

On New Generalized Fuzzy Directed Divergence Measure and Its Application in Decision Making Problem

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Abstract The concept of fuzzy sets presented by Zadeh has conquered an enormous achievement in numerous fields. Uncertainty in real world is ubiquitous. Entropy is an important tool with uncertainty and fuzziness. In this article, we propose new measure of directed divergence on fuzzy set. The extension of the fuzzy sets and one that integrated with other theories have been applied by some researchers. To prove the validity of measure, some axioms are proved. Using the proposed measure, we generate a method about decision making criteria and give a suitable method. In this article, we describe directed divergence measure for fuzzy set. Properties of proposed measure are discussed. In the real world, the multicriteria decision making is a very practical method and has a wide range of uses. By using multicriteria decision making, we can find best choice among the given criteria. In recent years, many researchers extensively apply fuzzy directed divergence for multicriteria decision making. Also some researchers defined the application of parameterized Hesitant Fuzzy Soft Set theory in decision making. In this article, we shall investigate the multiple criteria decision making problem under fuzzy environment. Application of introduced measure is given for decision making problem. A numerical example is given for decision making problem. In a fuzzy multicriteria problem, the analysis is given by an illustration example of the new

define approach regarding admission preference of a student for post graduate course of science stream.

Keywords Fuzzy Set, Divergence Measure, Directed Divergence, Decision Making

1. Introduction

The study of problem concerning which includes information, dispensation, storage retrieval and decision making are dealt with the help of information theory and fuzzy theory. First time Shannon [1] gives an idea that the measure of information theory is termed as entropy. After that Kullback and Liebler [2] evaluate that measure of information associated with the two probability distribution p_i and q_i of discrete random variable, is given as

$$D(p // q_i) = \sum_{i=1}^n p_i \log \frac{p_i}{q_i}$$

known as directed divergence. L. Zadeh [3] introduced the fuzzy set theory and the concept of fuzzy set theory is used in different areas of science and technology e.g. image

processing, pattern recognition, decision making etc. A fuzzy distance measure between two fuzzy set was proposed by Bhandari and Pal [4] using the concept of fuzzy measure conditioning corresponding to Kullback and Leibler [2] probabilistic directed divergence. Bhandari and Pal [4] introduced a measure

$$I(A, B) = \sum_{i=1}^n \mu_A(x_i) \log \frac{\mu_A(x_i)}{\mu_B(x_i)} + (1 - \mu_A(x_i)) \log \frac{(1 - \mu_A(x_i))}{(1 - \mu_B(x_i))}$$

Later, Fan and Xie [5] proposed discrimination of fuzzy information of fuzzy set A against B , corresponding to exponential fuzzy entropy given by Pal and Pal [6]. Kapur [7] introduced a generalized directed divergence measure corresponding to Havrda and Charvat [8]. Hooda and Bajaj [9] proposed directed divergence measure along with R-norm directed divergence. Bhatia and Singh [10] gave some measure of directed divergence of two set A and B . Tomar and Ohlan and Priya and Tomar [11, 14] proposed some measure of fuzzy directed divergence. Zahari Md Rodzi, Abd Ghafur Ahmad [15] defined the application of parameterized Hesitant Fuzzy Soft Set theory in decision making. Keeping in mind above literature we propose directed divergence and some important properties are also studied. It is shown that propose measure has wide application in pattern. In section II, a brief study about fuzzy set, measure and directed divergence is given. In section III, a new directed divergence measure is discussed. In section IV, properties are described with their proof. Application of proposed measure is discussed in section V. At last, the conclusion of all above work is given in VI section.

2. Preliminaries

In this section we define some definitions and notations about fuzzy set and directed divergence measure. We will

present those aspects of fuzzy set and its measure which will be used in our next discussion.

Definition 1. Let $\Gamma_n = \{P = (p_1, p_2, \dots, p_n) / p_i \geq 0\}$ is the set of all complete finite discrete probability distribution then measure of information was defined firstly by Shannon as

$$H(P) = \sum_{i=1}^n p_i \log p_i, P \in \Gamma_n$$

Definition 2. Let $X = \{x_1, x_2, \dots, x_n\}$ be universe of discourse then $A = \{x, \mu_A(x) \mid x \in X\}$ is called fuzzy set where $\mu_A(x): X \rightarrow [0, 1]$ is a membership function defined as follows

$$\mu_A(x_i) = \begin{cases} 0 & \text{if } x \notin A \\ 1 & \text{if } x \in A \\ 0.5 & \text{if } x \notin A \text{ or } x \in A \end{cases}$$

Some notation for two fuzzy set

1. $A \cup B = \{x, \max(\mu_A(x), \mu_B(x)) \mid x \in X\}$
2. $A \cap B = \{x, \min(\mu_A(x), \mu_B(x)) \mid x \in X\}$
3. $A = B = \{x, \mu_A(x) = \mu_B(x) \mid x \in X\}$
4. $A.B = \{x, \mu_A(x), \mu_B(x) \mid x \in X\}$
5. $A^c = \{x, \mu_A(x) = 1 - \mu_A(x) \mid x \in X\}$

Definition 3. Let $X = \{x_1, x_2, \dots, x_n\}$ be universe of discourse and $F(X)$ be the set of all family subset. A mapping $I: F(X) \times F(X) \rightarrow R$ is called divergence measure between fuzzy sets if

- i. $I(A: B) \geq 0$
- ii. $I(A: B) = I(B: A)$
- iii. $I(A: B) = 0$ iff $A = B$
- iv. $\text{Max}\{I(A \cup C, B \cup C), I(A \cap C, B \cap C)\} \leq I(A: B)$

Bajaj et.al.[12] defined the measure of fuzzy directed divergence as

$$I_{\alpha}(A : B) = \frac{1}{\alpha - 1} \sum_{i=1}^n \log \left[\mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \right]$$

$$I_{\alpha,\beta}(A : B) = \frac{1}{1 - 2^{\beta-1}} \sum_{i=1}^n \left(\left[\mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \right]^{\frac{\beta-1}{\alpha-1}} - 1 \right)$$

Prakash et. al. [13] introduced entropy measure on fuzzy set as

$$H_{\alpha}^{\beta}(A) = \frac{1}{(1 - \alpha)\beta} \sum_{i=1}^n \left(\left[\mu_A^{\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} \right]^{\beta} - 1 \right) ; \alpha > 0, \alpha \neq 1, \beta \neq 0$$

3. New Directed Divergence Measure

Corresponding to Prakash et. al. [13] we propose the measure of fuzzy directed divergence as follow

$$H_{\alpha,\beta}(A, B) = \frac{1}{(\alpha - 1)\beta} \sum_{i=1}^n \left(\left[\begin{array}{l} \mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \end{array} \right]^{\beta} - 1 \right) ; \alpha > 0, \alpha \neq 1, \beta \neq 0 \quad (1)$$

Theorem1. Show that $H_{\alpha,\beta}(A, B)$ is valid measure of fuzzy directed divergence.

Proof. To show that proposed measure in (1) is valid we have to prove following axioms

- I. We can clearly check in figure that $H_{\alpha,\beta}(A, B)$ is non-negative.

Figure 1. $H_{\alpha,\beta}(A, B)$

- II. $H_{\alpha,\beta}(A, B) \neq H_{\alpha,\beta}(B, A)$

But we know that $J_{\alpha,\beta}(A, B) = H_{\alpha,\beta}(A, B) + H_{\alpha,\beta}(B, A)$ is symmetric.

- III. $H_{\alpha,\beta}(A, B) = 0$ if $A = B$

- IV. We have to check the convexity of $H_{\alpha,\beta}(A, B)$.

So now,

$$\frac{\partial H_{\alpha,\beta}}{\partial \mu_A(x_i)} = \left\{ \alpha\beta \left[\begin{array}{l} \mu_A^{\alpha-1}(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1 - \mu_A(x_i))^{\alpha-1} (1 - \mu_B(x_i))^{1-\alpha} \end{array} \right] \left[\begin{array}{l} \mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \end{array} \right]^{\beta-1} \right\}$$

$$\frac{\partial^2 H_{\alpha,\beta}}{\partial \mu_A^2(x_i)} = \left\{ \begin{aligned} & \left[\alpha(\alpha-1)\beta \left[\mu_A^{\alpha-2}(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha-2}(1-\mu_B(x_i))^{1-\alpha} \right] \right. \\ & \left. \left[\mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha} \right]^{\beta-1} \right. \\ & \left. + \alpha(\beta-1)\beta \left[\mu_A^{\alpha-1}(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha-1}(1-\mu_B(x_i))^{1-\alpha} \right] \right. \\ & \left. \left[\mu_A^\alpha(x_i)\mu_B^{1-\alpha}(x_i) + (1-\mu_A(x_i))^\alpha(1-\mu_B(x_i))^{1-\alpha} \right]^{\beta-2} \right\} \\ \Rightarrow & \frac{\partial^2 H_{\alpha,\beta}}{\partial \mu_A^2(x_i)} > 0 \text{ for } \alpha > 0, \beta > 0, \alpha \neq 1, \beta \neq 1, 2, \end{aligned} \right.$$

Similarly we can show that $\Rightarrow \frac{\partial^2 H_{\alpha,\beta}}{\partial \mu_B^2(x_i)} > 0 \text{ for } \alpha > 0, \beta > 0, \alpha \neq 1, 2, \beta \neq 1,$

So axiomatically it is clear that proposed measure are valid.

4. Some Important Properties

Assume that the family of all fuzzy set of universe X , is denote by $FS(X)$ and $A, B, C \in FS(X)$ is given

$$\begin{aligned} A &= [\langle x, \mu_A(x) \rangle / x \in X] \\ B &= [\langle x, \mu_B(x) \rangle / x \in X] \\ C &= [\langle x, \mu_C(x) \rangle / x \in X] \end{aligned}$$

and we have $\Delta_1 = [x_i / x_i \in X, \mu_A(x_i) \geq \mu_B(x_i)]$

$$\Delta_2 = [x_i / x_i \in X, \mu_A(x_i) < \mu_B(x_i)]$$

Theorem2. Prove that proposed measure in (1) satisfies the following properties:

- I. $H_{\alpha,\beta}(A \cup B, A) + H_{\alpha,\beta}(A \cap B, A) = H_{\alpha,\beta}(B, A)$
- II. $H_{\alpha,\beta}(A, A \cap B) = H_{\alpha,\beta}(A \cup B, B)$
- III. $H_{\alpha,\beta}(A, A \cup B) = H_{\alpha,\beta}(A \cap B, B)$
- IV. $H_{\alpha,\beta}(A \cup B, C) + H_{\alpha,\beta}(A \cap B, C) = H_{\alpha,\beta}(A, C) + H_{\alpha,\beta}(B, C)$
- V. $H_{\alpha,\beta}(A \cup B, A \cap B) = H_{\alpha,\beta}(A \cup B, B) + H_{\alpha,\beta}(B, A \cap B)$
- VI. $H_{\alpha,\beta}(A, A^c) = H_{\alpha,\beta}(A^c, A)$
- VII. $H_{\alpha,\beta}(A^c, B^c) = H_{\alpha,\beta}(A, B)$
- VIII. $H_{\alpha,\beta}(A, B^c) = H_{\alpha,\beta}(A^c, B)$
- IX. $H_{\alpha,\beta}(A, B) + H_{\alpha,\beta}(A^c, B) = H_{\alpha,\beta}(A^c, B^c) + H_{\alpha,\beta}(A, B^c)$

Proof.

$$\begin{aligned} \text{I. } & \left[\begin{aligned} & H_{\alpha,\beta}(A \cup B, A) \\ & + H_{\alpha,\beta}(A \cap B, A) \end{aligned} \right] = \frac{1}{(\alpha-1)\beta} \sum_{i=1}^n \left(\left[\begin{aligned} & \left[\mu_{A \cup B}^\alpha(x_i)\mu_A^{1-\alpha}(x_i) \right. \\ & \left. + (1-\mu_{A \cup B}(x_i))^\alpha(1-\mu_A(x_i))^{1-\alpha} \right]^\beta \right. \\ & \left. + \left[\mu_{A \cap B}^\alpha(x_i)\mu_A^{1-\alpha}(x_i) \right. \right. \\ & \left. \left. + (1-\mu_{A \cap B}(x_i))^\alpha(1-\mu_A(x_i))^{1-\alpha} \right]^\beta - 2 \right) \\ & = \frac{1}{(\alpha-1)\beta} \left\{ \begin{aligned} & \left[\sum_{\Delta_1} \left(\left[\begin{aligned} & \left[\mu_A^\alpha(x_i)\mu_A^{1-\alpha}(x_i) \right. \right. \\ & \left. \left. + (1-\mu_A(x_i))^\alpha(1-\mu_A(x_i))^{1-\alpha} \right]^\beta \right) \right. \\ & \left. - 1 \right) \right] \right\} + \left\{ \begin{aligned} & \left[\sum_{\Delta_1} \left(\left[\begin{aligned} & \left[\mu_A^\alpha(x_i)\mu_A^{1-\alpha}(x_i) \right. \right. \\ & \left. \left. + (1-\mu_A(x_i))^\alpha(1-\mu_A(x_i))^{1-\alpha} \right]^\beta \right) \right. \right. \\ & \left. \left. + \sum_{\Delta_2} \left(\left[\begin{aligned} & \left[\mu_B^\alpha(x_i)\mu_A^{1-\alpha}(x_i) \right. \right. \\ & \left. \left. + (1-\mu_B(x_i))^\alpha(1-\mu_A(x_i))^{1-\alpha} \right]^\beta \right) \right. \right. \\ & \left. \left. - 1 \right) \right] \right\} \end{aligned} \right\} \end{aligned} \end{aligned}$$

$$= \frac{1}{(\alpha-1)\beta} \sum_{i=1}^n \left(\left[\begin{array}{l} \mu_B^\alpha(x_i) \mu_A^{1-\alpha}(x_i) \\ + (1-\mu_B(x_i))^\alpha (1-\mu_A(x_i))^{1-\alpha} \end{array} \right]^\beta - 1 \right)$$

Hence we can say that

$$H_{\alpha,\beta}(A \cup B, A) + H_{\alpha,\beta}(A \cap B, A) = H_{\alpha,\beta}(B, A)$$

$$\text{II. } H_{\alpha,\beta}(A, A \cap B) = \frac{1}{(\alpha-1)\beta} \sum_{i=1}^n \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_{A \cap B}^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_{A \cap B}(x_i))^{1-\alpha} \end{array} \right]^\beta - 1 \right)$$

$$= \frac{1}{(\alpha-1)\beta} \left\{ \sum_{\Delta_1} \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\} + \sum_{\Delta_2} \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_A^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_A(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\}$$

$$= \frac{1}{(\alpha-1)\beta} \left\{ \sum_{\Delta_1} \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\}$$

$$H_{\alpha,\beta}(A \cup B, B) = \frac{1}{(\alpha-1)\beta} \sum_{i=1}^n \left(\left[\begin{array}{l} \mu_{A \cup B}^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_{A \cup B}(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta - 1 \right)$$

$$= \frac{1}{(\alpha-1)\beta} \left\{ \sum_{\Delta_1} \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\} + \sum_{\Delta_2} \left(\left[\begin{array}{l} \mu_B^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_B(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\}$$

$$= \frac{1}{(\alpha-1)\beta} \left\{ \sum_{\Delta_1} \left(\left[\begin{array}{l} \mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) \\ + (1-\mu_A(x_i))^\alpha (1-\mu_B(x_i))^{1-\alpha} \end{array} \right]^\beta \right) - 1 \right\}$$

Hence we can say that

$$H_{\alpha,\beta}(A, A \cap B) = H_{\alpha,\beta}(A \cup B, B)$$

All other properties can be proved as above.

5. Application of Directed Divergence Measure in Decision Making

In the real world, the multicriteria decision making is a very practical method and has wide use. By using multicriteria decision making we can find best choice among the given criteria. In recent years, many researchers extensively apply fuzzy directed divergence for multicriteria decision making. In this article we shall investigate the multiple criteria decision making problem under fuzzy environment. For multicriteria decision making problem, we have a set of strategies say $A_1, A_2, A_3, \dots, A_n$ and suppose that each strategy has varied degree of effectiveness w. r. t. cost set $C_1, C_2, C_3, \dots, C_n$.

Step -1. First we arrange the preference of decision makers in the form of fuzzy decision making matrix for each alternative $A_j (j=1, 2, \dots, n)$ w. r. t. cost set $C_k (k=1, 2, \dots, m)$ as follows

$$D_{n \times m} [a_{ij}] = \begin{matrix} & C_1 & C_2 & \dots & C_m \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & & \\ \vdots & \vdots & & \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & & a_{nm} \end{bmatrix} & \end{matrix}$$

Step-2. We determine the ideal solution A^* from all alternative corresponding to their cost set as

$$A^* = \{H_1^*, H_2^*, \dots, H_n^*\} \text{ where } H_i^* = \max\{H_i^*\}$$

Step -3. Therefore we calculate the divergence using $H_{\alpha, \beta}(A : B)$ given as

$$H_{\alpha, \beta}(A, B) = \frac{1}{(\alpha - 1)\beta} \sum_{i=1}^n \left[\left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} \right]^\beta - 1 \right]$$

Step -4. To give ranking we take

$$\min\{H_{\alpha, \beta}(A_j, A^*)\}; \text{ where } 1 \leq j \leq n$$

Select the most desirable alternative according with descending order of their function.

Numerical Example

In a fuzzy multicriteria problem we analyze an illustration example of the new define approach regarding admission preference of a student for post graduate course of science stream. Suppose that the student wants to take admission in Indian Institute of Technology (IIT) and he wants to select an institute from five options

- $A_1 = IIT \text{ Delhi}$
- $A_2 = IIT \text{ Bombay}$
- $A_3 = IIT \text{ Madras}$
- $A_4 = IIT \text{ Kanpur}$
- $A_5 = IIT \text{ Roorki}$

These are the most valuable institute for science course. The student wants to choose institute on the following basis

- $C_1 = Placement$
- $C_2 = Ranking$
- $C_3 = Faculty$
- $C_4 = Facility$
- $C_5 = Fee$

Step-1. Arranging the value of all alternative we arrange the preference in matrix $M_{n \times m} [m_{ij}]$.

$$M_{n \times m} [m_{ij}] = \begin{matrix} & C_1 & C_2 & C_3 & C_4 & C_5 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{matrix} & \begin{bmatrix} 0.5 & 0.7 & 0.8 & 0.3 & 0.1 \\ 0.7 & 0.9 & 0.3 & 0.2 & 0.4 \\ 0.4 & 0.8 & 0.6 & 0.7 & 0.5 \\ 0.3 & 0.1 & 0.5 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0.4 & 0.8 & 0.6 \end{bmatrix} & \end{matrix}$$

Step-2. Optimum solution from above matrix is

$$A^* = \{0.7, 0.9, 0.8, 0.8, 0.6\}$$

Step-3. We calculate the divergence of A^* w. r. t. each alternative as

Table1. Different values of entropy at different parameters

α	β	$H_{\alpha, \beta}(A_1, A^*)$	$H_{\alpha, \beta}(A_2, A^*)$	$H_{\alpha, \beta}(A_3, A^*)$	$H_{\alpha, \beta}(A_4, A^*)$	$H_{\alpha, \beta}(A_5, A^*)$
0.1	0.1	0.14	0.14	0.03	0.31	0.25
0.9	0.1	1.25	1.36	0.34	2.81	2.33
1.1	2.1	1.56	1.74	0.43	3.68	3.05
5	2.1	30.25	72.54	2.60	1960.1	960.5

Step- 4. From the table-1 we find that $H_{\alpha,\beta}(A_3, A^*)$ has minimum value for all parameters so we can easily estimate that the best alternative is A_3 . So the student should take admission in IIT Madras.

Hence this shows that introduced fuzzy measure divergence is very suitable measure to solve the multicriteria decision making problem.

6. Conclusions

In this article we describe directed divergence measure for fuzzy set. Properties of proposed measure are discussed. Application of introduced measure is given for decision making. A numerical example is given for decision making.

Conflict of Interest

There is no any conflict of interest.

Ethical Approval

In present article, the authors have no any study with human participants or animals.

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