 1216 weekly observations from September 6, 1962 to December 26, 1985. They introduced the variance ratio and found a significant positive serial correlation for weekly and monthly holding-period returns, rejecting the random walk hypothesis. On the other hand, [12] confirmed that 25% to 40% of returns variation can be estimated using past information [13] examined the Kuala Lumpur and Singapore equity markets over the 1973-1978 and did not find any evidence in favor of random walk hypothesis. [14] examined Indian equity market using data from 1987 to 1994 and found the Indian market is weak-form inefficient. [15] investigated the nonlinear dependence in the Portuguese stock index for the period 1990-1997. The results from nonlinear tests confirm the inefficiency of the Portuguese market. The random walk hypothesis was evaluated by [16] for Amman Stock Exchange using daily index return. The statistical evidences from BDS test and GARCH(p,q) model are not sufficient to accept the random walk hypothesis for Jordan stock market.

[17] studied the Dhaka Stock Exchange covering from January 1990 to April 2001 using daily, weekly, and monthly data. Results imply that the behavior of stock prices does not reflect the random walk model. Another study [18] analyzed the efficiency of DSE index applying different parametric and nonparametric tests over 1994-2005. The results show that the Dhaka equity market is inefficient. [19] test efficient market hypothesis for Pakistan equity market using daily prices of KSE-100. They apply run tests, ADF unit root, and ACF test and report no evidence of market efficiency.The Colombo Stock Exchange was also found weak form inefficient from the study of [20] covering the index data from January 2010 to December 2014. The Vietnam stock index(V.N. Index)conducted the random walk hypothesis [21], measuring the autocorrelation, run test, and variance ratio test. Their results conclude that the efficient market hypothesis does not hold. [22] examined G7 and 3 European (Portugal, Spain, and Greece) equity markets. They applied nonlinear techniques like, BDS test, Engle test and detrended fluctuated analysis. And found the evidence of nonlinear trend for all the markets.

[23] study market efficiency of Dhaka Stock Exchange using daily, weekly, and monthly data of DGEN index applying the autocorrelation, run test, and variance ratio. Their overall findings confirm that DSE returns do not follow a random walk.[24] sought evidence on whether DSE index returns are independent and follow the random walk model. They measure Kolmogorov-Smirnov normality test, run test, and ARIMA models using daily prices and document that the random walk hypothesis does not hold. [25] tested efficiency of Dhaka Stock Exchange using daily closing prices of banking sector companies in addition to the market indices data (DSI, DSE-20). They applied run test and ADF unit root test, and their findings go against the random walk model. [26] used ADF test, run test, autocorrelation, and variance ratio test to investigate the Dhaka stock exchange market efficiency. He analyzes the DSI, DGEN, and DSE-20 index using daily closing prices. Findings suggest that none of the return series accepts the random walk hypothesis implying that the Bangladesh stock market is inefficient. [27]The NSE Nifty and BSE Sensex are used to test the Indian market efficiency measuring autocorrelation, run, unit root, and variance ratio tests. It found that the Indian equity market is not efficient. [28] empirically studied weak-form efficiency of Chittagong Stock Exchange (CSE) from 2006 to 2015. They applied parametric tests [Augmented Dickey-Fuller (ADF), Autocorrelation function] and nonparametric tests [Variance ratio, Kolmogorov Smirnov K-S, Run test]. They observed that CSE does not follow the random walk. [29] empirically tests random walk hypothesis for Pakistan stock market using monthly data of 83 companies from 26 different sectors for the dataset from 2009 to 2015. Based on ADF, Phillip-Perron, and run tests, he confirmed that RWH is rejected, and thus KSE-100 proves to be weak-form inefficient. [30] employed six different unit root tests on ten emerging Asian stock markets to detect random walk-in stock prices for the sample span 2001-2015. They evidenced that eight out of ten Asian stock markets during the whole sample period and all ten stock markets during the pre-crisis period (2001-2007) follow the random walk. Kuwait Stock Exchange was not weak-form efficient examined by [31]over the period 1998-2011. They use ten filter rules on weekly data of 42 listed stocks on KSE. [32] tested random walk hypothesis of Dhaka stock exchange for DSE-30 and DESX indices and found that market is inefficient, similar to the other studies done in Bangladesh context. [33] tested weak form efficiency of Dhaka stock exchange applying parametric and nonparametric techniques and found no evidence of market efficiency. They apply daily returns data of DSEX and DSE 30 indices covering June 1, 2004 to March18, 2018. Another study, [34]measures weak form efficiency of both Dhaka and Chittagong Stock Exchanges in Bangladesh over 2013-2017. They also applied parametric and nonparametric tests, and findings confirm the absence of weak-form efficiency for both equity markets. [35] examined efficient market hypothesis for 15 major African stock markets adopting wavelet unit root analysis tool with varying frequency band. Findings suggest that past information can predict future price movements in all African stock markets, confirming the absence of weak-form efficiency.

3. Data Structure and Methodologies

In the study, we employ the closing prices of the Dhaka

Stock Exchange benchmark index known as DSEX index over the period from January 01, 2000 to December 31, 2020. The data for closing prices of the index have been collected from the Dhaka Stock Exchange library. The daily market returns are calculated as follows:

$$r_t = \ln (p_t / p_{t-1}) * 100$$

Pt and pt-1 are the closing price of the DSEX index for the current day and the previous day, respectively, and log is Natural Logarithm.

3.1. Unit Root Test

The stationarity of DSEX return series is tested using the three popular models. These are: Augmented Dickey-Fuller (ADF) test, the Phillips-Peron (PP.) test, and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. One underlying assumption of any time series analysis is that the data should follow the Stationary process.

3.1.2. Breusch–Godfrey (BG) LM Test

The BG test is a common test for detecting autocorrelation in the error of the retur series. The null hypothesis of the model: No serial correlation in the residuals up to any specific orders.

3.2. Nonlinear Tests

3.2.1. BDS Independence Test

The BDS is a time-based non-linear dependence test developed by Brock, Dechert, and Scheinkman. The primary objective of the technique is to detect the nonlinear correlations in the residuals of return series after having fitted the linear Autoregressive moving average (ARMA) model. The null hypothesis is that increments are independently and identically distributed.

3.2.2. Engle's ARCH LM Test

ARCH is a LM test for measuring autoregressive conditional heteroscedasticity in the residuals of return series.

The null hypothesis of the model: No ARCH effects up to any specific order in the residuals of index returns.

3.2.3. Mutual Information Test

Mutual information is a measure of global dependence between variables and is based on Shannon’s entropy concept. It has the ability to capture both linear and nonlinear dependence, being cross-section or time series variables. In terms of mathematical definition, the mutual information of two random variables X_1 and X_2 is given by:

$$I(X_1, X_2) = \int x_1 \int x_2 \, p(x_1, x_2) \log \frac{p(x_1, x_2)}{p(x_1) p(x_2)} dx_2 dx_1 \quad (2)$$

Mutual information is a nonnegative concept. It will be null when the variables under study are statistically independent, and positive when there are signs of any type of dependence between those.

Since $0 \leq I(X_1, X_2) \leq +\infty$, it becames necessary to normalize its values between 0 and 1 in order to better compare results with the linear correlation coefficient and simplify respective interpretations. In order to achieve this goal, we estimate the global correlation coefficient [36]; [37], that is given by:

$$\lambda = \{1 - Exp[-2I(X_1, X_2)]\}^{1/2} \quad (3)$$

λ has the capacity to capture the global dependence between the variables under study and relies between 0 and 1, being comparable to the Pearson correlation coefficient. [37] construct an entropy-based independence test, using the hypothesis:

$$H_0 : I(X_1, X_2) = 0$$
$$H_1 : I(X_1, X_2) => 0 \quad (4)$$

The null of independence is rejected when mutual information is positive and statistically significant (the critical values could be found in [37]). The simulated critical values for the null distribution are based on a percentile approach [37]. In the Appendix, we present the 3 levels of significance of the empirical distribution for the mutual information using a white noise as base.

A major difficulty in the estimation of mutual information from empirical data lies in the fact that the underlying probability density function is unknown. According to [38], the most effective and efficient approach to estimate mutual information is based on equiprobable histogram-based estimators using marginal equiquantisation for the construction of cells (see e.g. Darbellay [39]). The marginal equiquantisation, has advantages, because it maximizes the mutual information estimates [39][40].

3.2.4 Lyapunov Exponent Model

The Lyapunov exponent has the ability to test and detect the possible chaotic behavior of a financial time series. This exponent measures the exponential rate at which two nearby orbits are moving apart and estimates the level of dependence on initial conditions (giving signs of the possible evidence of nonlinear dynamics). In this research work [42] approach, and we focus on the largest exponent, that could be defined by:

$$\lambda_i = \lim_{n \to \infty} \frac{1}{n\tau} \sum_{k=1} n \ln \left| \frac{\delta x_i}{\delta x_0} \right| \quad (5)$$

Where, the delta terms indicate the given distance within trajectories in the phase space for the two time periods. In terms of interpretation, a positive Lyapunov exponent evidences a possible divergence between nearby trajectories, meaning high sensitivity face to initial conditions.

There are several methods to estimate Lyapunov exponents. In this research, we highlight 2:

- Using the mathematical model ODEs;
- Using empirical time-series; reconstructing the trajectories in the phase space, consisting of equally sampled values of an observable variable.

The second option is faster in computational terms, but there are some critical topics in the respective implementation, namely the selection of the sampling time.

Our method is described by PDEs. We discretised model 5 using a finite difference scheme. Our model is based on $2N$ ODEs, where N is the number of nodes used in the finite difference scheme. We need to solve $2N(2N+1)$ ODEs for a time $K_\tau \gg 1$. In order to minimize the computational efforts, we only estimate λ_1.

The ODEs system to be solved are:

$$\begin{aligned} \dot{\underline{x}} &= f(\underline{x}) \\ \dot{\underline{\underline{\Phi}}} &= \underline{\underline{J}}.\underline{\underline{\Phi}} \end{aligned} \tag{6}$$

where $\underline{x}$ is the state vector, $\underline{\underline{J}}$ is the jacobian matrix and $\underline{\underline{\Phi}}$ is the transition matrix, and

$$\begin{aligned} \dot{\underline{x}} &= f(\underline{x}) \\ \delta \dot{\underline{x}} &= \underline{\underline{J}}.\underline{\delta x} \end{aligned} \tag{7}$$

[41] estimated the Lyapunov exponent by averaging the observed orbits divergence rates. Some authors, however, argue that this method is less reasonable when we are dealing with noisy systems. Therefore [42] and [43] formulated different approaches applying Jacobian methods to control any upward bias can arise from estimating Lyapunov exponents.

4. Analysis and Findings

We start with descriptive statistics of the return series presented in Table 1 to glimpse the nature of the distribution.

The results from the table indicate that the return series of DSE benchmark index in Bangladesh are not found to be normally distributed, confirmed by Jarque-Bera results. Thus, we reject the null hypothesis that DSEX returns series are not normal.

4.1. Unit Root Test Results

Results from Table 2 highlight that the results from all three models namely ADF, P.P. and KPSS indicate that DSEX returns are stationary at the level form and do not support the necessary condition of the random walk hypothesis.

4.2. BG Test Results

Table 3 states the BG test results.

Breusch–Godfrey test statistics results clearly reject the null hypothesis of no autocorrelation and confirm that the random walk hypothesis does not hold for the Dhaka stock exchange benchmark index.

Table 1. Descriptive Statistics of DSEX Return Index

Mean	**SD**	**Maximum**	**Minimum**	**Skewness**	**Kurtosis**	**Jarque-Bera**
0.1436	2.0542	24.0001	-13.2124	1.0120	19.6908	20210 (0.000)

Notes: P value is given in parenthesis.

Table 2. Unit Root Test Results for DSEX Returns

ADF-t Statistics	PP-t Statistics	KPSS-t Statistics
-68.37 [0.000]	**-68.52** [0.000]	0.**2930**

Note: ADF (Augmented Dickey-Fuller), P.P. (Phillips-Perron Test), and KPSS (Kwaitkowski-Phillips-Schmidt- Shin Test). Critical Value for KPSS: At 1% level=.7390, At 5% level=.4630.

Table 3. Breusch-Godfrey Serial Correlation LM Test

F-statistic = 7.677307 Prob. F(4,5189)= [0.0000]	Obs*R-squared=30.55797 Prob. Chi-Square(4)= [0.0000]

Note: p value is in parenthesis.

Table 4. Results from BDS Independence Test

	m(dimension)=2	m=3	m=4	m=5	m=6
ε =.3	0.0173 [0.0000]	0.0514 [0.0000]	0.0543 [0.0000]	0.0494 [0.0000]	0.0426 [0.0000]
ε =.5	0.0348 [0.0000]	0.0165 [0.0000]	0.0115 [0.0000]	0.0071 [0.0000]	0.0043 [0.0000]
ε =.7	0.0386 [0.0000]	0.07562 ([0.0000]	0.1018 [0.0000]	0.1173 [0.0000]	0.1259 [0.0000]

Note: the m and ε represent the dimension and distance, respectively. P value is given in parentheses.

4.3. BDS Test Results

Table 4 presents the results of the BDS test using the embedding dimension (m) of 2,3,4,5,6, and the value of ε (distance between points measured in terms of the number of standard deviations of the raw data) in case of specific value of 0.3,0.5, and 0.7.

The BDS test results clearly suggest the DSEX returns series possess the non-linear dependency element. That means the return observations are not independent.

4.4. GARCH(p,q) Effect Estimates

We apply a GARCH(p,q) variation to model the best fitted specification to the returns data set to justify the presence of non-linear dependency.

Table 5. GARCH(2,2) Model Estimates

	Coefficient	Probability Value
Constant	4.1623	[0.0000]*
Residual^2(-1)	0.1768	[0.0000]*
Residual^2(-2)	0.0868	[0.0000]*
F-Statistics	43.002	[0.0000]*

*Significant at 1% level.

The GARCH (2,2) specification is chosen as the best fitted model. The results fail to accept the null hypothesis of homoscedasticity in the DSEX data series. Therefore, we can, once again, confirm the presence of non-linear structure in the Dhaka equity market return series.

4.5. Engle's Test Results

As the best fitted GARCH(2,2) specification recognized the existence of non-linear structure in the DSEX returns series. We further examined the ARCH effects of the selectedmodel.

Table 6. Engle Heteroscedasticity Test

F-statistic	**20.34**	Prob. F(4,5184)= [0.0000]
Obs*R-squared	**18.25**	Prob.Chi-Square(4)= **[.0000]**

The results from ARCH effects strongly confirmed the presence of heteroscedasticity in the returns series. The results are consistent with the findings of the other techniques presented in earlier part. Therefore, we, once again, conclude that the Bangladesh equity market is not efficient.

4.6. Mutual Information Test Results

Global correlation coefficient developed from mutual information is a component used to determine the presence of global dependence. This actually detects the extent of serial dependence in the return series. The Table 7 summarizes the outcome of its values.

Table 7. Mutual Information and global correlation coefficients

Lags	Mutual Information	Global Correlation Coefficient	R
t-1	0.0838**	0.3928	0.0522
t-2	0.0575**	0.3296	-0.0371
t-3	0.059**	0.3336	0.0214
t-4	0.0452**	0.2940	0.0342
t-1	0.0479**	0.3022	0.0349
t-2	0.0314**	0.2467	-0.0127
t-3	0.0325**	0.2509	-0.0126
t-4	0.0408**	0.2799	0.0119
t-1	0.0287**	0.2362	0.0289
t-2	0.0337**	0.2553	0.0064

Note: **Sig **Significant at 5% level

We calculate the mutual information for daily DSEX return series using different lags and values are stated in table 7. we can see, all the coefficients are statistically significant, leading to the rejection of the null hypothesis of mutual independence in return data. Besides that, we can verify that the level of global dependence does not fall significantly in the last lags, which indicates the absence of fast decay of dependence. Based on the global correlation coefficients, it is observed that these coefficients are significantly higher for all the lags values compared to any lineal coefficients found in BG correlation test presented earlier. This finding authenticates that non-linear structure exists in the returns series of benchmark index. The comparisons of both coefficients are provided in the following diagram.

Figure 1 tries to compare two correlation coefficients: Pearson which is linear and GCC (Global correlation coefficient) which has the ability to capture the nonlinearities. GCC is expected to be higher than

Pearson's linear correlation coefficient, as it captures every type of correlation. Even so, it must be taken into account that the difference between these does not necessarily correspond to the purely non-linear correlation part. The green bar indicates the global correlation coefficients and purple bars measures the linear correlation coefficients. By looking at both correlation coefficients, it can be stated that nonlinear behavior may exist in the return observations of the Dhaka Stock Exchange benchmark index.

4.7. Lyapunov Exponents Results

The algorithms of Lyapunov exponent are estimated from MATLAB software based on the Runge-Kutta algorithm integrator of orders 2 and 3. We can recognize its integration of the ODEs system (4) usually conducted through $N = 50$, for $k\tau = 1. \times 10^5$ taking advantage of the fast C-Mex coding for SIMULINK, and it took about 10^4s.

The condition of such kind of calculation is presented in figure 2 where the variables remained constant with the exception that we can change only γ:

According to Figure 2, we can see that the majority of the Lyapunov exponents are positive, being the maximum Lyapunov exponent positive too . These results point clearly to the existence of sensitivity to initial conditions and the presence of chaos.

In order to evaluate the robustness of these findings, we split the whole sample into two sub-periods. As the Bangladesh stock market experienced a massive crash in 2011, the study segregated the two sub-period as before market crash period (From January1, 2000 to December 31, 2010) and after market crash period (From January 1, 2011 to December 31, 2020). The outcome from the two newly adopted methodology is presented in this section.

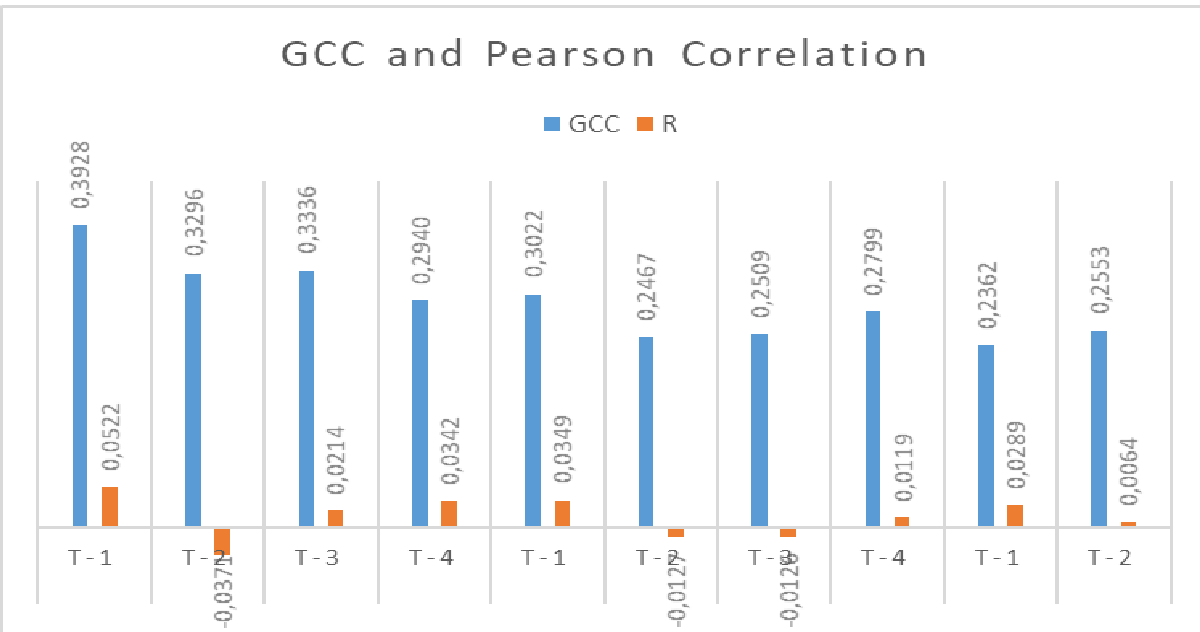

Figure 1. Global and Linear Correlation Coefficients

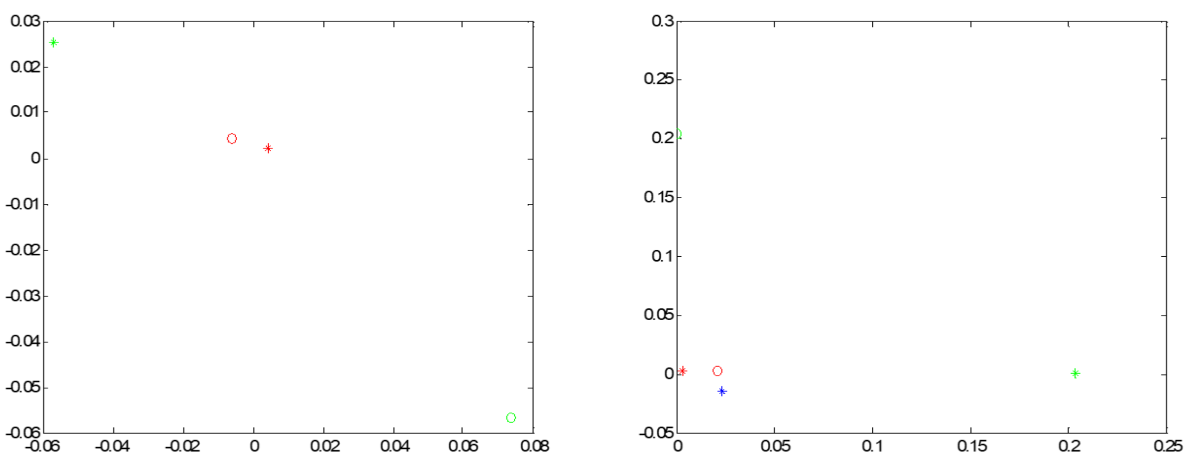

Figure 2. Maximum Lyapunov exponent.

4.8. Sub-periods Analysis: Period 1 (2000-2010)

4.8.1. Mutual Information Results

Table 8. Values from Mutual Information up to 10 Leg for the first

Period	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag9	Lag10
Coefficients	0.0555**	0.0324**	0.0256**	0.0192**	0.0185**	0.0069**	0.0066**	0.0098**	0.0128**	0.0076**

**Significant at 1% level

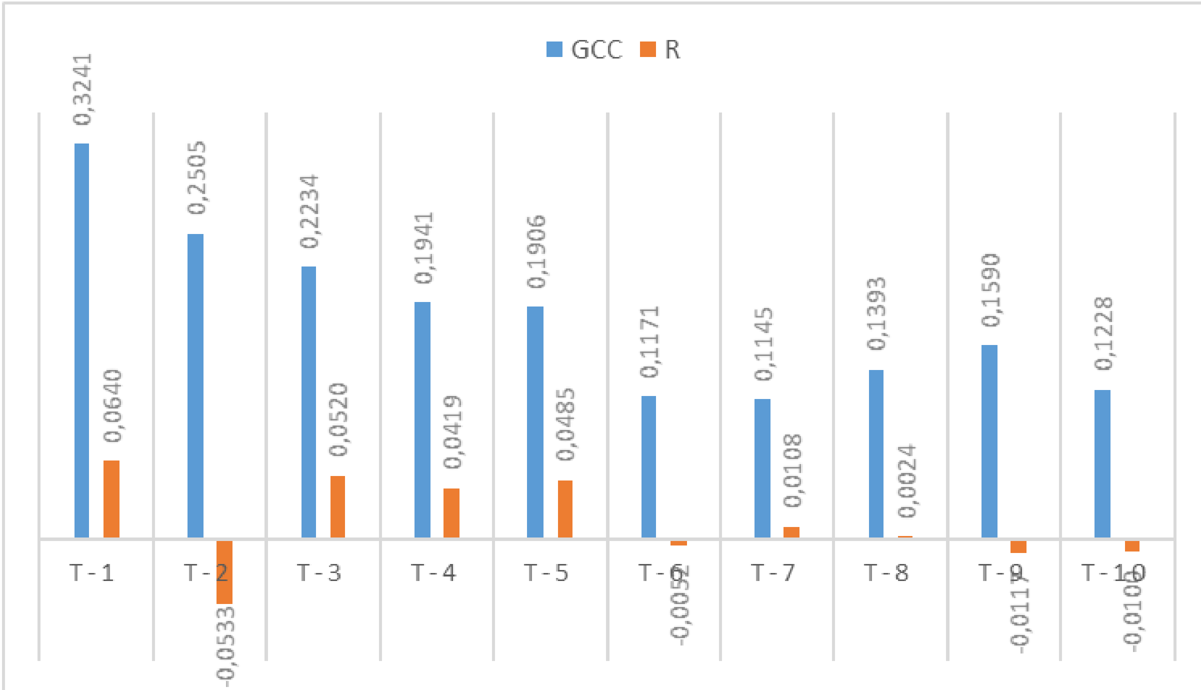

Figure 3. Global and Linear Correlation Coefficients for the first sub-period

4.8.2. Lyapunov Exponents Results

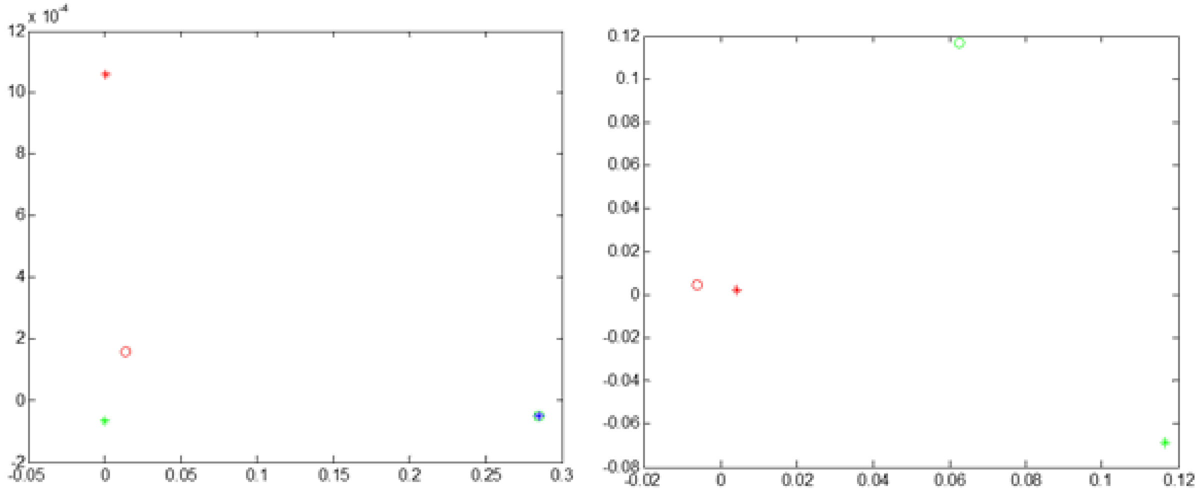

Figure 4. Maximum Lyapunov exponent

For the first sub-period, the results do not differ qualitatively from the ones presented to the whole period (Figure 2). The Lyapunov exponents obtained allow us to conclude about the presence of trajectories that start at two almost identical state vectors and diverge exponentially along time.

4.9. Sub-periods Analysis: Period 2 (2011-2020)

4.9.1. Mutual Information Result

Table 9. Mutual Information Values Corresponding to Different Legs

Period	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Lag 7	Lag 8	Lag9	Lag10
Coefficients	0.1093**	0.0758**	0.0853**	0.0683**	0.0825**	0.0561**	0.0622**	0.0625**	0.0612**	0.0686**

**Significant at 1% level

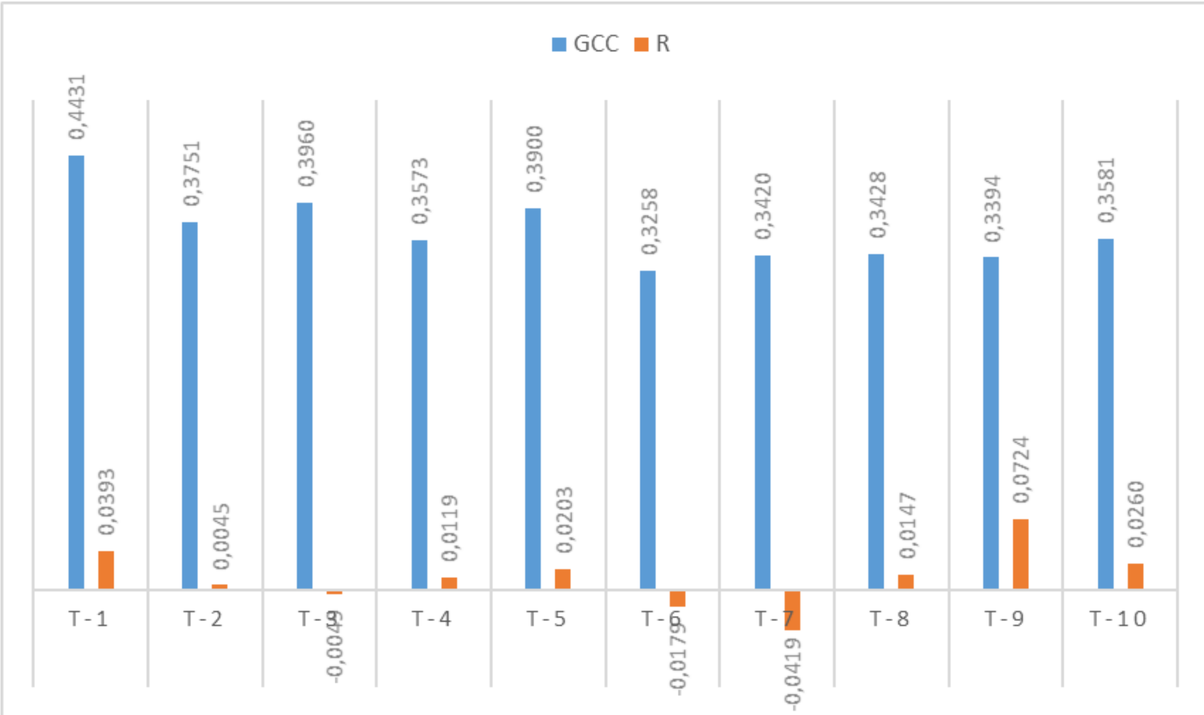

Figure 5. Global and Linear Correlation Coefficients for the second sub-period

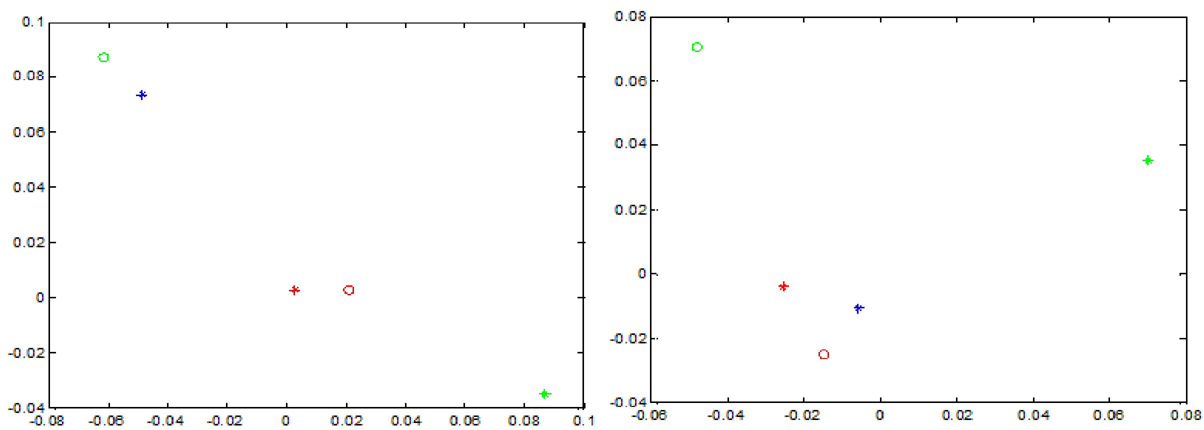

Figure 6. Maximum Lyapunov exponent

4.9.2. Lyapunov Exponents Results

For the second sub-period (after crisis), we denote a decreasing of the Lyapunov results, indicating that the behaviour of the series is more stochastic now. From the first period to the second one, we denote an increase of global serial dependence, especially for higher lags. Besides this, the deterministic part of that dependence shows a slight decrease, pointing to the possibility of an increase of the stochastic behaviour of the stock market under study.

5. Predicting the Market Return Applying Trading Rule

In this study, we found that the Dhaka stock market is weak-form inefficient. In this part, we tried to use the two most popular and effective technical indicators (Moving average convergence-divergence and Relative strength index) to justify the findings. Moving average convergence-divergence (MACD) is an indicator can be conducted for any period developed by Gerrald Appel during the late 1970s. On the other hand, the Relative Strength Index (RSI) is the versatile momentum indicator used to measure the speed and change in price movements developed by Welles Wilder(1978).

Using the moving average convergence divergence chart, in this report, holding period returns (HPRs) and holding period abnormal returns (HPARs) from the buy signal day to the next ten consecutive days are calculated and given below (Table 10).

Figure 7. Moving Average Convergence-Divergence (MACD)

Table 10. The Holding period average returns (HPRs) and holding period abnormal returns (HPARs) using MACD Indicator

Holding Period	HPRs(%)	T Statistics	HPARs(%)	T Statistics
(0,1)	3.50*	6.2154	2.36	2.356
(0,2)	4.1*	7.245	3.54	2.954
(0,3)	4.96*	6.354	3.87	2.354
(0,4)	5.32*	8.124	4.24	2.964
(0,5)	5.99*	6.356	4.23	3.256
(0,6)	6.12*	7.254	4.57	3.9659
(0,7)	8.68*	7.278	6.14	3.124
(0,8)	17.12*	7.365	11.02	3.024
(0,9)	8.12*	6.354	6.05	2.965
(0,10)	19.35*	6.698	13.25	3.958
Buy to Sell Signal	32.34*	9.245	27.12	6.258

Note: *Significant at 5% level. HPRs is calculated as $\frac{(V_t - V_{t-1}) + CF_t}{V_t}$ where V_t is , the current value of the the investment, V_{t-1} is the investment cost, and CF_t is the cash inflows. HPARs is the holding period returns in excess of market returns over the same period. HPARs is calculated as $HPR_j(t_1, t_2) - HPR_m(t_1, t_2)$ where $HPR_m(t_1, t_2$ is the holding period return of market portfolio from period 1 to period 2.

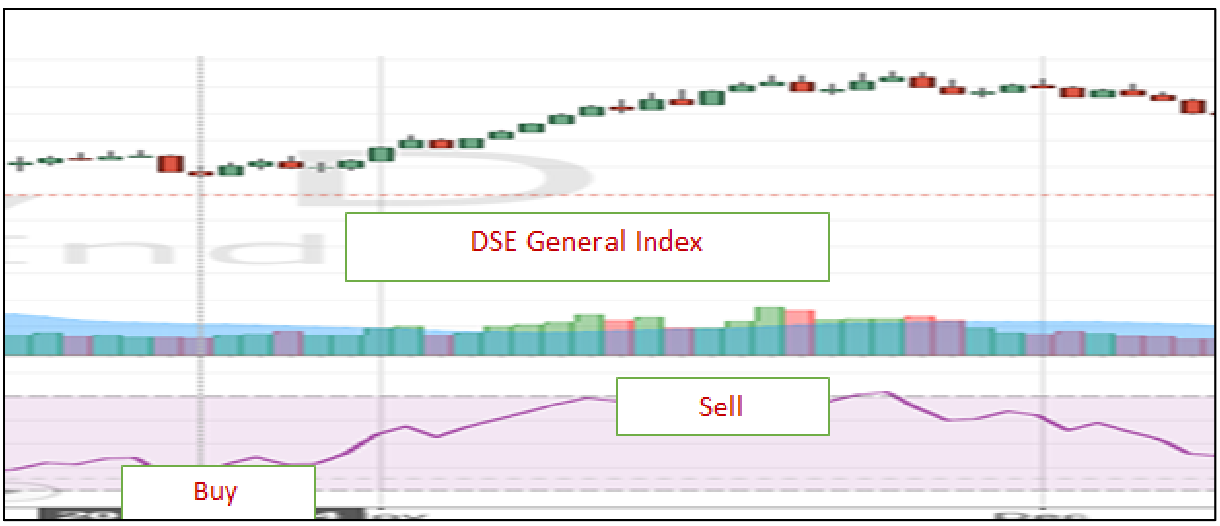

Figure 8. Relative Strength Index (RSI)

Table 11. The Holding period average returns (HPRs) and holding period abnormal returns (HPARs) using RSI Indicator

Holding Period	HPRs(%)	T Statistics	HPARs(%)	T Statistics
(0,1)	2.50*	4.224	1.98	2.145
(0,2)	3.56*	6.354	2.68	2.658
(0,3)	4.02*	5.236	3.65	3.101
(0,4)	6.11*	7.145	6.74	3.124
(0,5)	6.58*	5.365	4.32	2.154
(0,6)	7.35*	6.840	3.65	3.125
(0,7)	8.26*	6.980	5.24	3.245
(0,8)	16.54*	6.347	9.45	3.564
(0,9)	9.81*	5.642	6.21	2.154
(0,10)	18.26*	6.125	11.25	2.264
Buy to Sell Signal	31.54*	8.124	27.12	4.125

Note:**Significant at 5% level.

Returns information in the above table indicates if investors buy shares from buy signal, they can maximum earn 18 percent returns, but if they can hold until the sell signal the maximum returns will be 31.54%, and the average holding period in this would be 28 days. The results from two trading indicators demonstrate that investors can earn statistically significant abnormal holding period returns, which is not supported by the random walk hypothesis. Thus, we can say that the Dhaka stock market is not weak-form efficient also endorsed by various econometric models applied in the earlier part of the study.

6. Conclusions

The behavior of equity markets has been a fascinating subject in academic research over the last thirty years. In the study, we investigated the market efficiency dynamics and chaotic behavior for an emerging market like Bangladesh. In the first part, we employed the traditional serial time dependence techniques to recognize whether the Dhaka Stock Exchange benchmark index displays any predictable patter. All linear and nonlinear tests reveal that linear and nonlinear dependency is detected for the DSEX return series, confirming that the random walk process does not hold for the Dhaka stock exchange. In addition to the traditional techniques, we employed two new non-linear econometric methodologies: mutual information and lyapunov exponent. The findings from both the approaches precisely characterize the presence of non-linear dependency in the DSEX return series.

The exciting part of the study is that we developed two technical trading indicators to explore whether the stock market has a predictable pattern. The findings understandably substantiate that investors can earn abnormal holding period returns for the Bangladesh stock market, particularly in the short-run, the evidence against the efficient market hypothesis.

The practical implications of the results suggest that investors (individual, institutional, and foreigners) active in the Bangladesh stock markets should be more vigilant and foresighted in reshuffling their investment choices as markets displayed a trend pattern. It will also have a consequential effect on the decision-making process of professional fund managers like mutual funds, hedge funds since this will induce them to apply any trading rule to make profitable investment strategies. The results will give room for academic researchers to discover the kinds of anomalies in Bangladesh stock markets, focusing on investors' behavioral aspects to justify weak-form inefficiency. The study concludes that the regulator would find it necessary to initiate strategies for more interventions in operational procedures of stock trading to ensure that security prices are approaching fair value.

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Appendix

Tables of critical values for testing serial independence through mutual information for data. 5000 replications were computed. D.F. are the degrees of freedom for the mutual information, which correspond to the dimension (d) of the analysed vectors.

N=100			
	Percentiles		
D.F	90	95	99
2	0.0185	0.0323	0.0679
3	0.1029	0.1232	0.1933
4	0.1059	0.1260	0.1722
5	0.2290	0.2580	0.3261
6	0.6639	0.7528	0.9663
7	0.8996	0.9731	1.1586
8	1.3384	1.3839	1.5024
9	1.9030	1.9352	2.0142
10	2.5266	2.5571	2.6181

N=500			
	Percentiles		
D.F	90	95	99
2	0.0037	0.0070	0.0144
3	0.0222	0.0369	0.0501
4	0.0680	0.0788	0.1128
5	0.1756	0.2066	0.2712
6	0.3084	0.3514	0.4390
7	0.4920	0.5391	0.6339
8	0.4477	0.4843	0.5659
9	0.6661	0.6941	0.7594
10	1.0884	1.1082	1.1483

N=1000			
	Percentiles		
D.F	90	95	99
2	0.0019	0.0041	0.0071
3	0.0133	0.0191	0.0311
4	0.0340	0.0399	0.0568
5	0.0708	0.0865	0.1128
6	0.2119	0.2430	0.3046
7	0.3635	0.3954	0.4688
8	0.4041	0.4414	0.5252
9	0.3865	04114	0.4640
10	0.6418	0.6585	0.6942

N=2000			
	Percentiles		
D.F	90	95	99
2	0.0009	0.0019	0.0033
3	0.0061	0.0094	0.0147
4	0.0169	0.0203	0.0278
5	0.0701	0.0804	0.1030
6	0.1370	0.1549	0.1940
7	0.2496	0.2733	0.3224
8	0.4497	0.4864	0.5508
9	0.3036	0.3298	0.3858
10	0.3530	0.3669	0.3996

N=2500			
	Percentiles		
D.F	90	95	99
2	0.0008	0.0015	0.0030
3	0.0054	0.0078	0.0129
4	0.0134	0.0171	0.0251
5	0.0556	0.0648	0.0797
6	0.1203	0.1376	0.1738
7	0.2181	0.2418	0.2884
8	0.3938	0.4217	0.4719
9	0.3175	0.3409	0.4024
10	0.2931	0.3124	0.3477

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