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Choice of Strata Boundaries for Allocation Proportional to Stratum Cluster Totals in Stratified Cluster Sampling

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Abstract In survey planning, sometimes, there arises situation to use cluster sampling because of nature of spatial relationship between elements of population or physical feature of land over which elements are dispersed or unavailability of reliable list of elements. At the same time, there requires technique and strategy for ensuring precision of the sample in representing the parent population. Although several theoretical cum practical works have been done in cluster sampling, stratified sampling and stratified cluster sampling, so far, the problem of stratified cluster sampling for a study variable based on an auxiliary variable, which is required in practice, has never been approached. For the first time, this paper deals with the problem of optimum stratification of population of clusters in cluster sampling with clusters of equal size of a characteristic y under study based on highly correlated concomitant variable x for allocation proportional to stratum cluster totals under a super population model. Equations giving optimum strata boundaries (OSB) for dividing population, in which sampling unit of the population is a cluster, are obtained by minimising sampling variance of the estimator of population mean. As the equations are implicit in nature, a few methods of finding approximately optimum strata boundaries (AOSB) are deduced from the equations giving OSB. In deriving the equations, mathematical tools of calculus and algebra are used in addition to statistical methods of finding conditional expectation of variance. All the proposed methods of stratification are empirically examined by illustrating in live data, population of villages in Lunglei and Serchhip districts of Mizoram State, India, and found to perform efficiently in stratifying the population. The proposed methods may provide practically feasible solution in planning socio-economic survey.

Keywords Allocation, Gamma Probability Density Function, Cluster Size, Optimum Strata Boundaries, Stratified Cluster Sampling, Stratification Variable

1. Introduction

In stratified sampling, a heterogeneous population is divided into a number of groups called strata which are within strata homogeneous and sample is selected from strata using suitable sample selection method; the method is used for administrative convenience and enhancing the precision of representation of the sample for the parent population. On the other hand, when the availability of reliable list of elements (units) of population is difficult or the elements are spatially dispersed in such a way that there requires lots of energy, time and cost while surveying the elements selected by simple random sampling, cluster sampling or area sampling is employed by grouping the contiguous elements or elements, which can be conveniently surveyed together without much extra effort, into clusters; then, the clusters are taken as sampling units of population while selecting sample from the population. The strategy used in cluster sampling for enhancing its precision is to make the population within cluster as heterogeneous as possible and increase inter cluster homogeneity as much as possible. Formation of cluster primarily depends on the spatial relationships between elements in terms of geographical contiguity, good connectivity and convenience in surveying together, less energy, resource and time while surveying the elements within cluster, in addition to scheming for increasing intra-cluster heterogeneity and inter-cluster homogeneity. When the clusters are considered as sampling units of population and then stratified by methods of stratified sampling, the inter cluster homogeneity is increased within strata of clusters which in turn serves the purpose of scheming in cluster sampling for enhancing the precision of representation of sample for the parent population.

In stratified sampling, ever since Dalenius [1] introduced the problem of finding optimum strata boundaries (OBS) based on Tschuprow [2] and Neyman [3] optimum allocation (TNOA) for enhancing homogeneity within strata, the vastness of research in the area has been increasing as a number of researchers, inter alia, Dalenius and Gurney [4], Mahalanobis [5], Hansen et al. [6], Dalenius and Hodges [7,8], Ekman [9], etc., embarked on the work who initially used study variable as stratification variable. As the use of study variable as stratification variable is unrealistic, many workers mostly in the later years extended the work of finding OSB and AOSB by using an auxiliary variable which is highly correlated with the study variable. Dalenius [10], Taga [11], Singh and Sukhatme [12], Singh [13-16], Singh and Prakash [17], Yadava and Singh [18], etc., to mention a few among many, worked on the problem of finding OSB and AOSB based on auxiliary variable for various allocations under different sampling designs. The problem of optimum stratification was again considered from the perspective of more than one study variable by, inter alia, Ghosh [19], Gupta and Seth [20], Rizvi et al. [21,22] etc., whereas Danish and Rizvi [23] approached the problem from the perspective of two auxiliary variables having one study variable.

It is pertinent to mention that in the direction of development of allocation of sample size to strata in stratified sampling, ever since Tschuprow [2] and Neyman [3] proposed TNOA based on study variable, it is Hanurav [24] and Rao [25] who introduced using auxiliary variable under a superpopulation model considered by them. Gupt and Rao [26] obtained allocation of sample size to strata for probability proportional to size under the superpopulation model. Gupt [27,28] modified the aforesaid superpopulation model into a more general form and hence obtained a few generalised model-based allocations; Gupt and Ahamed [29,30] obtained a few methods of stratification for some of the generalised model-based allocations under simple random sampling with and without replacement (SRSWR and SRSWOR) in the form of equations giving OSB and solutions to the equations giving AOSB. Gupt et al. [31] also obtained methods of stratification giving OSB and AOSB for auxiliary variable optimum allocation (AOSB) obtained by Hanurav [24].

In the area of stratified cluster sampling, Mehta and Mandowara [32] considered problem of finding OSB and AOSB in stratifying population based on study variable for TNOA, proportional and equal allocation under SRSWOR design.

For the first time, we have introduced in this paper the problem of optimum stratification for a characteristic under study y based on a highly correlated auxiliary variable x in stratified cluster sampling with clusters of equal size under the following superpopulation model which is a modified form of the model used by Hanurav [24] and Rao [25].

$$\left.\begin{array}{l}\text{(i) } \xi(y_i|x_i) = \alpha + \beta x_i \\ \text{(ii) } \mathrm{V}(y_i|x_i) = \sigma^2 x_i \\ \text{(iii) } \varsigma(y_i, y_j|x_i, x_j) = 0\end{array}\right\} \quad (1)$$

where α , β and σ^2 are the superpopulation parameters with $\sigma^2 > 0$ and the scripts ξ , V and ς denote conditional expectation, variance and covariance given x's respectively.

Here in this paper, the crux of the work is to simultaneously address the inevitable conditions of spatial relationship of elements leading to the use of cluster sampling and scheming for increasing inter-cluster homogeneity and intra-cluster heterogeneity to increase precision of the sampling.

We use information on the auxiliary variable x which is highly correlated with study variable y to stratify population whose units are clusters whereas clusters are formed by grouping the elements in the way discussed above elaborately; the allocation and sample selection procedure used in this work are allocation proportional to stratum total and SRSWR, which will hold true for SRSWOR too when finite population correction is neglected.

The paper has six sections. Section 2 deals with obtaining conditional expectation of population variance between cluster means. In section 3, the derivation of equations giving OSB is presented. In section 4, a few methods of finding AOSB are presented. In Section 5, empirical illustration of all the proposed methods of stratification is carried out in live data and results are discussed. Section 6 gives the conclusion.

2. Expression for Conditional Expectation of Population Variance between Clusters Means

Considering a population consists of N clusters of M

elements each and a sample of n clusters is to be selected from N clusters by SRSWR. Let Y_{ij} be the value of characteristic under study for the j^{th} element in the i^{th}cluster, $j=1,2,...,M$; $i=1,2,...,N$. Then, mean square between the cluster means, $\sigma_{by}^2=\frac{1}{N}\sum_{i=1}^{N}\left(\bar{Y}_i-\bar{\bar{Y}}\right)^2$, where $\bar{Y}_i$ and $\bar{\bar{Y}}$ are the means of the i^{th}cluster and cluster means. σ_{by}^2 can again be expressed as $\sigma_{by}^2=\frac{1}{M^2}\sigma_T^2$, where $\sigma_T^2=\frac{1}{N}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$ is the mean square of cluster totals and $\bar{Y}$ is the mean of cluster totals

$$\Rightarrow \sigma_{by}^2=\frac{1}{NM^2}\sum_{i=1}^{N}\left(\sum_{j=1}^{M}Y_{ij}\right)^2-\frac{1}{M^2}\left(\frac{\sum_{i=1}^{N}Y_i}{N}\right)^2$$

$$=\frac{1}{NM^2}\sum_{i=1}^{N}\sum_{j=1}^{M}Y_{ij}^2+\frac{1}{NM^2}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}Y_{ij}Y_{ij'}$$

$$-\frac{1}{N^2M^2}\left(\sum_{i=1}^{N}Y_i^2+\sum_{i=1}^{N}\sum_{\substack{i'=1\\ i\neq i'}}^{N}Y_iY_{i'}\right)$$

$$=\frac{1}{NM^2}\sum_{i=1}^{N}\sum_{j=1}^{M}Y_{ij}^2+\frac{1}{NM^2}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}Y_{ij}Y_{ij'}$$

$$-\frac{1}{N^2M^2}\sum_{i=1}^{N}\left\{\sum_{j=1}^{M}Y_{ij}^2+\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}Y_{ij}Y_{ij'}\right\}$$

$$-\frac{1}{N^2M^2}\sum_{i=1}^{N}\sum_{\substack{i'=1\\ i\neq i'}}^{N}\left(\sum_{j=1}^{M}Y_{ij}\right)\left(\sum_{k=1}^{M}Y_{ik}\right)$$

$$\Rightarrow \sigma_{by}^2=\frac{1}{NM^2}\left(1-\frac{1}{N}\right)$$

$$\left\{\sum_{i=1}^{N}\sum_{j=1}^{M}Y_{ij}^2+\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}Y_{ij}Y_{ij'}\right\}$$

$$-\frac{1}{N^2M^2}\sum_{i=1}^{N}\sum_{\substack{i'=1\\ i\neq i'}}^{N}\sum_{j=1}^{M}\sum_{k=1}^{M}Y_{ij}Y_{i'k} \qquad (2)$$

Taking conditional expectation of (2) given x's

$$\xi\left(\sigma_{by}^2|x\right)=\frac{1}{NM^2}\left(1-\frac{1}{N}\right)$$

$$\left\{\sum_{i=1}^{N}\sum_{j=1}^{M}V\left(Y_{ij}|X_{ij}\right)+\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\xi\left(Y_{ij}|X_{ij}\right)\right)^2\right\}$$

$$+\frac{1}{NM^2}\left(1-\frac{1}{N}\right)\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}\left\{\xi\left(Y_{ij}|X_{ij}\right)\xi\left(Y_{ij'}|X_{ij'}\right)+\varsigma\left(Y_{ij},Y_{ij'}|X_{ij},X_{ij'}\right)\right\}-\frac{1}{N^2M^2}$$

$$\sum_{i=1}^{N}\sum_{\substack{i'=1\\ i\neq i'}}^{N}\sum_{j=1}^{M}\sum_{k=1}^{M}\left\{\xi\left(Y_{ij}|X_{ij}\right)\xi\left(Y_{i'k}|X_{i'k}\right)+\varsigma\left(Y_{ij},Y_{i'k}|X_{ij},X_{i'k}\right)\right\} \qquad (3)$$

Using (1) in (3), we get

$$\xi\left(\sigma_{by}^2|x\right)=\frac{1}{NM^2}\left(1-\frac{1}{N}\right)$$

$$\left\{\sum_{i=1}^{N}\sum_{j=1}^{M}\sigma^2X_{ij}+\sum_{i=1}^{N}\sum_{j=1}^{M}\left(\alpha+\beta X_{ij}\right)^2\right\}$$

$$+\frac{1}{NM^2}\left(1-\frac{1}{N}\right)\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{\substack{j'=1\\ j\neq j'}}^{M}\left(\alpha+\beta X_{ij}\right)\left(\alpha+\beta X_{ij'}\right)$$

$$-\frac{1}{N^2M^2}\sum_{i=1}^{N}\sum_{\substack{i'=1\\ i\neq i'}}^{N}\sum_{j=1}^{M}\sum_{k=1}^{M}\left(\alpha+\beta X_{ij}\right)\left(\alpha+\beta X_{i'k}\right)$$

On simplification, we get

$$\xi\left(\sigma_{by}^2|x\right)=\beta^2\sigma_{bx}^2+\frac{\sigma^2(N-1)}{N^2M^2}\sum_{i=1}^{N}\sum_{j=1}^{M}X_{ij}, \qquad (4)$$

where $\sigma_{bx}^2=\frac{1}{N}\sum_{i=1}^{N}\left(\bar{X}_i-\bar{\bar{X}}\right)^2$

3. Derivation of Methods of Finding Optimum Strata Boundaries

The conditional expectation of σ_{by}^2 given x in (4) can be expressed as

$$\xi\left(\sigma_{by}^2|x\right)=\frac{\beta^2}{M^2}\sigma_{xT}^2+\frac{\sigma^2(N-1)}{NM^2}\mu_{xT}, \qquad (5)$$

where $\sigma_{xT}^2=\frac{1}{N}\sum_{i=1}^{N}(X_i-\bar{X})^2$ is the mean square of the cluster totals and μ_{xT} is the mean of cluster totals of the population for the x variable.

From this step onwards, we consider the notations y and x as the study variable and auxiliary variable respectively for cluster totals as unit of population, then we can rewrite (5) as

$$\xi\left(\sigma_{by}^2|x\right)=\frac{\beta^2}{M^2}\sigma_x^2+\frac{\sigma^2(N-1)}{NM^2}\mu(x) \qquad (6)$$

For stratification purpose, we divide the population of N units into L number of strata such that $\sum_{h=1}^{L}N_h=N$ and a sample of size n_h is selected by SRSWR from h^{th}stratum of size N_h such that $\sum_{h}^{L}n_h=n$.

Sampling variance of the estimator of the population mean in stratified sampling for the study variable is $V(\bar{y}_{st})=\sum_{h=1}^{L}\frac{W_h^2\sigma_{hy}^2}{n_h}$, where W_h is the weight of h^{th} stratum.

$$\Rightarrow \xi\{V(\bar{y}_{st}|\boldsymbol{X})\}=\sum_{h=1}^{L}\frac{W_h^2\xi\left(\sigma_{hy}^2|\underline{X}_h\right)}{n_h} \qquad (7)$$

where $\boldsymbol{X}=\left(\underline{X}_1,\underline{X}_2,...,\underline{X}_N\right)$

and $\underline{X}'_h=\left(X_{h1}\ X_{h2}\ ...\ X_{N_h}\right)$.

Since, the allocation proportional to stratum total of the auxiliary variable is considered, we have

$$n_h\propto X_h$$

$$\Rightarrow n_h=\frac{nW_h\mu_h(x)}{\bar{X}} \qquad (8)$$

where $\mu_h(x)$ is the mean the h^{th} stratum and $\bar{X}$ is the population mean of the x variable.

From (6), (7) and (8), we get

$$n\xi\{V(\bar{y}_{st}|\boldsymbol{X})\}=\bar{X}\sum_{h=1}^{L}\left\{\frac{\beta^2}{M^2}\frac{W_h^2\sigma_{hx}^2}{W_h\mu_h(x)}+\frac{\sigma^2W_h^2(N_h-1)\mu_h(x)}{N_hM^2W_h\mu_h(x)}\right\}$$

$$\Rightarrow n\xi\{V(\bar{y}_{st}|\boldsymbol{X})\}=\frac{\bar{X}\beta^2}{M^2}\sum_{h=1}^{L}\frac{W_h\sigma_{hx}^2}{\mu_h(x)}+\frac{\sigma^2\bar{X}(N-L)}{NM^2}. \qquad (9)$$

If $f(x)$ is probability density function, we can get

$$\left.\begin{aligned} W_h &= \int_{x_{h-1}}^{x_h} f(t)dt \\ W_h\mu_h(x) &= \int_{x_{h-1}}^{x_h} tf(t)dt \\ \{\sigma_{hx}^2+\mu_h^2(x)\}W_h &= \int_{x_{h-1}}^{x_h} t^2 f(t)dt \end{aligned}\right\} \quad (10)$$

Minimising $n\xi\{V(\bar{y}_{st}|\boldsymbol{X})\}$ in (9) is equivalent to minimising $\sum_{h=1}^{L}\frac{W_h\sigma_{hx}^2}{\mu_h(x)}$,

Therefore, we take $\frac{\delta}{\delta x_h}\sum_{h=1}^{L}\frac{W_h\sigma_{hx}^2}{\mu_h(x)}=0$.

All other terms except $\frac{\delta}{\delta x_h}\left(\frac{W_h\sigma_{hx}^2}{\mu_h(x)}\right)$ & $\frac{\delta}{\delta x_h}\left(\frac{W_{h+1}\sigma_{(h+1)x}^2}{\mu_{h+1}(x)}\right)$ will vanish, therefore, we get

$$\frac{\delta}{\delta x_h}\left(\frac{W_h\sigma_{hx}^2}{\mu_h(x)}\right)+\frac{\delta}{\delta x_h}\left(\frac{W_{h+1}\sigma_{(h+1)x}^2}{\mu_{h+1}(x)}\right)=0. \quad (11)$$

Considering the first term of (11) and using (10)

$$\frac{\delta}{\delta x_h}(W_h\sigma_{hx}^2)=\{x_h-\mu_h(x)\}^2 f(x_h), \text{ and}$$

$$\frac{\delta}{\delta x_h}\left\{\frac{W_h\sigma_{hx}^2}{\mu_h(x)}\right\}=\frac{\{x_h-\mu_h(x)\}\{\mu_h(x)x_h-\mu_h^2(x)-\sigma_{hx}^2\}}{\mu_h^2(x)}f(x_h). \quad (12)$$

Similarly, we can get the second term of (11) as

$$\frac{\delta}{\delta x_h}\left\{\frac{W_{h+1}\sigma_{(h+1)x}^2}{\mu_{h+1}(x)}\right\}=\frac{\{x_h-\mu_{h+1}(x)\}\{\mu_h+1(x)x_h-\mu_{h+1}^2(x)-\sigma_{(h+1)x}^2\}}{\mu_{h+1}^2(x)}\{-f(x_h)\}. \quad (13)$$

From (11), (12) and (13), we get

$$\frac{\{x_h-\mu_h(x)\}\{\mu_h(x)x_h-\mu_h^2(x)-\sigma_{hx}^2\}}{\mu_h^2(x)}=\frac{\{x_h-\mu_{h+1}(x)\}\{\mu_h+1(x)x_h-\mu_{h+1}^2(x)-\sigma_{(h+1)x}^2\}}{\mu_{h+1}^2(x)}. \quad (14)$$

Equations (14) give OSB for the estimation variable y based on auxiliary variable x. We call (14) as exact equations.

4. Derivation of Methods of Finding AOSB Corresponding to the Exact Equations

4.1. Approximation Based on Series Expansion

Since the exact equations (14) are implicit, i.e., equations consist of parameters which are the functions of OSB, it is difficult in solving for OSB in stratifying populations. Therefore, in this section, we carry out derivation for obtaining the solutions of equations (14) which give AOSB. Singh and Sukhatme [12] developed a technique to use Ekman's [33] identity for obtaining series expansion of conditional mean and variance. Gupt and Ahamed [29,30], Gupt et al. [31] used the technique for obtaining series expansion of conditional mean and variance for some functions. Here, in this paper too, the same technique is used for which we assume the existence of continuity and first three partial derivatives of $f(x)$ with respect to x, $\forall x \in (x_{h-1}, x_{h+1})$ for all the values of h. For expanding, right hand side of equations (14) about the point x_h, we take $k_{h+1}=x_{h+1}-x_h$ and all the derivatives are evaluated at x_h. Thus series expansion of conditional mean and variance are obtained as follows:

$$\mu_{(h+1)x}=x_h+\frac{k_{h+1}}{2}+\frac{f'}{12f}k_{h+1}^2+\frac{ff''-f'^2}{24f^2}k_{h+1}^3+\frac{9f'''f^2-25ff'f''+15f'^3}{720f^3}k_{h+1}^4+O(k_{h+1}^4). \quad (15)$$

$$\sigma_{(h+1)x}^2=\frac{k_{h+1}^2}{12}+\frac{2ff''-5f'^2}{720f^2}k_{h+1}^4+O(k_{h+1}^5). \quad (16)$$

From (15), we get

$$\frac{x_h}{\mu_{(h+1)x}}=x_h\mu_{(h+1)x}^{-1}$$

$$=x_hx_h^{-1}\left\{1+\frac{k_{h+1}}{2x_h}+\frac{f'}{12fx_h}k_{h+1}^2+\frac{ff''-f'^2}{24f^2x_h}k_{h+1}^3+\frac{9f'''f^2-25ff'f''+15f'^3}{720f^3x_h}k_{h+1}^4+O(k_{h+1}^5)\right\}^{-1}$$

$$\Rightarrow\frac{x_h}{\mu_{(h+1)x}}=1-\frac{k_{h+1}}{2x_h}+\frac{3f-f'x_h}{12fx_h^2}k_{h+1}^2+\frac{2ff'x_h-3f^2-ff''x_h^2+f'^2x_h^2}{24f^2x_h^3}k_{h+1}^3$$
$$+\frac{\left\{\begin{matrix}45f^3-25ff'^2x_h^2-45f^2f'x_h+30f^2f''x_h^2\\-9f'''f^2x_h^3+25ff'f''x_h^3-15f'^3x_h^3\end{matrix}\right\}}{720f^3x_h^4}k_{h+1}^4+O(k_{h+1}^5) \quad (17)$$

Again from (15) and (16), we get

$$\frac{\sigma_{(h+1)x}^2}{\mu_{(h+1)x}^2}=\left\{\frac{k_{h+1}^2}{12}+\frac{2ff''-5f'^2}{720f^2}k_{h+1}^4+O(k_{h+1}^5)\right\}$$
$$\left\{x_h+\frac{k_{h+1}}{2}+\frac{f'}{12f}k_{h+1}^2+\frac{ff''-f'^2}{24f^2}k_{h+1}^3+\frac{9f'''f^2-25ff'f''+15f'^3}{720f^3}k_{h+1}^4+O(k_{h+1}^4)\right\}^{-2}$$

$$=x_h^{-2}\left\{\frac{k_{h+1}^2}{12}-\frac{k_{h+1}^3}{12x_h}+\frac{9f-2f'x_h}{144fx_h^2}k_{h+1}^4+\frac{2ff''-5f'^2}{720f^2}k_{h+1}^4\right\}$$
$$+O(k_{h+1}^5)$$

$$\Rightarrow \frac{\sigma^2_{(h+1)x}}{\mu^2_{(h+1)x}}$$

$$= \frac{k^2_{h+1}}{12x_h^2} - \frac{k^3_{h+1}}{12x_h^3} + \frac{45f^2 - 10ff'x_h + 2ff''x_h^2 - 5{f'}^2x_h^2}{720f^2x_h^4}k^4_{h+1} + O(k^5_{h+1}). \quad (18)$$

Using (15), (17) and (18) in the exact equations (14), we can get as follows:

Right Hand Side of equations (14)

$$= \left(x_h - \mu_{(h+1)x}\right)\left(\frac{x_h}{\mu_{(h+1)x}} - 1 - \frac{\sigma^2_{(h+1)x}}{\mu^2_{(h+1)x}}\right)$$

$$= \left\{-\frac{k_{h+1}}{2} - \frac{f'}{12f}k^2_{h+1} - \frac{ff'' - {f'}^2}{24f^2}k^3_{h+1} - \frac{9f'''f^2 - 25ff'f'' + 15{f'}^3}{720f^3}k^4_{h+1}\right\}$$

$$\left[-\frac{k_{h+1}}{2x_h} + \frac{3f - f'x_h}{12fx_h^2}k^2_{h+1} + \frac{2ff'x_h - 3f^2 - ff''x_h^2 + {f'}^2x_h^2}{24f^2x_h^3}k^3_{h+1} + \frac{\begin{Bmatrix}45f^3 - 25f{f'}^2x_h^2 - 45f^2f'x_h + 30f^2f''x_h^2 \\ -9f'''f^2x_h^3 + 25ff'f''x_h^3 - 15{f'}^3x_h^3\end{Bmatrix}}{720f^3x_h^4}k^4_{h+1} - \frac{k^2_{h+1}}{12x_h^2} + \frac{k^3_{h+1}}{12x_h^3} - \frac{45f^2 - 10ff'x_h + 2ff''x_h^2 - 5{f'}^2x_h^2}{720f^2x_h^4}k^4_{h+1}\right]$$

$$+ O(k^5_{h+1})$$

$$= \frac{k^2_{h+1}}{4x_h} + \frac{f'x_h - 2f + f'x_h}{24fx_h^2}k^3_{h+1} + \frac{\begin{Bmatrix}3ff''x_h^2 - 3{f'}^2x_h^2 - 2ff'x_h + {f'}^2x_h^2 \\ -6ff'x_h + 3f^2 + 3ff''x_h^2 - 3{f'}^2x_h^2\end{Bmatrix}}{144f^2x_h^3}k^4_{h+1} + O\left(k_i^5\right)$$

$$= \frac{k^2_{h+1}}{4x_h}\left\{1 + \frac{f'x_h - f}{fx_h}\frac{k_{h+1}}{3} + \frac{\begin{pmatrix}6ff''x_h^2 - 5{f'}^2x_h^2 \\ -8ff'x_h + 3f^2\end{pmatrix}}{36f^2x_h^2}k^2_{h+1}\right\}$$

$$+ O(k^5_{h+1})$$

$$= \frac{k^2_{h+1}}{4x_h}\left\{1 + \frac{\frac{\delta}{\delta x_h}\left(\frac{f}{x_h}\right)}{\left(\frac{f}{x_h}\right)}\frac{k_{h+1}}{3} + O(k^2_{h+1})\right\}.$$

Thus, we can rewrite the right hand side of equations (14) as

$$\text{RHS} = \frac{k^2_{h+1}}{4x_h}\left\{1 + \frac{\frac{\delta}{\delta x_h}\left(\frac{f(x_h)}{x_h}\right)}{\left(\frac{f(x_h)}{x_h}\right)}\frac{k_{h+1}}{3} + O(k^2_{h+1})\right\}.$$

Similarly, we can obtain the left hand side of equations (14) as

$$\text{LHS} = \frac{k_h^2}{4x_h}\left\{1 - \frac{\frac{\delta}{\delta x_h}\left(\frac{f(x_h)}{x_h}\right)}{\left(\frac{f(x_h)}{x_h}\right)}\frac{k_h}{3} + O(k_h^2)\right\},$$ where $k_h = x_h - x_{h-1}$

Therefore, equations (14) can be reduced to

$$\frac{k_h^2}{4x_h}\left\{1 - \frac{\{g(t)\}'}{g(t)}\frac{k_h}{3} + O(k_h^2)\right\}$$

$$= \frac{k^2_{h+1}}{4x_h}\left\{1 + \frac{\{g(t)\}'}{g(t)}\frac{k_{h+1}}{3} + O(k^2_{h+1})\right\},$$

where $g(t) = \frac{f(t)}{t}$ and writing t in place the variable x_h.

On raising power $\frac{3}{2}$ on both sides and expanding by binomial theorem,

$$k_h^3\left\{1 - \frac{\{g(t)\}'}{g(t)}\frac{k_h}{2} + O(k_h^2)\right\}$$

$$= k^3_{h+1}\left\{1 + \frac{\{g(t)\}'}{g(t)}\frac{k_{h+1}}{2} + O(k^2_{h+1})\right\}$$

$$\Rightarrow k_h^3\, g(t)\left\{1 - \frac{\{g(t)\}'}{g(t)}\frac{k_h}{2} + O(k_h^2)\right\}$$

$$= k_i^3\, g(t)\left\{1 + \frac{\{g(t)\}'}{g(t)}\frac{k_{h+1}}{2} + O(k^2_{h+1})\right\} \quad (19)$$

The identity proposed by Singh and Sukhatme [12] and used by, inter alia, Gupt and Ahamed [29,30] and Gupt et al. [31] is as follows:

$$\left[\int_m^{\text{n}} \sqrt[\alpha]{f(t)}\,dt\right]^{\alpha} = k^{\alpha}f(\text{m})\left[1 + \frac{k_i}{2}\frac{f'(\text{m})}{f(\text{m})} + O(k^2)\right]$$

$$= k^{\alpha-1}\int_{\text{m}}^{\text{n}} f(t)dt\left(1 + O(k^2)\right) \quad (20)$$

where $k = n - m$

Using the identity (20) in (19), we proceed as follows:

$$\Rightarrow k_h^2 \int_{x_{h-1}}^{x_h} g(t)dt\{1 + O(k_h^2)\}$$

$$= k^2_{h+1}\int_{x_h}^{x_{h+1}} g(t)dt\{1 + O(k^2_{h+1})\}.$$

Since $h = 1,2,3,. . . ,L.$, the equality holds for all the strata, therefore, we get

$$k_h^2\int_{x_{h-1}}^{x_h} g(t)dt = C_1 \quad (21)$$

$$\Rightarrow \int_{x_{h-1}}^{x_h} \sqrt[3]{g(t)}dt = C_2 \quad (22)$$

The AOSB corresponding to exact equations (14) are given by the two equivalent methods (21) and (22). The values of constants C_1 and C_2 can be approximately evaluated by $C_1 = \frac{1}{L}(b-a)^2 \int_a^b g(t)dt$ and $C_2 = \frac{1}{L}\int_a^b \sqrt[3]{g(t)}dt$ respectively, where we assume b and a are upper and lower bounds of points of stratification $x_h's$, i.e., $a \le x_h \le b$.We can use (21) and (22) in finding AOSB, i.e., $x_h's$ by fixing lower boundary x_{h-1} every time.

Thus, the above analytical study has led to arrive at the following theorem.

Theorem 1: *If the function $g(x)$ is bounded and possesses first two partial derivatives for all values of x in (a, b), for a given number of strata, taking equal intervals on the cumulative of $\sqrt[3]{g(x)}$ gives AOSB.*

4.2. Other Approximations Using Assumptions on Coefficient of Variation

Again, we deduce some more methods of approximation from exact equations (14), these approximation methods are still implicit but easy to use. We proceed as follows:

Equations (14) can be rewritten as

$$\{x_h - \mu_h(x)\}\left\{\frac{x_h}{\mu_h(x)} - 1 - c_{hx}^2\right\} = \{x_h - \mu_{h+1}(x)\}\left\{\frac{x_h}{\mu_{h+1}(x)} - 1 - c_{(h+1)x}^2\right\}, \quad (23)$$

where c_{hx}^2 and $c_{(h+1)x}^2$ are the square of coefficients of variation of h^{th} and $(h+1)^{th}$strata.

If we consider square of coefficients of variation are approximately equal for every two successive strata, i.e., $c_{hx}^2 \approx c_{(h+1)x}^2$ and approximately equal to arithmetic mean of square of coefficients of variation of the consecutive strata, then AOSB are given by

$$x_h = \sqrt{\left(1 + \bar{C}_{\left(h+\frac{1}{2}\right)x}^2\right)\mu_h(x)\mu_{h+1}(x)}\ , \quad (24)$$

where $\bar{C}_{\left(h+\frac{1}{2}\right)x}^2 = \frac{c_{hx}^2 + c_{(h+1)x}^2}{2}$

If we consider the square of coefficients of variation for every two successive strata are approximately equal to geometric mean of the square of coefficients of variation the two consecutive strata, AOSB are given by

$$x_h = \sqrt{\left(1 + \tilde{C}_{\left(h+\frac{1}{2}\right)x}\right)\mu_h(x)\mu_{h+1}(x)} \quad (25)$$

where $\tilde{C}_{\left(h+\frac{1}{2}\right)x} = c_{hx}c_{(h+1)x}$

If we consider square of coefficients of variation are negligibly small relative to unity, AOSB are given by

$$x_h = \sqrt{\mu_h(x)\mu_{h+1}(x)} \quad (26)$$

Thus, we have obtained (24), (25) and (26) as approximations to equations (14) to give AOSB.

5. Empirical Illustration

The proposed exact equations (14) and methods of approximation (22), (24), (25) and (26) are illustrated in stratifying population of two districts, Lunglei and Serchhip districts, of Mizoram, a state of India, in which villages are taken as elements of cluster. The data of villages is taken from Village Profile & Development Indicators [34]. There are 193 villages in the two districts. We take number of households of a village as the study variable y and population of village as the auxiliary variable x.The correlation between the study variable y and stratification variable x is sufficiently high, i.e., 0.9604. The data is shown in appendix I.

Mizoram is a hilly state of India, 88.93% of the total geographical area is covered by hilly forests; the villages and towns are spread over the hilly terrain of the state. There are rolling hills, tiny valleys, rivers and lakes in the state. Villages are connected by mostly minor and a few major hilly roads. In many cases, the geographical distance between any two villages may be short but they are separated by rivers, lakes, streams, and swamps in the thick rainforest. Therefore, the road transport connecting the two villages may be extremely long requiring lots of energy and time to travel from one to the other. Considering physical feature of the land and pattern of distribution of villages, stratified cluster sampling may be an appropriate sampling design in survey planning. Therefore, we use Google Earth pro and Geographic Information System to locate the villages, rivers, minor roads, major roads, hill ranges and altitudes while forming the clusters. The clusters are formed not only by combining the villages connected by the shortest roads but also taking in account other constraints like variation in altitude and separation by rivers, lakes and thick forest cover. The formation of clusters is shown in appendix II.

In the case of illustrating methods of approximation (21) and (22), since we have theoretically proven both the methods are equivalent, we conveniently use (22). While using (22), we require a Probability Density Function (PDF) that the auxiliary variable x follows. For fitting a suitable distribution, we use the data of x variable in which each value of the variable is divided by 1000. We have two sets of data, one is when each cluster is made of two villages, i.e., cluster size, $M = 2$ and the other is when each cluster is made of three villages, i.e., cluster size, $M = 3$.

We try to find the most suitable PDF that the x variable of the live data follows. The fitdistrplus package in R-software is used in fitting a number of known PDFs in data of x variable of both the populations by using the methods - Maximum Likelihood Estimation (MLE), Moment Matching Estimation (MME) and Quantile Matching Estimation (QME) one after another.

Of all the various PDFs we have tried to fit to the data, Gamma Probability Density Function (GPDF) is found to be fitting best to the data of both populations; the decision of best fitting is made by taking into consideration simultaneously the values of LL (log likelihood), AIC (Akaike Information Criteria), BIC (Bayesian Information Criteria) and standard errors ($s.e$) of parameters.

Thus, the PDF followed by the x variable is as follows:

$$f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \tag{27}$$

where $\alpha > 0,\ \beta > 0,\ \ \forall x \in (0,\ \infty)$

The two populations are characterised as follows in fitting GPDF to them.

a. In the data for $M = 2$, shape parameter $\alpha = 2.24436$, rate parameter $\beta = 1.49746$, $s.e(\alpha) = 0.302965$, $s.e(\beta) = 0.226425$, $LL = -120.354$, $AIC = 244.7089$, $BIC = 249.8376$, which are estimated by MLE method.

b. In the data for $M = 3$, shape parameter $\alpha = 3.29280$, rate parameter $\beta = 1.464706$, $s.e(\alpha) = 0.551335$, $s.e(\beta) = 0.266752$, $LL = -97.53638$, $AIC = 199.0728$, $BIC = 203.3905$, which are estimated by MLE method.

We use the above PDF (27) in illustrating approximation method (22), along with the estimates of parameters, in stratifying both the populations. Numerical integration and differentiation methods are used in working out the approximation method in stratifying the populations.

We examine the performance of all the proposed methods of stratification in the stratified cluster sampling design in the two sampling frames. At first, we illustrate the methods in the population in which cluster is formed by the combination of two villages and secondly in the population in which cluster is formed by combination of three villages; the results are shown in Tables 1, 2, 3 and 4. For population of clusters of size two, we present points of stratification due to all the proposed methods of stratification in Table 1 and the variances and relative efficiencies due to the methods in Table 2. Similarly, for population of clusters of size 3, we present the said results in the same way in Tables 3 and 4. Each of the two populations is stratified into two, three, four, five, and six strata by using the all the proposed stratification methods and equal interval stratification. The efficiencies of the proposed methods are compared with that of equal interval stratification in both the populations for each considered number of strata, L =2, 3, 4, 5 and 6.

Table 1. Points of stratification in the population of clusters of size two, $M = 2$

L	Points of stratification					
	Equal Interval Stratification	Stratification due to Exact equations (14)	Stratification due to approx. method (22)	Stratification due to approx. method (24)	Stratification due to approx. method (25)	Stratification due to approx. method (26)
2	2606	1948.76	1670.25	1956.62	1948.17	1576.24
3	1737.33	1195.17	1054.43	1197.12	1181.24	1138.76
	3474.37	2562.68	2446.52	2562.16	2551.44	2488.94
4	1303	1141.11	785.83	1143.14	1134.86	938.54
	2606	2170.56	1670.23	2170.90	2170.72	1910.28
	3909	3714.61	2930.7	3715.32	3714.08	3421.17
5	1042.40	1037.59	633.96	1039.08	1023.56	665.25
	2084.80	1867.38	1286.33	1868.23	1859.03	1212.51
	3127.20	2871.18	2110.77	2871.44	2870.71	2063.53
	4169.60	4083.96	3267.01	4084	4083.99	3513.50
6	868.67	685.69	535.91	763.70	680.49	665.25
	1737.33	1224.06	1054.43	1300.39	1224.04	1212.51
	2606	1965.15	1670.23	2005.12	1964.47	1952.09
	3474.67	2901.17	2446.52	2901.25	2901.16	2890.63
	4343.33	4083.96	3516.02	4084	4083.99	4071.54

Table 2. Variance and Relative Efficiencies in the population of clusters of size two, $M = 2$

L	Variances and Relative Efficiencies (RE)										
	Equal Interval $nV(\bar{y}_{st})$	Exact equations (14) $nV(\bar{y}_{st})$	RE	Approx. method (22) $nV(\bar{y}_{st})$	RE	Approx. method (24) $nV(\bar{y}_{st})$	RE	Approx. method (25) $nV(\bar{y}_{st})$	RE	Approx. method (26) $nV(\bar{y}_{st})$	RE
2	3837.94	3481.67	110.23	3769.89	101.78	3481.67	110.23	3481.67	110.23	3777.08	101.61
3	2657.10	1747.45	152.06	1401.19	189.63	1747.45	152.06	1673.32	158.79	1542.09	172.31
4	1929.32	1303.18	148.05	1389.96	138.80	1303.18	148.05	1303.18	148.05	1327.32	145.35
5	1177.71	1110.60	106.04	997.14	119.75	1135.11	103.75	1157.05	101.79	917.41	128.37
6	1198.35	904.22	132.53	693.87	179.28	1030.96	116.24	908.45	131.91	908.45	131.91

Table 3. Points of stratification in the population of clusters of size three, $M = 3$

L	Points of stratification					
	Equal Interval Stratification	Stratification due to Exact equations (14)	Stratification due to approx. method (22)	Stratification due to approx. method (24)	Stratification due to approx. method (25)	Stratification due to approx. method (26)
2	2760.5	2252.81	2267.28	2250.97	2250.16	2158.41
3	1840.33 3680.67	1882.76 3739.34	1630.44 3054.09	1884.89 3739.45	1881.83 3739.44	1843.76 3689.25
4	1380.25 2760.50 4140.75	1281.16 2206.93 4033.18	1344.91 2267.28 3543.94	1281.67 2206.61 4034.38	1279.91 2206.42 4032.78	1264.84 2187.85 4004.84
5	1104.2 2208.4 3312.6 4416.8	1242.71 2125.28 3277.32 4614.26	1180.22 1872.54 2714.75 3886.22	1243.07 2125.77 3277.28 4614.92	1242.34 2125.17 3277.25 4612.71	1227.82 2078.20 3120.91 4472.03
6	920.17 1840.33 2760.5 3680.68 4600.83	1190.62 1854.31 2650.97 3515.14 4765.48	1072.32 1630.44 2267.28 3054.09 4141.62	1191.09 1854.06 2651.25 3515.14 4765.61	1189.21 1853.64 2649.21 3515.14 4765.44	1179.09 1806.41 2562.61 3474.63 4755.21

Table 4. Variance and Relative Efficiencies in the population of clusters of size three, $M = 3$

L	Variances and Relative Efficiencies (RE)										
	Equal Interval $nV(\bar{y}_{st})$	Exact equations (14) $nV(\bar{y}_{st})$	RE	Approx. method (22) $nV(\bar{y}_{st})$	RE	Approx. method (24) $nV(\bar{y}_{st})$	RE	Approx. method (25) $nV(\bar{y}_{st})$	RE	Approx. method (26) $nV(\bar{y}_{st})$	RE
2	2759.67	2097.53	117.27	2097.52	117.26	2097.53	117.27	2097.53	117.27	2097.53	117.27
3	1025.14	1025.14	100	1078.06	95.09	1025.14	100	1025.14	100	1025.14	100
4	1081.06	923.48	117.06	529.32	204.23	923.48	117.06	923.48	117.06	923.48	117.06
5	675.81	580.56	116.41	555.01	121.76	580.56	116.41	580.56	116.41	633.51	106.68
6	678.61	512.77	132.34	554.45	122.39	533.59	127.18	533.59	127.18	523.16	129.71

It is seen that in the population of clusters of size two, the exact equations (14) perform with higher efficiencies at $L = 2$, 5 and much higher efficiencies at $L = 3$, 4 and 6 than that of equal interval stratification. In the same way, approximation methods (22), (24), (25) and (26) too perform when compared with equal interval stratification. Approximation methods (22) and (26) are found to be relatively better in overall performances than all the other four proposed methods of stratification.

In the population of clusters of size three, the exact equations (14), approximation methods (22), (24), (25) and (26) perform with considerably higher efficiencies than that of equal interval stratification at all the considered number of strata except when $L = 3$ at which all the proposed methods of stratification other than (22) perform with same efficiency with equal interval stratification; method (22) performs with slightly lower efficiency than that of equal interval stratification at $L = 3$. However, in all other number of strata, method (22) performs with strikingly high efficiencies; whereas all other proposed methods of stratification are performing with more or less same efficiency with that of method (22).

But, although the proposed methods of stratification perform well in both the populations, it is found that the methods perform relatively better in the population of clusters of size two than in the population of clusters of size three.

6. Conclusions

It is seen in stratified cluster sampling with clusters of equal size, the proposed methods of stratification are performing excellently. The inevitability of the use of cluster sampling due to the nature of spatial relationship between elements of a population or unavailability of proper sampling frame and strategy for ensuring the precision of estimator of population parameter are simultaneously addressed in stratified cluster sampling design presented in this paper. The exact equations (14) and approximation methods (24), (25) and (26) are all performing with more or less same efficiencies, rather interestingly, the approximation method (26), i.e., AOSB are given by the geometric mean of means of consecutive strata, is found to be performing slightly better than other three proposed methods of stratification- (14), (24) and (25). The approximation method (22) which is in the form definite integral of a defined function according to the population used performs best in the overall. Although the methods of approximation (24), (25) and (26) are implicit, they are easy to use. Therefore, all these proposed methods of stratification may be useful in the practical application of survey planning for socio-economic survey.

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Appendix I

Village wise population and number of households, Lunglei and Serchhip districts, Mizoram

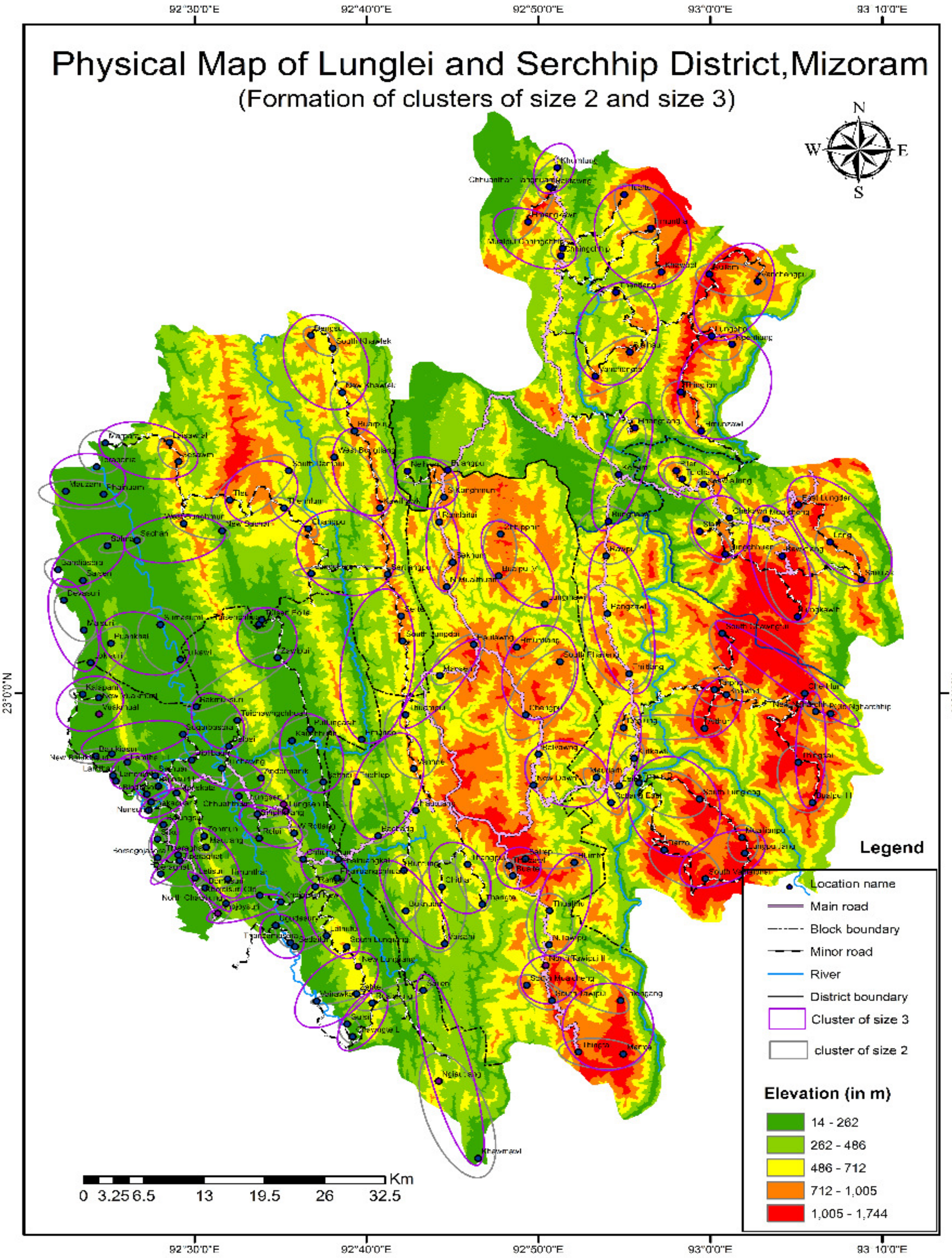
Sl. No.	Name of village	Population	No. of household	Sl. No.	Name of village	Population	No. of household
1	Thinglian	504	82	62	Sertlangpui	560	110
2	Thentlang	780	147	63	Sesawm	428	89
3	Sialhau	600	109	64	Thenhlum	1266	250
4	Rullam	650	112	65	Terabonia	525	104
5	Khumtung	1340	254	66	Tleu	107	23
6	Khawbel	757	131	67	Tuikawi	949	217
7	Lungpho	1005	182	68	Belthei	631	105
8	Mualpui Chhingchhip	1750	350	69	Belpei	1186	262
9	Vanchengte	126	27	70	Balukiasury	193	37
10	Vanchengpui	1000	170	71	Balungsury	356	79
11	Keitum	2150	435	72	Bindiasora	490	115
12	Neihloh	380	72	73	Borsegojasora	305	72
13	Ngentiang	676	130	74	Bomasury	430	86
14	Hmunzawl	538	110	75	Chhumkhum	212	53
15	Hmuntha	1028	160	76	Chhuahthum	291	51
16	Hmawngkawn	165	40	77	Chawngte L	1053	254
17	Hualtu	1140	208	78	Chawilung North	369	65
18	Hriangtlang	640	120	79	Diblibagh	1860	382
19	Bungtlang	2263	435	80	Gulsil	227	54
20	Baktawng	2050	336	81	Hmunthar	186	39
21	Buangpui	485	105	82	Kalapani	556	107

22	Chhuanthar Tlangnuam	2420	280	83	Kauchhuah	639	146
23	Chhingchhip	2092	750	84	Khawmawi	1665	297
24	Sialsir	344	57	85	Khojoysuri	410	82
25	Sailulak	964	192	86	Letisury	386	58
26	Lungchhuan	780	145	87	Lalnutui	304	64
27	Khawlailung	2672	562	88	Lamthai I	354	80
28	Piler	505	100	89	Lamthai II	763	168
29	Leng	839	180	90	Lamthai III	281	51
30	Lungkawlh	900	175	91	Lungsen I	1255	251
31	Mualcheng	1510	300	92	Lungsen II	1716	322
32	Tuichang	27	8	93	Muriskata	267	55
33	Bawktlang	350	74	94	Mautlang	263	56
34	Chekawn	350	54	95	Nunsury II	680	136
35	East Lungdar	3700	735	96	Ngiautlang	204	44
36	Bandisora	765	137	97	New Balukiasury	378	67
37	Bunghmun	1232	230	98	New Khojoysury	803	195
38	Bungtlang	255	56	99	New Vuakmual	315	62
39	Changpui	457	103	100	New Lungrang	750	270
40	Dampui	36	12	101	Nunsury I	852	186
41	Dengsur	616	127	102	Old Khojoysury	490	123
42	Devasora	835	145	103	Phairuangchhuah	244	48
43	Hmundo	180	35	104	Phairuangkai	1200	308
44	Kawlhawk	146	34	105	Putlungasih	1758	250
45	Lungchem	401	85	106	Rangte	840	167
46	Lokhi Sury	256	54	107	Rolui	543	95
47	Laisawral	578	109	108	Rotlang West	750	154
48	Marpara South	2691	571	109	Rualalung	380	82
49	Mauzam	715	143	110	S. Lungrang	332	72
50	Malsury	900	170	111	Sailen	132	30
51	New Sachan	114	222	112	Samuksury	975	214
52	New Khawlek	164	33	113	Sedailui	220	68
53	Puankhai	1182	229	114	Silkur	345	73
54	Phainuam	218	44	115	Sihphirtlang	126	28
55	South Khawlek	140	30	116	Sugorbasora	537	111
56	Sumasumi	487	103	117	Tipperagath I	1171	202
57	Salmur	272	55	118	Tipperaghat II	636	122
58	Saisen	255	55	119	Tipperaghat III	633	118
59	Sachan	600	112	120	Thanzamasora	385	75
60	S. Lungdai	280	53	121	Thekaduar	347	75
61	Serte	446	89	122	Tuichawngchhuah	221	49

Continued

Sl. No.	Name of village	Population	No. of household	Sl. No.	Name of village	Population	No. of household
123	Tuichawng	3419	583	159	Thualthu	661	152
124	Tuisenchhuah	330	85	160	Thuampui	467	115
125	Tuisen Bolia	793	153	161	Thingfal	1789	323
126	Undermanik	777	106	162	Thangte	170	38
127	Ugudasury	491	109	163	Thehlep	28	8
128	Vuakmual	6	1	164	Tawipui South	1388	216
129	Vairawkai	185	35	165	Thaizawl	400	83
130	Zehtet	596	118	166	Thangpui	154	32
131	Zawlpui	1766	352	167	Tawipui North I	712	167
132	Zohmun	487	96	168	Tawipui North II	874	201
133	Serhuan	597	135	169	Vaisam	357	81
134	Buknuam	393	81	170	Vanhne	820	201
135	Buarpui	1450	360	171	Aithur	220	36
136	Bualpui V	720	112	172	Bualpui H	1114	205
137	Bualte	446	96	173	Cherhlun	2492	486
138	Chengpui	180	38	174	Denlung	130	24
139	Chithar	220	52	175	Darzo	1590	310
140	Chhipphir	1411	287	176	Khawhri	430	94
141	Dawn	430	72	177	Kutkawk	5	3
142	Haulawng	2245	527	178	Lungpuitlang	204	50
143	Hlumte	210	37	179	Leite	785	170
144	Hmuntlang	122	24	180	Maudarh	74	18
145	Kanghmun South	537	135	181	Muallianpui	1214	219
146	Mamte	852	127	182	New Ngharchhip	757	103
147	Mausen	300	70	183	Old Ngharchhip	168	65
148	Mualcheng South	960	167	184	Pangzawl	3170	503
149	Mualthuam North	1519	330	185	Rawpui	1050	174
150	Lungmawi	272	52	186	Rotlang E	732	161
151	Pachang	39	13	187	South Vanlaiphai	1991	390
152	Phaileng South	330	63	188	S. Lungleng	157	31
153	Ramlaitui	657	158	189	S. Chawngtui	696	140
154	Ralvawng	520	76	190	Tuipui D	945	202
155	Runtung	211	34	191	Thiltlang	1364	249
156	Sairep	264	52	192	Thingsai	2536	503
157	Sekhum	384	85	193	Tarpho	450	95
158	Thlengang	240	43				

Appendix II


Physical Map of Lunglei and Serchhip District,Mizoram
(Formation of clusters of size 2 and size 3)
N
W
E
S
92°30'0"E
92°40'0"E
92°50'0"E
93°0'0"E
93°10'0"E
23°0'0"N
Legend
Location name
Main road
Block boundary
Minor road
River
District boundary
Cluster of size 3
cluster of size 2
Elevation (in m)
14 - 262
262 - 486
486 - 712
712 - 1,005
1,005 - 1,744
0 3.25 6.5 13 19.5 26 32.5 Km

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