Mathematics and Statistics 9(5): 669-684, 2021
DOI: 10.13189/ms.2021.090507 http://www.hrpub.org

Hesitant Fuzzy Network Approach for Alternatives Selection with Incomplete Weight Information

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Received November 18, 2020; Revised July 5, 2021; Accepted July 22, 2021

Cite This Paper in the following Citation Styles

***(a):** [1] Shahira Shafie, Abdul Malek Yaakob , "Hesitant Fuzzy Network Approach for Alternatives Selection with Incomplete Weight Information," Mathematics and Statistics, Vol. 9, No. 5, pp. 669 - 684, 2021. DOI: 10.13189/ms.2021.090507.*

***(b):** Shahira Shafie, Abdul Malek Yaakob (2021). Hesitant Fuzzy Network Approach for Alternatives Selection with Incomplete Weight Information. Mathematics and Statistics, 9(5), 669 - 684. DOI: 10.13189/ms.2021.090507.*

Abstract Networked rule bases in fuzzy system, acknowledged as fuzzy network, carries multiple stages of development in decision making processes that involves the uncertainty in the data used as medium in various field. Fuzzy network promotes transparency in multicriteria decision making (MCDM) whereby the criteria are divided into subsystems of cost and benefit to ensure good assessment performance. By considering Hesitant fuzzy sets (HFS), which gives the permission of a set of possible values to present the membership degree of an element, we develop a novel approach that applies fuzzy network and the maximizing deviation method in solving MCDM problem. Fuzzy network addresses transparency in the formulation and maximizing deviation method can restore weight information in MCDM problems whether partially known or fully unknown. The proposed method is applied in case study of stock evaluation that carries opinion evaluated by several decision makers and compared in terms of performance using Spearman rho correlation.

Keywords Merging Operations, MCDM, Maximizing Deviation, Stock Selection, Transparency, Vertical Merging, Maximizing Deviation Method

1. Introduction

Decision makers often face struggles in evaluating alternatives in accordance to criteria due to different understanding towards problem domain and the way of thinking thus, resulting in different opinions and evaluations. In the context of alternatives selection, decision makers also can describe their intuition on risk and profit through the values of optimism coefficient [1]. Hence, vagueness in data and uncertainty in evaluating alternatives dealt by decision makers can be managed by implying hesitant fuzzy approach in the formulation. Vicenc Torra [2] developed hesitant fuzzy set-in order to overcome uncertainty and vagueness in decision making. As mentioned in [3], hesitant fuzzy sets allow the membership degree of an element of a set to be represented by several possible values. For example, a group of associates are assigned to decide on relevant degree an alternative should achieve according to criteria set. Most of them chose 0.6, certain with 0.7 and a couple of associates chose 0.9. The associates cannot discuss with each other during the evaluation. Thus, the degree that should be met by the alternative in accordance to criterion set can be presented as $\{0.6,0.7,0.9\}$. Compared to interval valued fuzzy set, hesitant fuzzy set able to interpret the evaluation objectively. Thus, evaluations made by decision makers are more valid and explanatory to be carried in decision making.

According to Adel, Teh and Raja [4], a model that could describe the relation of input and output applied comprehensibly is a system that has applied transparency coherently. Fuzzy network delivers transparency in fuzzy mathematical formalism of MCDM by subdividing criteria involved into cost and benefit factors. Therefore, the decision makers can assess the performance of each alternative during the decision-making process [5]. Facilitation of a fuzzy network to an identical linguistic fuzzy system is essential in formal models. The formal models hold mainly compacted information that are formed into nodes in fuzzy networks. As established formal models of fuzzy system imply rule bases of if-then rules, the input from formal model represents as nodes in a fuzzy network with links as connections. The linguistics terms that are used in the models as the inputs and the outputs are commonly adapted from liked scales and represented in the form of positive integers.

Maximizing deviation method is applied in order to generate information on weight of criteria whether partially known or fully unknown. As cited in [6] lack of attention given on weights of criteria due to low expertise level, limited knowledge in the area of problem and time constraint. In decision making, weights of attributes indicate the importance of attributes towards making better decision in evaluating alternatives. Different weights bring different influence towards the rank of alternatives as end result. Hence, maximizing deviation method is implied in the proposed hesitant fuzzy network to find weights of attributes. The case study of stock selection has been considered for the purpose of applicability and the validation of the proposed method. The structure of the paper proceed as section 2 explains the formulation of hesitant fuzzy network thoroughly as section 3 implies the method of hesitant fuzzy network in stock evaluation. Further analysis and conclusion are discussed in the last section 4.

2. Method Formulation

The proposed method is the Fuzzy Network system that incorporates Hesitant Fuzzy Elements (HFN) in which decision makers generate opinion independently due to different background in the area that will affect the decisions made. In addition, decision makers are allowed to present several possible opinions towards the alternatives according to criteria. The criteria implemented in the evaluations are categorized into criteria of cost and benefit. Benefit Level (BL) and Cost Level (CL) are assigned as medium to the categories in the fuzzy system that indicates the value of each category and produce Alternative Level (AL) as end result. As MCDM problems involve distinct set of alternatives, $A = \{A_1, A_2, \dots, A_m\}$,

Step 1

Construct decision matrices where decisions, $h_{el,k}$ and $j_{fl,k}$, made by decision makers are Hesitant Fuzzy Elements (HFEs) according to the alternatives using Figure 1 and 2. where $h_{el,k}$ and $j_{fl,k}$ are decided by kth group of decision makers. As $k = 1, 2, 3, \dots, K$. These matrices are then categorized according to the criteria that are determined in Benefit System and Cost System which is Benefit Criteria and Cost Criteria.

$$D_k^B = \begin{matrix} B_1 \\ B_2 \\ \vdots \\ B_e \end{matrix} \begin{bmatrix} h_{11,k} & h_{12,k} & \cdots & h_{1l,k} \\ h_{21,k} & h_{22,k} & \cdots & h_{2l,k} \\ \vdots & \vdots & \ddots & \vdots \\ h_{e1,k} & h_{e2,k} & \cdots & h_{el,k} \end{bmatrix} \quad (1)$$

$$D_k^C = \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_f \end{matrix} \begin{bmatrix} j_{11,k} & j_{12,k} & \cdots & j_{1l,k} \\ j_{21,k} & j_{22,k} & \cdots & j_{2l,k} \\ \vdots & \vdots & \ddots & \vdots \\ j_{f1,k} & j_{f2,k} & \cdots & j_{fl,k} \end{bmatrix} \quad (2)$$

for $k = 1,2, \dots, k$

The matrices D_k^B that represents the decision matrix of benefit and D_k^C represents the decision matrix of cost. $h_{el,k}$ represents the alternative's rating with respect to the benefit criteria, $B_e(e = 1,2, \dots, e)$, as $j_{fl,k}$ denoted as the rating of alternatives to the cost criteria, $C_f(f = 1,2, \dots, f)$

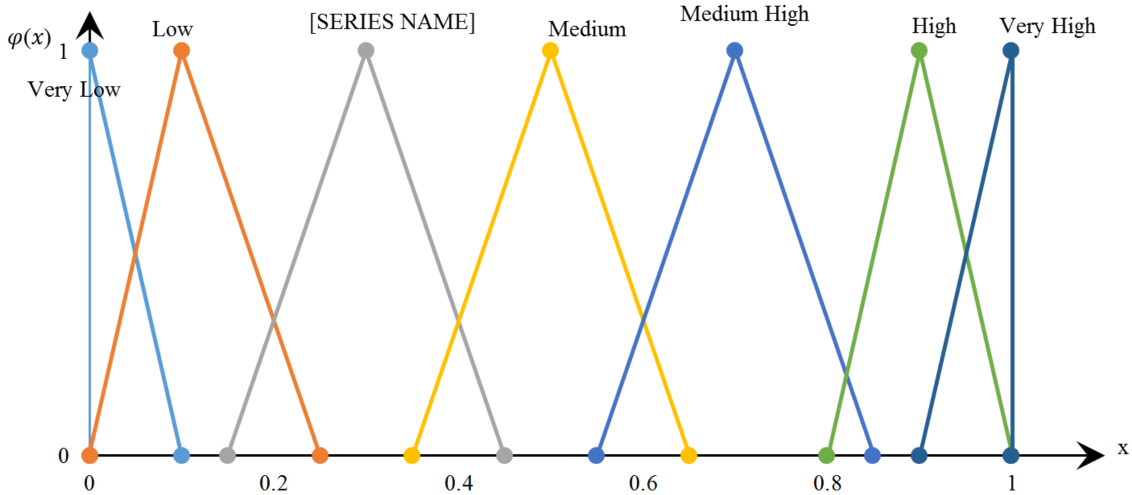

Figure 1. Linguistic Terms for the Importance Weight of Each Criterion

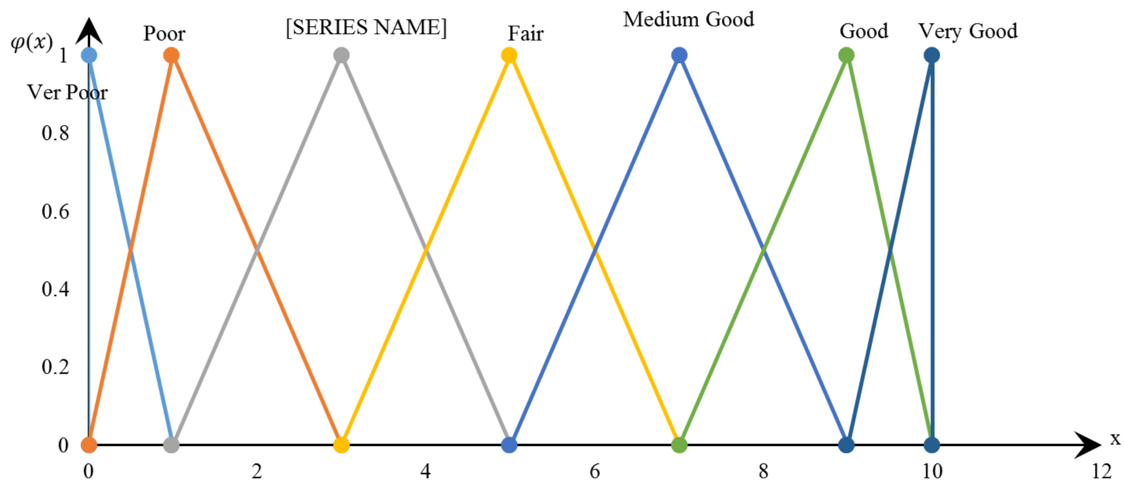

Figure 2 Linguistic Terms for the Ratings of Alternatives

Step 2

Utilize the maximizing deviation method to find the weight of criteria. According to Xu and Zhang [6], the Model 1 denoted as (M-1) derived is specifically to find the weight vector of the criteria assigned to alternatives that are completely unknown.

$$(M-1)=\begin{cases} \max \quad D(w)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} w\sqrt{Z} \\ s.t. \quad w_j \geq 0,\ j=1,2,\ldots,n,\ \sum_{j=1}^{n} w_j^2 = 1 \end{cases} \quad (3)$$

whereas $Z = \frac{1}{l}\sum_{\lambda=1}^{l}\left|h_{ij}^{\sigma(\lambda)} - h_{kj}^{\sigma(\lambda)}\right|^2$ and the interval for $i = 1,2,\ldots m$ and $j = 1,2,\ldots,n$ (4)

Step 3

By referring to Eq. (5) and (6), derive the hesitant fuzzy positive initial solution (PIS) A^+ and hesitant fuzzy negative fuzzy initial solution (NIS) A^- .

$$A^+ = \left\{x_j, \max \left\langle h_{ij}^{\sigma(\lambda)}\right\rangle, j = 1,2,\ldots n\right\}$$

$$= \left\{\begin{array}{l} \left\langle x_1, \left(\left(h_1^1\right)^+, \left(h_1^2\right)^+, \ldots, \left(h_1^l\right)^+\right)\right\rangle \\ \times \left\langle x_2, \left(\left(h_2^1\right)^+, \left(h_2^2\right)^+, \ldots, \left(h_2^l\right)^+\right)\right\rangle \\ \times \ldots \left\langle x_n, \left(\left(h_n^1\right)^+, \left(h_n^2\right)^+, \ldots, \left(h_n^l\right)^+\right)\right\rangle \end{array}\right\} \quad (5)$$

$$A^- = \left\{x_j, \min \left\langle h_{ij}^{\sigma(\lambda)}\right\rangle, j\ 1,2,\ldots n\right\}$$

$$= \left\{\begin{array}{l} \left\langle x_1, \left(\left(h_1^1\right)^-, \left(h_1^2\right)^-, \ldots, \left(h_1^l\right)^-\right)\right\rangle \\ \times \left\langle x_2, \left(\left(h_2^1\right)^-, \left(h_2^2\right)^-, \ldots, \left(h_2^l\right)^-\right)\right\rangle \\ \times \ldots \left\langle x_n, \left(\left(h_n^1\right)^-, \left(h_n^2\right)^-, \ldots, \left(h_n^l\right)^-\right)\right\rangle \end{array}\right\} \quad (6)$$

Step 4

Determine the separation measure of alternatives, d_i^+ and d_i^- from the hesitant fuzzy PIS A+ and NIS A- by incorporating the hesitant fuzzy Euclidean distance presented by Xu and Xia [7]

$$d_i^+ = \sum_{j=1}^{n} d(h_{ij}, h_j^+) w_j = \sum_{i=1}^{n} w_j \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l} \left| h_{ij}^{\sigma(\lambda)} - \left(h_j^{\sigma(\lambda)}\right)^+ \right|^2} \quad (7)$$

$$i = 1,2,\dots n$$

$$d_i^- = \sum_{j=1}^{n} d(h_{ij}, h_j^-) w_j = \sum_{i=1}^{n} w_j \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l} \left| h_{ij}^{\sigma(\lambda)} - \left(h_j^{\sigma(\lambda)}\right)^- \right|^2} \quad (8)$$

$$i = 1,2,\dots n$$

Step 5

Determine the relative closeness coefficient, C_i of alternatives with respect to hesitant fuzzy PIS A^+ separately according to the level of criteria, benefit and cost criteria, using the formula:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (9)$$

Step 6

Referring to Yaakob [8], find the Influence Closeness Coefficient (ICC) by adopting the influence degree of decision makers to each stocks measured. The method is then continued by normalizing the ICC (NICC) by assigning the maximum value of ICC to be divided to each ICC according to stocks.

$$\sigma_k = \frac{\theta_k}{\sum_{i=1}^{k} \theta_k} \quad (10)$$

$$For\ k = 1, 2, \dots, K$$

Supposedly θ is denoted as the influence degree of decision maker k where the value sits in between 0 that is verified as not influential to 10 that signifies as most influential [8]. σ resembles the normalized influence degree of kth decision makers. The method is carried on by assigning the normalized influence degree to each correlation coefficient of stocks in accordance.

$$ICC_{j,k}^{B} = \sigma_k \times CC_{j.k}^{B} \quad (11)$$

$$ICC_{j,k}^{C} = \sigma_k \times CC_{j.k}^{C} \quad (12)$$

According to j = 1, 2, … m and k = 1, 2, …, K

The $ICC_{j,k}^{B}$ and $ICC_{j,k}^{C}$ is then normalized as equation shown in order to obtain values between 0 to 1.

$$NICC_{j.k}^{B} = {ICC_{j,k}^{B}}\Big/{\max_j ICC_{j,k}^{B}} \quad (13)$$

$$NICC_{j.k}^{C} = {ICC_{j,k}^{C}}\Big/{\max_j ICC_{j,k}^{C}} \quad (14)$$

According to j = 1, 2, … m and k = 1, 2, …, K

The normalized influenced closeness coefficient is then converted into linguistic terms that are determined from Figure 3 to evaluate the level of alternatives performance.

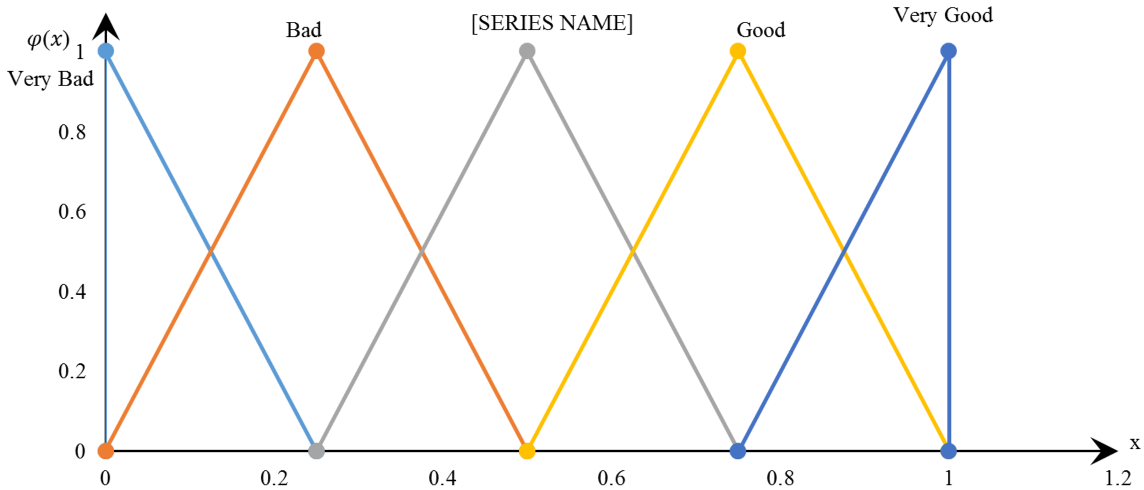

Figure 3. Linguistic Terms for the Level of Alternatives

Step 7

Build the antecedent and consequent matrices according to the NICC calculated in Step 6, with respect to the benefit and cost system. Referring to the Eq. (15) and (16), build the antecedent matrices of BS and CS with respect to groups, k:

$$x_k = \begin{matrix} B_1 \\ B_2 \\ \vdots \\ B_{m,k} \end{matrix} \begin{bmatrix} x_{11,k} & x_{12,k} & \cdots & x_{1m,k} \\ x_{21,k} & x_{22,k} & \cdots & x_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{e1,k} & x_{e2,k} & \cdots & x_{em,k} \end{bmatrix} \quad (15)$$

$$y_k = \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_{l,k} \end{matrix} \begin{bmatrix} y_{11,k} & y_{12,k} & \cdots & y_{1l,k} \\ y_{21,k} & y_{22,k} & \cdots & y_{2l,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{e1,k} & y_{e2,k} & \cdots & y_{el,k} \end{bmatrix} \quad (16)$$

for $k = 1,2,\dots,K$

$x_{em,k}$ and $y_{el,k}$ are linguistic terms reflecting the views of decision makers. Consequent matrices are built as the compliment to previous antecedent matrices. $\lambda_{m,k}$ and $\psi_{m,k}$ represent the coefficients in consequent matrices for benefit system and cost system respectively.

$$\Lambda_k = \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \quad (17)$$

$$\Psi_k = \begin{bmatrix} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{bmatrix} \quad (18)$$

as $k = 1,2,\dots,K$

where Λ_k and Ψ_k are linguistic terms representing the output of the BS and CS systems, reflecting the values of $NICC^B_{j.k}$ and $NICC^C_{j.k}$.

The benefit system consists of K matrix decision rules presented in Eq. (19)

$$\text{If } x_k = \begin{bmatrix} x_{11,k} & x_{12,k} & \cdots & x_{1m,k} \\ x_{21,k} & x_{22,k} & \cdots & x_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{e1,k} & x_{e2,k} & \cdots & x_{em,k} \end{bmatrix},$$

$$\text{then } \Lambda_k = \begin{bmatrix} \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \quad (19)$$

Previous matrices can best be interpreted in rule bases as

$$\begin{matrix} \text{Rule 1}: \textit{If } B_1 \textit{ is } x_{11,k} \textit{ and} \cdots \textit{and } B_e \textit{ is } x_{e1,k} \textit{ then } BL \textit{ is } \lambda_{1,k} \\ \vdots \qquad \vdots \qquad \vdots \\ \text{Rule } m: \textit{If } B_1 \textit{ is } x_{1m,k} \textit{ and} \cdots \textit{and } B_e \textit{ is } x_{em,k} \textit{ then } BL \textit{ is } \lambda_{m,k} \end{matrix} \quad (20)$$

where BL is the benefit level of alternatives, for $j = 1,\dots,m$ and , for $k = 1,\dots,K$. The same thing is carried on the cost system of K matrix decision rules whereas

$$\text{If } Y_k = \begin{bmatrix} y_{11,k} & y_{12,k} & \cdots & y_{1m,k} \\ y_{21,k} & y_{22,k} & \cdots & y_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{e1,k} & y_{e2,k} & \cdots & y_{em,k} \end{bmatrix},$$

then

$$\Psi_k = \begin{bmatrix} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{m,k} \end{bmatrix} \quad (21)$$

For matrices k=1, 2, …, K

Previous matrices can best be interpreted in rule bases as

$$\begin{matrix} \text{Rule 1}: \textit{If } C_1 \textit{ is } x_{11,k} \textit{ and} \cdots \textit{and } C_f \textit{ is } x_{f1,k} \textit{ then } CL_1 \textit{ is } \psi_{1,k} \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ \text{Rule } m: \textit{If } C_1 \textit{ is } x_{1m,k} \textit{ and} \cdots \textit{and } C_f \textit{ is } x_{fm,k} \textit{ then } CL_m \textit{ is } \psi_{m,k} \end{matrix} \quad (22)$$

With the cost level of alternatives represented by CL , for $j = 1,\dots,m$ and , for $k = 1,\dots,K$

Step 8

Define the Alternative System (AS), M_k. AS can be constructed based on the antecedent and consequent matrices built.

$$M_k = \begin{matrix} BL \\ CL \end{matrix} \begin{bmatrix} \lambda_{1,k} \cdots \lambda_{1,k} \cdots \lambda_{m,k} \cdots \lambda_{m,k} \\ \psi_{1,k} \cdots \psi_{m,k} \cdots \psi_{1k} \cdots \psi_{m,k} \end{bmatrix} \quad (23)$$

As k= 1,2,…..K

By criteria of benefit and cost, the measured levels of the same alternative j is represented by each row of inputs for this case. Hence, the AS antecedent matrices, M_k, are developed with the size of $2 \times m$ as in Eq. (24)

$$M_k = \begin{matrix} BL \\ CL \end{matrix} \begin{bmatrix} \lambda_{1,k}, \lambda_{2,k}, \lambda_{3,k} \cdots \lambda_{m,k} \\ \psi_{1,k}, \psi_{2,k}, \psi_{3,k} \cdots \psi_{m,k} \end{bmatrix} \quad (24)$$

In order to complement the antecedent matrices of AS, the AS consequent matrices are retrieved accordingly:

(i). Calculate the aggregation $\xi_{j,k}$ of weighted $NICC^B_{j.k}$ and $NICC^C_{j.k}$ under the equation:

$$\xi_{j,k} = \frac{NICC^B_{j.k} \times \left(\frac{e}{e+f}\right) + NICC^C_{j.k} \times \left(\frac{f}{e+f}\right)}{2} \quad (25)$$

According to $j = 1, 2, \dots m$ and $k = 1, 2, \dots, K$

(ii). Normalize the values of $\xi_{j,k}$ in order to certain they lies in between [0,1].

$$N\xi_{j,k} = \xi_{j,k} \big/ \max_j \xi_{j,k} \quad (26)$$

According to $j = 1, 2, \dots m$ and $k = 1, 2, \dots, K$

(iii). The normalized value of $\xi_{j,k}$ is then translated into linguistic terms as listed in Figure 3 in order to represent the alternative levels. Then the K for AS consequent matrices, in this case of size 1× m rather than 1 × m·m , are described in Eq. (27)

$$N_k = AL\left[N\xi_{1,k}, N\xi_{2,k} \cdots N\xi_{m,k}\right] \text{ for } k = 1,\dots,K \quad (27)$$

Alternatives level is categorised as AL. Decision rules in K matrices represents the system of alternatives:

If

$$M_k = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} \lambda_{1,k} & , & \lambda_{2,k} & , & \lambda_{3,k} & \cdots & \lambda_{m,k} \\ \psi_{1,k} & , & \psi_{2,k} & , & \psi_{3,k} & \cdots & \psi_{m,k} \end{bmatrix},$$

then

$$N_k = AL\left[N\xi_{1,k}, N\xi_{2,k} \cdots N\xi_{m,k}\right] \tag{28}$$

for $k = 1, \ldots, K$
and can best be interpreted in rule bases

$$\begin{aligned} &\text{Rule } 1: \textit{If } BL_1 \textit{ is } \lambda_{1,k} \textit{ and } \cdots \textit{and } CL_1 \textit{ is } \psi_{1,k} \textit{ then } AL_1 \textit{ is } N\xi_{1,k} \\ &\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \\ &\text{Rule } m: \textit{If } BL_m \textit{ is } \lambda_{m,k} \textit{ and } \cdots \textit{and } CL_m \textit{ is } \psi_{m,k} \textit{ then } AL_m \textit{ is } N\xi_{m,k} \end{aligned} \tag{29}$$

With respect to $k = 1, \ldots, K$

Step 9

Interpret derived rules into generalised Boolean matrix. Generalised Boolean matrix could depict the overall system represented by BS, CS and AS systems. Referring to the evaluations of decision makers,

$$\begin{array}{c|ccc} & \lambda_{j,1} & \cdots & \lambda_{j,K} \\ x_{1j,1}\cdots x_{ej,1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x_{1j,k}\cdots x_{ej,K} & 0 & \cdots & 1 \end{array}, \text{ for } j = 1, 2, \ldots m \tag{30}$$

The possible transposition of the benefit system rule base are depicted by the row and columns of the Boolean matrix previous. The transposition implement linguistic terms as in Figure 1 and 2 in order to represent the input and linguistic terms in Figure 3 to acknowledge the output.

$$\begin{array}{c|ccc} & \psi_{j,1} & \cdots & \psi_{j,k} \\ y_{1j,1}\cdots y_{ej,1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ y_{1j,K}\cdots y_{ej,K} & 0 & \cdots & 1 \end{array} \tag{31}$$

For $j = 1,2, \ldots, m$

Same method is carried on CS where the same linguistic terms also implied as BS. In order to form a generalized Boolean matrix that includes BS generalized Boolean matrices and CS generalized Boolean matrices, vertical merging is implied in the form of Eq. (32).

$$\begin{array}{c|ccc} & \lambda_{j,i} & \cdots & \lambda_{j,K} \\ & \psi_{j,1} & \cdots & \psi_{j,K} \\ \begin{matrix} x_{1j,1}\cdots x_{ej,1} \\ y_{1j,1}\cdots y_{ej,1} \end{matrix} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \begin{matrix} x_{1j,K}\cdots x_{ej,K} \\ y_{1j,K}\cdots y_{ej,K} \end{matrix} & 0 & \cdots & 1 \end{array} \tag{32}$$

for $j = 1, 2, \ldots, m$

Construct AS generalized Boolean matrix based on each j alternatives accordingly as in Eq. (33)

$$\begin{array}{c|ccc} & N\xi_{j,1} & \cdots & N\xi_{j,K} \\ \lambda_{j,1}\cdots\psi_{j,1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{j,K}\cdots\psi_{j,K} & 0 & \cdots & 1 \end{array} \tag{33}$$

As end result, resultant generalized Boolean matrix is formed as in Eq. (34)

$$\begin{array}{c|ccc} & N\xi_{j,1} & \cdots & N\xi_{j,k} \\ \begin{matrix} x_{1j,1}\cdots x_{ej,1} \\ y_{1j,1}\cdots y_{fj,1} \end{matrix} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \begin{matrix} x_{1j,K}\cdots x_{ej,K} \\ y_{1j,K}\cdots y_{ej,K} \end{matrix} & 0 & \cdots & 1 \end{array} \tag{34}$$

Step 10

Use the generalized Boolean matrix from Eq. (34) to construct the rules of alternatives as referred in Eq. (35) for $j = 1, 2, \ldots m$

$$\begin{aligned} &\textit{Rule } 1: \textit{if } B_1 \textit{ is } x_{1j1} \textit{ and } \cdots \textit{and } B_e \textit{ is } x_{e,j,1} \textit{ and } C_1 \textit{ is } y_{1j1} \textit{ and } \cdots \textit{and } C_f \textit{ is } y_{fj1} \textit{ then } AL \textit{ is } N\xi_{j,1} \\ &\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ &\textit{Rule } n_j: \textit{if } B_1 \textit{ is } x_{1jk} \textit{ and } \cdots \textit{and } B_e \textit{ is } x_{e,j,k} \textit{ and } C_1 \textit{ is } y_{1jK} \textit{ and } \cdots \textit{and } C_f \textit{ is } y_{fj,K} \textit{ then } AL \textit{ is } N\xi_{j,K} \end{aligned} \tag{35}$$

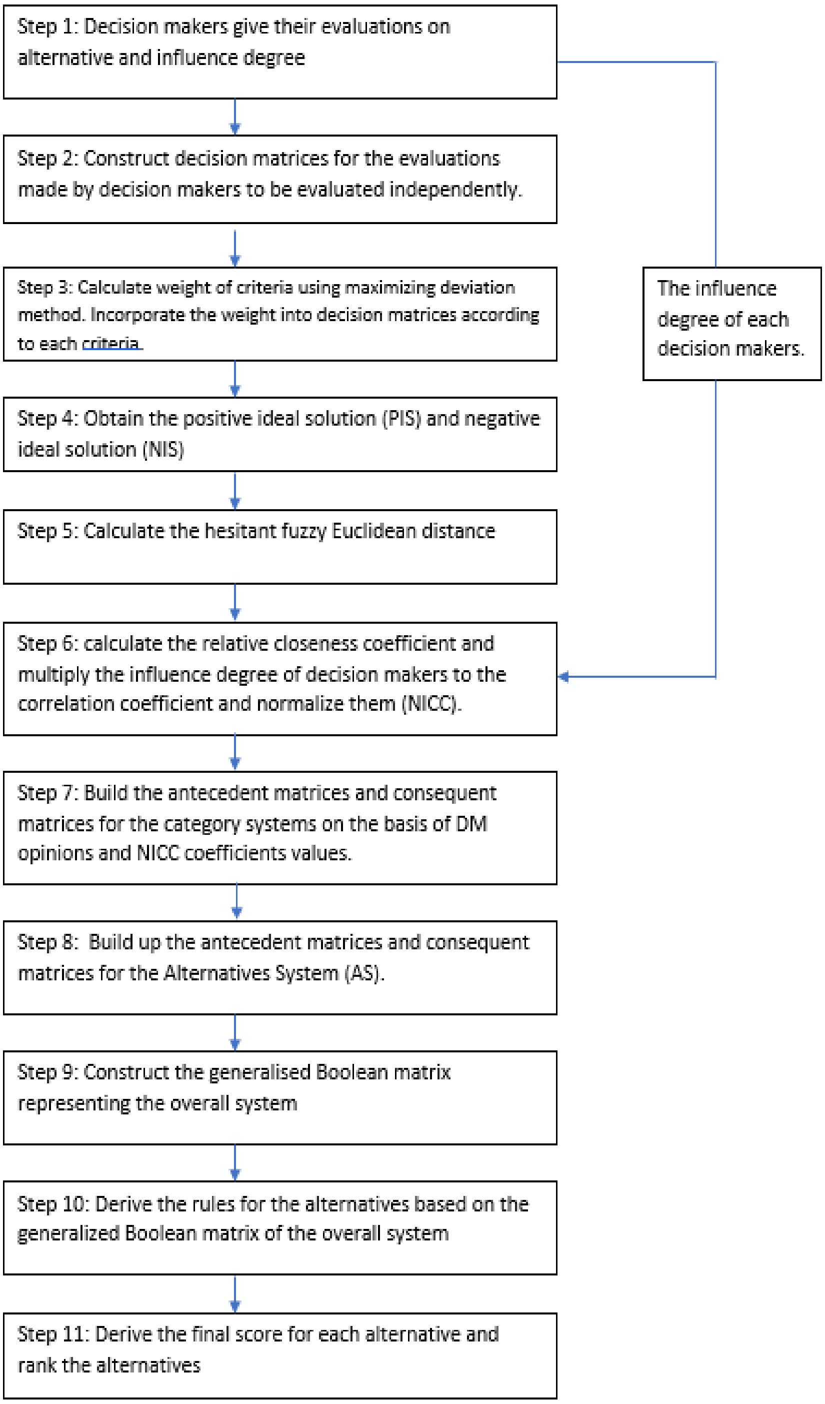

Figure 4. Proposed framework of HFN method

Step 11

Derive a final score for each alternative.

Final score for each alternative, Ω_j can be derived by multiplying the aggregate membership value of the consequent part of the n_j rules, $N\xi_{j,k}$, to the summation of benefit and cost NICC, $NICC_{j,k}^{B}$ and $NICC_{j,k}^{C}$ and perform averaging based on the number of rules and decision makers. Thus, rank every alternative accordingly based on the final score attained. Figure 4 illustrates the framework of proposed method. In order to evaluate the performance of method, we consider implementing the method in stock evaluation that considers 30 stocks to be evaluated.

$$\Omega_j = \frac{\sum_{Rule1}^{n} \sum_{k=1}^{K} N\xi_{j,K} \times \left(NICC_{j,k}^{B} + NICC_{j,k}^{C}\right)}{n \cdot K} \quad (36)$$

For $j = 1, 2, \ldots, m$

3. Case Study: Stocks Evaluation

This case study involves 30 stocks as alternatives to be evaluated by decision makers from 3 groups. The decisions from decision makers are merged according to groups into HFEs. There are six criteria that are categorised into four benefits and two cost. The process of ranking stocks follows the proposed methods in Section 2.

Step 1

Based on Table 1, 2 and 3, the arguments from experts of 3 groups has applied HFE earlier in order to construct decision matrices for the benefit and cost systems based on Eq. (1). The rating of each criterion for each equity is notified and the importance of criteria is completely unknown.

Table 1. Stock Evaluation by Group 1

GROUP 1	**B1**	**B2**	**C1**	**B3**	**B4**	**C2**
Stocks	MVF	ROE	D/E	CR	MV/NS	P/E
S1	VG	P	F	P	F	VG
S2	VG	MG	MP	MP	F	F
S3	MP	MP	MG	MP	F	MG
S4	G	VG	G	MP	MG	MG
S5	MP	P	MP	P	F	G
S6	MP	MG	F	MP	MG	MG
S7	F	P	G	MG	G	G
S8	MG	MP	G	MG	MG	G
S9	MG	MP	MG	P	P	G
S10	VG	MP	MG	P	G	VG
S11	MP	MP	VG	MG	P	P
S12	F	F	F	G	MG	MG
S13	VG	MP	VG	MG	P	G
S14	MG	F	G	MG	G	MG
S15	P	F	MG	P	VP	G
S16	F	MG	MP	MP	F	MG
S17	MP	MP	G	F	P	G
S18	P	F	VG	VG	MG	MG
S19	G	F	G	MP	VG	MP
S20	MG	MG	VG	F	VP	MG
S21	VG	MP	VG	MG	MP	G
S22	MP	MG	MG	P	P	G
S23	G	F	MG	P	MG	VG
S24	VG	F	F	MG	G	G
S25	MP	F	G	G	G	G
S26	MP	F	MG	MG	F	MG
S27	P	MG	MG	MG	G	VG
S28	G	MG	MG	G	VG	G
S29	VG	MG	MP	MP	VG	VG
S30	G	G	F	F	VP	MG

Table 2. Stock Evaluation by Group 2

GROUP 2	B1	B2	C1	B3	B4	C2
Stocks	MVF	ROE	D/E	CR	MV/NS	P/E
S1	MG	VP	MP	VP	MP	MG
S2	VG	G	F	F	MG	MG
S3	MP	P	MG	MG	MP	F
S4	VG	VG	VG	G	G	G
S5	P	P	P	P	MP	MG
S6	MP	F	F	MP	MP	F
S7	MG	MP	G	MG	F	G
S8	MG	F	G	MG	MG	G
S9	MG	F	G	MG	F	G
S10	VG	F	G	MG	G	VG
S11	G	MG	VG	G	VG	G
S12	F	MG	MG	G	MG	G
S13	VG	F	G	G	F	G
S14	MG	MG	G	G	VG	G
S15	P	MG	F	MP	P	MG
S16	F	G	MG	MG	MG	G
S17	P	P	F	MP	VP	MG
S18	P	MG	G	F	G	G
S19	G	G	VG	F	VG	MG
S20	G	G	VG	MG	MP	G
S21	G	F	VG	MG	F	MG
S22	F	G	G	MP	MP	MG
S23	F	MP	F	P	F	G
S24	G	MG	MG	MG	MG	G
S25	F	MG	MG	G	MG	G
S26	P	MG	MP	F	MP	F
S27	P	F	MP	F	F	G
S28	MG	F	MP	MG	MG	MG
S29	VG	G	MG	F	VG	VG
S30	MG	MG	P	MP	P	F

Table 3. Stock Evaluation by Group 3

GROUP 3	**B1**	**B2**	**C1**	**B3**	**B4**	**C2**
Stocks	MVF	ROE	D/E	CR	MV/NS	P/E
S1	MG	VP	MP	VP	P	G
S2	VG	G	MG	MG	MG	MG
S3	MP	P	F	MP	F	MG
S4	VG	VG	VG	MG	G	G
S5	P	VP	P	MP	MP	MG
S6	F	F	MP	F	F	F
S7	MG	MP	MG	G	MG	G
S8	G	F	MG	G	MG	G
S9	G	F	G	F	MG	G
S10	VG	MG	VG	MG	G	VG
S11	G	G	VG	G	G	VG
S12	MG	MG	MG	G	G	MG
S13	G	MG	G	MG	F	G
S14	G	G	G	MG	G	G
S15	MP	MG	F	P	P	MG
S16	G	G	MG	MG	MG	G
S17	P	P	F	MP	P	MG
S18	MP	MG	G	G	MG	MG
S19	VG	G	VG	MG	VG	G
S20	G	G	VG	MG	F	G
S21	VG	MG	G	MG	MG	G
S22	F	G	G	F	F	MG
S23	MG	F	F	P	F	G
S24	VG	MG	G	G	MG	G
S25	F	MG	MG	G	MG	MG
S26	P	F	F	F	MP	F
S27	P	F	F	F	MG	G
S28	MG	F	MG	MG	G	MG
S29	VG	G	MG	MG	VG	VG
S30	F	F	F	MP	VP	MG

Table 4. Ranking Based On Proposed HFN Method

Stocks	Actual Rank	Final Score	Proposed Method Rank
S1	30	0.1271	28
S2	9	0.3478	20
S3	21	0.1841	23
S4	2	0.8320	1
S5	24	0.0196	30
S6	23	0.1515	27
S7	18	0.5101	14
S8	14	0.6457	9
S9	12	0.5079	15
S10	4	0.7181	5
S11	1	0.6614	8
S12	8	0.4107	17
S13	13	0.7252	4
S14	11	0.7091	6
S15	20	0.1634	25
S16	5	0.3622	19
S17	27	0.1672	24
S18	19	0.5862	11
S19	7	0.7281	3
S20	6	0.6755	7
S21	10	0.7386	2
S22	15	0.3894	18
S23	22	0.3181	21
S24	16	0.6127	10
S25	17	0.5612	12
S26	26	0.1524	26
S27	28	0.2674	22
S28	25	0.4478	16
S29	3	0.5572	13
S30	29	0.1029	29

Step 2

Calculate the optimal weight vector by utilizing the maximizing deviation method in Eq. (2):

$$W = (0.20715, 0.16523, 0.16091, 0.17266, 0.19771, 0.09634)$$

Step 3

Utilize the Eq. (3) and (4) in determining the hesitant fuzzy PIS (A^{+}) and NIS (A^{-}) according to decision makers separately

$$A^{+} = \{\langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle\}$$

$$A^{-} = \{\langle 0.1,0.3,0.3,0.5\rangle, \langle 0,0.1,0.1,0.3\rangle, \langle 0,0,0,0.1\rangle, \langle 0,0.1,0.1,0.3\rangle, \langle 0.9,1,1,1\rangle, \langle 0.9,1,1,1\rangle\}$$

Step 4

Calculate the separation measures d_i^{+} and d_i^{-} according to alternatives, A_i from the A^{+} and A^{-} using the Eq. (5)

$$d_1^{+} = 0.16923, \; d_1^{-} = 0.14162,$$

$$d_2^+ = 0.153454, d_2^- = 0.159245,\ d_3^+ = 0.199047,\ d_3^- = 0.114849,$$

$$d_4^+ = 0.091328,\ d_4^- = 0.223299$$

Step 5

Calculate the relative closeness coefficient Ci of each alternative A_i to the hesitant fuzzy PIS, A^+.
$C_1 = 0.45558$, $C_2 = 0.50926$, $C_3 = 0.365882$, $C_4 = 0.70973$

Step 6

Calculate the Influence Closeness Coefficient (ICC) and Normalized Influence Closeness Coefficient (NICC) by referring to Eq. (8) until Eq. (11). To exemplify, influence degree of G1 is $\theta_1 = 5$ as can be seen in Table 5. To calculate the influence degree of G1,

$$\sigma_k = \frac{5}{\sum_{i=1}^{3}\theta_3} = 0.277778$$

Table 5. Level of Expertise

	Level of Expertise (θ_i)	Influence degree
G1	5	0.277778
G2	6	0.333333
G3	7	0.388889

Multiply the influence degree of each group to the closeness coefficient of alternatives accordingly as referred to Eq. (8) to Eq. (9) in order to generate ICC. Then in accordance to Eq. (10) and Eq. (11), find the NICC of each alternative with respect to the criteria divided.

Step 7

Construct the rule base for the Benefit System (BS) and Cost System (CS) based on the NICC calculated.

Transform the NICC obtained into linguistic terms referring to Figure 3 in order to form the antecedent and consequent matrices of both BS and CS as perform in Eq. (15) – (22).

$$NICC_{1,1}^{B} = 0.48479 = R$$

$$NICC_{1,1}^{C} = 0.63189 = G$$

$$M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} \lambda_{1,1} & , & \lambda_{2,1} & , & \lambda_{3,1} & \cdots & \lambda_{30,1} \\ \psi_{1,1} & , & \psi_{2,1} & , & \psi_{3,1} & \cdots & \psi_{30,1} \end{bmatrix} = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & , & B & \cdots & G \\ G & , & B & , & G & \cdots & R \end{bmatrix}$$

Rule 1 : *If* BL_1 *is* R *and* $\cdots$ *and* CL_1 *is* G *then* AL_1 *is* $N\xi_{1,k}$

⋮ ⋮ ⋮ ⋮

Rule m : *If* BL_m *is* $\lambda_{m,k}$ *and* $\cdots$ *and* CL_m *is* $\psi_{m,k}$ *then* AL_m *is* $N\xi_{m,k}$

Step 8

The antecedent matrices, M_k, of the Alternatives System (AS) of each DM k are constructed based on the Benefit Level (BL) and Cost Level (CL), which are the outputs of the benefit system BS and cost system CS, respectively. Based on opinion of G1,

$$M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} \lambda_{1,1} & , & \lambda_{2,1} & , & \lambda_{3,1} & \cdots & \lambda_{30,1} \\ \psi_{1,1} & , & \psi_{2,1} & , & \psi_{3,1} & \cdots & \psi_{30,1} \end{bmatrix} = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & ,\cdots, & G \\ G & , & B & ,\cdots, & R \end{bmatrix}$$

The AS consequent matrices are derived as follows:

i. Calculate the aggregation $\xi_{j,1}$ of weighted $NICC_{j.1}^{B}$ and $NICC_{j.1}^{C}$

$$\xi_{1,1} = \frac{NICC_{1,1}^{B} \times \left(\frac{e}{e+f}\right) + NICC_{1,1}^{C} \times \left(\frac{f}{e+f}\right)}{2}$$

$$= \frac{0.484789 \times \left(\frac{4}{2+4}\right) + 0.631889 \times \left(\frac{2}{2+4}\right)}{2}$$

$$= 0.291428$$

ii. Normalize the values of $\xi_{j,k}$ in order to certain they lies in between [0,1].

$$N\xi_{1,1} = {}^{\xi_{1,1}}\!/_{\max_j \xi_{j,1}} = {}^{0.291428}\!/_{0.5} = 0.582856 = \text{R}$$

Match the value of $N\xi_{1,1}$ to linguistic terms listed in alternatives level.

iii. The AS consequent matrix N_1 for DM1 is constructed based on the values $N\xi_{j,1}$ or each alternatives, *j,* where AL stands for Alternative Level.

$$N_1 = AL[N\xi_{1,1}, N\xi_{2,1} \cdots N\xi_{30,1}] = AL[R, B \cdots R]$$

The alternatives system of G1 is presented

If $M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & ,\cdots, & G \\ G & , & B & ,\cdots, & R \end{bmatrix}$, then $N_1 = AL[R, B \cdots R]$

can best be interpreted in rule bases

Rule 1 : *if BL is R, and CL is G, then AL is R*

Rule 2 : *if BL is G, and CL is B, then AL is B*

⋮

Rule 30 : *if BL is G, and CL is R, then AL is R*

Step 9

Present BS, CS and AS derived rules in Boolean matrix form. The Boolean benefit system matrix for S1 is generated as shown in

	1	2	3	4	5
1111	0	0	0	0	0
:	:	:	:	:	:
2222	0	0	0	0	0
:	:	:	:	:	:
3333	0	0	0	0	0
:	:	:	:	:	:
4444	0	0	0	0	0
:	:	:	:	:	:
5 1 1 2	0	0	0	1	0
5 1 1 3	0	0	0	1	0
5555	0	0	0	0	0
:	:	:	:	:	:
6666	0	0	0	0	0
7 2 2 4	0	0	1	0	0

The Boolean cost system matrix for S1 is generated as shown in

	1	2	3	4	5
11	0	0	0	0	0
:	:	:	:	:	:
22	0	0	0	0	0
:	:	:	:	:	:
33	0	0	0	0	0
3 5	0	1	0	0	0
3 6	0	1	0	0	0
:	:	:	:	:	:
44	0	0	0	0	0
4 7	0	0	0	1	0
:	:	:	:	:	:
55	0	0	0	0	0

In order to form a generalized Boolean matrix that includes BS generalized Boolean matrices and CS generalized Boolean matrices, vertical merging is implied.

	11		22	23	24	25	32	34	35	42		44
4444/44	0	0	0	0	0	0	0	0	0	0	0	0
:	:	:	:	:	:	:	:	:	:	:	:	:
5555/55	:	:	:	:	:	:	0	0	:	0	:	0
5 1 1 2/35	:	:	:	:	:	:	0	0	:	1	:	0
5 1 1 2/36	:	:	:	:	:	:	0	0	:	1	:	0
5 1 1 2/47	:	:	:	:	:	:	0	0	:	0	:	1
5 1 1 3/35	:	:	:	:	:	:	0	0	:	1	:	0
5 1 1 3/36	:	:	:	:	:	:	0	0	:	1	:	0
5 1 1 3/47	:	:	:	:	:	:	0	0	:	0	:	1
7 2 2 4/35	:	:	:	:	:	:	1	0	:	0	:	0
7 2 2 4/36	:	:	:	:	:	:	1	0	:	0	:	0
7 2 2 4/47	:	:	:	:	:	:	0	1	:	0	:	0

The alternative system AS Boolean matrix for S1 is evaluated as follow

	1	2	3	4	5
11	0	0	0	0	0
:	:	:	:	:	:
22	:	0	0	:	:
24	:	0	0	:	:
32	:	0	0	:	:
33	:	0	0	:	:
34	:	0	1	:	:
3 5	:	0	0	:	:
:	:	:	:	:	:
42	:	1	:	:	:
44	:	0	:	:	:

The resulting Boolean matrix is generated as shown representing the overall system

	1	2	3	4	5
1111/11	0	0	0	0	0
5555/55	0	0	0	0	0
5 1 1 2/35	:	1	0	:	:
5 1 1 2/36	:	1	0	:	:
5 1 1 2/47/NA	:	0	:	:	:
5 1 1 3/35	:	1	0	:	:
5113/36	:	1	0	:	:
:	:	0	:	:	:
5 1 1 3/47	:	0	:	:	:
:	:	:	:	:	:
7224/47	:	:	1	:	:
7777/77/5	:	:	0	:	:

From the Boolean matrix previously, the rule basis for stock S1 are derived, as

Rule 1:	5 1 1 2/35/2	5112	35	2
Rule 2:	5 1 1 2/36/2	5112	36	2
Rule 3:	5 1 1 3/35/2	5113	35	2
Rule 4:	5113/36/2	5113	36	2
Rule 5:	7224/47/3	7224	47	3

Five rules are obtained that can be interpreted according to the linguistic terms on the level of rating as

Rule 1: If B_1 is MG, and B_2 is VP and B_3 is VP and B_4 is P and C_1 is MP and C_2 is MG then S_1 is P

Rule 2: If B_1 is MG, and B_2 is VP and B_3 is VP and B_4 is P and C_1 is MP and C_2 is G then S_1 is P

Rule 3: If B_1 is MG, and B_2 is VP and B_3 is VP and B_4 is MP and C_1 is MP and C_2 is MG then S_1 is P

Rule 4: If B_1 is MG, and B_2 is VP and B_3 is VP and B_4 is MP and C_1 is MP and C_2 is G then S_1 is P

Rule 5: If B_1 is VG, and B_2 is P and B_3 is P and B_4 is F and C_1 is MP and C_2 is MG then S_1 is MP

Step 10

For each alternative the final score is derived from Eq. (36). There are 6 active rules emitted for S1 as end result and the final score for S1 is obtained by implementing the equation. Then, the calculation continues on finding the average aggregate membership value in order to be infused to the influence multiplier.

$$\frac{0.3+0.3+0.3+0.3+0.3+0.5}{6}\cdot\frac{0.32856+0.41904}{2} = 0.1271$$

Higher position in ranking is defined by the highest final score attained. Thus, it is shown that in Table 4, Stock 4 has the highest rank compared to other stocks, which depicts that Stock 4 has the highest final score. As opposed earlier, Spearman rho correlation is adapted in order to evaluate the performance of proposed method compared to the actual rank of stocks.

$$\rho = 1-\frac{(6\times 1132)}{(30\times(30^2-1))} = 0.70$$

4. Analysis of Result

Proposed HFN is implied with maximizing deviation method that assist in determining weight of criteria when the weight is completely unknown. The validation of the proposed HFN is carried out with the comparison to established methods under Spearman rho correlation as shown in Table 6.

Table 6. Comparison between Proposed Method and Established Methods

Methods	Transparency	Hesitancy	Spearman Rho
HFN (proposed method)	Yes	Yes	0.70
Zeshui , X., & Xiaolu, Z. (2013) [6]	No	Yes	0.70
Yaakob et. Al. (2017) [8]	Yes	No	0.60

The Spearman rho of the proposed HFN is as same as established methods [6], which is 0.7. Both methods applied hesitant fuzzy set in dealing with vagueness in assessments and the proposed method applies fuzzy network as advantage to promote transparency in the formulation. Hence, proposed method performs well with the addition of fuzzy network in order to promote transparency in the formulation. As compared to [8], proposed method works better with the incorporation of hesitant fuzzy set in fuzzy network thus, achieved higher Spearman rho score.

5. Conclusions

This paper proposes a novel method of Hesitant Fuzzy Network TOPSIS (HFN), in which fuzzy network has improvised the transparency of decision making by thoroughly divide the factors into subsystems thus, creating dynamical communication among them. The method is then enhanced by implying the maximizing deviation method in finding the weight that is completely unknown from the beginning [6]. The proposed method from this research could be a great decision-making tool to make better decisions in solving decision making problem. The incorporation of hesitant fuzzy set to fuzzy network develop decision making method that promotes transparency in assessment and overcome vagueness in decision making. Due to the fluctuation financial scenarios, different experiments with various slot of time should be considered for testing the efficiency of the method. In addition, application of the new method would contribute towards the body of knowledge and wide range application area involving food security, energy security, water security, transportation, environmental, healthcare and technology.

Acknowledgements

This research is supported by Ministry of Higher Education (MOHE) of Malaysia through **Fundamental Research Grant Scheme (FRGS/1/2019/STG06/UUM/02/6)**. We also want to thank Government of Malaysia which provide MyBrain15 program for sponsoring this work under the self-fund research grant and L00022 from Ministry of Science, Technology and Innovation (MOSTI).

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