

# Multi-Product Economic Inventory Policy with Time Varying Power Demand, Shortages and Complete Backordering

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**Abstract** The present article elaborates a multi-product economic inventory model in which multiple items are considered with demand, which depends on time, abides power law, shortages, and complete backordering. The rate of production is greater than the rate of demand for each item. This type of demand pattern is accessible to various kinds of practical problems in the present modern era. In this paper, the production rate is assumed to be directly proportional to the rate of demand. In real situations, demand is always dependent on the price of any product, so it is also assumed that demand is depleted linearly with the price. This paper aims to fulfill the requirement of products and maximize the overall profit in any organization. Optimal values of involved costs, price, re-order point, and scheduling period are also calculated. In this work, the objective is to maximize the total profit and optimize production quantity. Optimization methods are used to obtain the maximum profit. Shortages are permitted and completely backordered. Total cost and production quantity models for deteriorating items have been developed. The study is validated with a numerical example, and sensitive analysis of the optimal solutions concerning main parameters is carried out using the software Mathematica. The tables are also shown for optimal values of the present model.

**Keywords** Inventory, Power Demand, Lot Size,

Multi-Items, Shortages, Backordering

## 1. Introduction

In practical situations, customers' demand depends on time, and hence researchers presented some varying time models. Barbosa and Friedman [1] proposed a time-varying model for decaying items. Silver and Peterson [2] concludes that retail-level sales are proportional to displayed inventory, and more quantity of presented products will attract more customers to purchase. Mitra et al. [3] suggested a pattern of demand that varies with time. Baker and Urban [4] presented an EOQ model involving a demand pattern for a stock of power. After few years, Mandal and Phaujdar [5] studied a policy for decaying items with the demand which depends on stock. Sarkar et al. [6] presented a time and price-dependent production model in the environment of reliability and inflation. Urban [7] studied periods of an inventory model, unlike the demand of stock time by a given quantity. Alfares [8] presented a policy where the demand relates to the inventory level and cost of holding, which relates to the period of storage. Datta and Pal [9] studied a fixed demand inventory policy with

inventory-level dependent demand for some fixed level and a non-variable rate of demand for the cycle (left) with an infinite time horizon. Lee and Wu [10] elaborated a policy optimal order quantity for decaying products with power demand and shortages. Singh et al. [11] introduced an EOQ model for decaying products with shortages and power pattern demand. Pal et al. [12] analyzed periods of an inventory distinct from the inventory demand by a known amount. Hwang and Hahn [13] introduced an inventory model of production for decaying items. Many researchers used holding cost always constant, but in some cases time- dependent holding cost is used. Singh and Mishra [14] introduced an EOQ scheme for perishable items with power pattern demand. Sicilia et al. [15] presented an optimum policy for lot size having demand of power pattern and production, which is directly proportional to demand rate.

Demand is always price dependent, so the price is one of the factors for any organization. Tripathi et al. [16] presented a system with a power pattern withholding cost and stock-related demand for two types of a business environment. Chaudhuri and Ray [17] introduced the money time value by studying an inventory policy with the stock-induced demand rate. Giri, Goswami, and Chaudhuri [18] studied a production model for perishable items with the influence of shortages. Banerjee and Sharma [19] studied an inventory policy with the price and time-dependent seasonal demand. Later again, Banerjee [20] introduced a combined economic lot-size system and supposed that lot-for-lot shipment scheme followed by the supplier with the retailer". In continuation, Goyal [21] analyzed Banerjee's policy by taking a batch having a number of same-shaped shipments, but the batch production had to be completed before the starting of shipment. Liu et al. [22] presented a joint investment policy for decaying products with quality and price-dependent demand. Hill [23] introduced an optimal lot-sizing inventory model of two-stage and inventory batching schemes. Zhu [24] introduced a dynamic replacement and production model by using random price-sensitive demand. Teng et al. [25] presented an increasing payment scheme. Goyal and Chang [26] introduced a model of ordering-transfer to obtain the retailer's optimal order amount and per order number of shifting from the warehouse. Yang and Wee [27] introduced an inventory policy of multi-lot-size production for decaying products. Heydari and Gholamian [28] presented a mixed integer probabilistic model by considering METRIC stochastic relation in a problem of location-routing. Hsieh et al. [29] presented the effect of shortages and delays in payment. Sarkar [30] presented sustainable ordering policies for non-instantaneous decaying products under carbon emission and multi-trade-credit-policies. Kumar et al. [31] in 2020 has developed a manufacturing reliability inventory model in which demand depends on advertising, time, and selling price.

In this study, we developed an inventory model based on demand pattern i.e., power demand, and it defines that type of demand, which varies with the rate of change in the form of a power function. This kind of demand pattern is used in situations of the high demand of products at the starting of the cycle or at the ending of the cycle.

## 2. Notations and Assumptions

### 2.1. Notations

In the present study the following notations are used:

- N: Number of products in the system
- T: Period of scheduling
- $Q_i$ : Quantity of production for product  $i$
- $W_i$ : Quantity of demand for inventory cycle for product  $i$
- $d_i$ : Average demand for product  $i$  and given as  $d_i = W_i/T$
- $r_i$ : Point of reorder for product  $i$
- $u_i, v_i, \lambda_i$ : constants
- $B_i$ : as the constant of proportionality
- $l_i$ : Length of period of production for product  $i$
- $p_i$ : Price of selling for product  $i$  (per unit)
- $c_i$ : Cost of unit production for product  $i$  (per unit)
- $\mu_i$ : Set up cost of production for product  $i$  (per replenishment)
- $x_i$ : Per unit per unit time cost of unit backlogging for product  $i$ .
- $y_i$ : Per unit per unit time cost of unit carrying for product  $i$ .
- $s_i$ : Per unit revenue of sales for product  $i$ .
- $C1_i$ : Per unit time cost of production for product  $i$ .
- $C2_i$ : Per unit time set up cost for product  $i$ .
- $C3_i$ : Per unit time backordering cost for product  $i$ .
- $C4_i$ : Per unit time holding cost for product  $i$ .
- $TP_i(p_i, r_i, T)$ : Per unit time total profit for product  $i$ .
- $\Omega(p^*, r^*, T)$ : Total profit for the whole inventory system

### 2.2. Assumptions

In the present study the following assumptions are summarized in whole study:

- (1). Shortages are permitted and completely backordered.
- (2). Infinite planning horizon.
- (3). Inventory system with multi-products.
- (4). Rate of demand is less than the rate of production for every product.
- (5). The rate of production  $P_i(t)$  is assumed as directly proportional to the rate of demand  $W_i(t)$  for each product  $i$  at time  $t$  ( $0 \leq t \leq l_i$ ) and given as  $P_i(t) = 2_i W_i(t)$  with  $2_i > 1$ .
- (6). The rate of demand is time related which depletes with price in linear way with time and obeys a power

pattern. Thus, the rate of demand can be assumed as follows:

$$W_i(t) = (u_i - v_i) \frac{\mu_i}{\lambda_i} (t/T)^{1/\lambda_i - 1}, \quad (0 \leq t \leq T) \text{ where } \mathbf{0} < \lambda_i < \infty, u_i > \mathbf{0}, v_i > \mathbf{0}$$

### 3. Mathematical Modeling and Solution of the Problem

In this study we consider a manufacturing industry which produces  $N$  types of products. There is an average demand  $d_i$  for each product and is related to time which depletes with price in linear way and obeys a power pattern. It is required to please the requirement of customers and also to optimize the overall profit at the same time. The pattern of inventory begins from  $s$  units of the total inventory at time  $t = 0$  and simultaneously the production starts at the initial time with rate of production  $P_i(t)$  at time  $t_{1i}$  which goes to zero and proceeds until  $t = l_i$  for each item  $i$ . In the same direction the replenishment quantity  $W_i$  is produced. During the cycle  $[0, l_i]$  the stock level of any product  $i$  increasing with rate  $P_i(t) - W_i(t)$ . Later the stock level depletes up to  $t = t_{2i}$  as per demand. The demand will be backlogged during the interval  $[t_{2i}, T]$ . Suppose  $I_1(t)$  and  $I_2(t)$  the levels of in hand inventory for any item  $i$  in the intervals  $[0, l_i]$  and  $[l_i, t]$  respectively. In this study there are three decision variables (i) Price, (ii) Size of back order, and (iii). Scheduling period. The modeling of the present study is as follows:

The demand during the period  $[0, T]$ , which is a scheduling period is given as

$$\int_0^T W_i dt = \int_0^T (u_i - v_i p_i) \frac{d_i}{\lambda_i} (t/T)^{1/\lambda_i - 1} dt = (u_i - v_i p_i) d_i T \quad (1)$$

The whole system associated with this model can be given as follows.

$$\begin{aligned} \frac{dI_1(t)}{dt} &= P_i(t) - W_i(t) = (\beta_i - 1)W_i(t) \\ &= (\beta_i - 1)(u_i - v_i p_i) \frac{d_i}{\lambda_i} (t/T)^{1/\lambda_i - 1} \\ I_1(0) &= r_i, \quad 0 \leq t \leq l_i \end{aligned} \quad (2)$$

$$\frac{dI_2(t)}{dt} = -W_i(t) = -(u_i - v_i p_i) \frac{d_i}{\lambda_i} (t/T)^{1/\lambda_i - 1}$$

$$I_2(T) = r_i, \quad l_i \leq t \leq T \quad I_2(T) = r_i, \quad l_i \leq t \leq T \quad (3)$$

Using the boundary conditions  $I_1(0) = I_2(T) = r_i$ , the solution of equations (2) and (3) are given as

$$\begin{aligned} I_1(t) &= r_i + (\beta_i - 1)(u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i}, \quad 0 \leq t \leq l_i \\ I_1(t) &= r_i + (\beta_i - 1)(u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i}, \quad 0 \leq t \leq l_i \end{aligned} \quad (4)$$

$$I_2(t) = r_i + (u_i - v_i p_i) d_i T - (u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i}, \quad l_i \leq t \leq T \quad (5)$$

A lot size production cycle is completed at  $t = l_i$ . By solving equations (4) and (5), the level of maximum inventory can be determined, which is given as follows:

$$I_i = T / \beta_i^{\lambda_i} \quad (6)$$

$$I_i(l_i) = r_i + \frac{\beta_i - 1}{\beta_i} (u_i - v_i p_i) d_i T \quad (7)$$

The lot size  $Q_i$  can be determined as follows,

$$\begin{aligned} Q_i \int_0^{l_i} P_i(t) dt &= \int_0^{l_i} \beta_i (u_i - v_i p_i) \frac{d_i}{\lambda_i} (t/T)^{1/\lambda_i - 1} dt = (u_i - v_i p_i) d_i T \\ &= (u_i - v_i p_i) d_i T \end{aligned} \quad (8)$$

As we had assumed that demand of period of scheduling is the same as lot quantity. So, it can be assumed that  $I(l_i) \geq \mathbf{0}$  and  $r_i > \mathbf{0}$ . Assume that at time  $t = t_{1i}$  the level of inventory goes to zero during the period of production. Using the condition  $I_1(t_{1i}) = 0$  to find  $t_{1i}$  from equation (4), we have

$$t_{1i} = \left( \frac{-s_i}{(\beta_i - 1)(u_i - v_i p_i) d_i T} \right)^n T \quad (9)$$

Here  $n$  is the number of cycles. The net level of inventory for the cycle  $[l_i, T]$  goes to zero at time  $t = t_{2i}$ . Using the condition  $I_2(t_{2i}) = 0$  to find  $t_{2i}$  from equation (5), we have

$$t_{2i} = \left( 1 + \frac{s_i}{(u_i - v_i p_i) d_i T} \right)^n T \quad (10)$$

Now, we calculate the four assumed costs for each product  $i$  by considering the fact that the mean number of runs of production is  $\frac{1}{T}$ .

The revenue of sales for product  $i$  is

$$RS_i = p_i \frac{Q_i}{T} = (u_i - v_i p_i) p_i d_i \quad (11)$$

The cost of production for product  $i$  is

$$C1_i = c_i \frac{Q_i}{T} = (u_i - v_i p_i) c_i d_i \quad (12)$$

The set-up cost for product  $i$  is

$$C2_i = \frac{\mu_i}{T} \quad (13)$$

The Backordering or Shortage Cost

$$\begin{aligned} C3_i &= \frac{-x_i}{T} \left[ \int_0^{t_{1i}} \{r_i + (\beta_i - 1)(u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i}\} dt + \int_{t_{2i}}^T \{r_i + (u_i - v_i p_i) d_i T - (u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i}\} dt \right] \end{aligned}$$

On simplification, we have

$$C3_i = \left[ \frac{(r_i + (u_i - v_i p_i) d_i T)^{\lambda_i + 1}}{(\lambda_i + 1)(u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} + \frac{(-r_i)^{\lambda_i + 1}}{(\lambda_i + 1)(\beta_i - 1)^{\lambda_i} (u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} - \frac{(u_i - v_i p_i) d_i T}{(\lambda_i + 1)} - r \right] x_i \quad (14)$$

The holding or carrying cost is

$$C4_i = \frac{y_i}{T} \left[ \int_{t_{1i}}^{t_i} \left\{ r_i + (\beta_i - 1)(u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i} \right\} dt + \int_{t_i}^{t_{2i}} \left\{ r_i + (u_i - v_i p_i) d_i T - (u_i - v_i p_i) d_i T (t/T)^{1/\lambda_i} \right\} dt \right]$$

On integrating, we have

$$C4_i = \left[ \frac{(r_i + (u_i - v_i p_i) d_i T)^{\lambda_i + 1}}{(\lambda_i + 1)(u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} + \frac{(-r_i)^{\lambda_i + 1}}{(\lambda_i + 1)(\beta_i - 1)^{\lambda_i} (u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} - \frac{(u_i - v_i p_i) d_i T}{(\lambda_i + 1)} \right] y_i - \frac{(u_i - v_i p_i) d_i T}{(\lambda_i + 1)} y_i \quad (15)$$

Now we can calculate the total profit for the product *i* as follows:

$$TP_i(p_i, r_i, T) = RS_i - C1_i - C2_i - C3_i - C4_i = (u_i - v_i p_i) p_i d_i - \frac{(r_i + (u_i - v_i p_i) d_i T)^{\lambda_i + 1} (x_i + y_i)}{(\lambda_i + 1)(u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} - \frac{(-r_i)^{\lambda_i + 1} (x_i + y_i)}{(\lambda_i + 1)(\beta_i - 1)^{\lambda_i} (u_i - v_i p_i)^{\lambda_i} d_i^{\lambda_i} T^{\lambda_i}} + \frac{(u_i - v_i p_i) d_i T}{(\lambda_i + 1)} \left( \frac{y_i}{\beta_i^{\lambda_i}} + x_i \right) - \frac{\mu_i}{T} - (u_i - v_i p_i) c_i d_i \quad (16)$$

Now define the functions  $p^* = (p_1, p_2, p_3, \dots, p_n)$ ,  $r^* = (r_1, r_2, r_3, \dots, r_n)$  and determine the total profit of inventory as follows:

$$\Omega(p^*, r^*, T) = \sum_{i=1}^N TP_i(p_i, r_i, T) \quad (17)$$

The essential conditions to optimize the total profit are as follows:

$$\frac{\partial \Omega(p^*, r^*, T)}{\partial r_i} = 0, \quad \frac{\partial \Omega(p^*, r^*, T)}{\partial p_i} = 0 \quad \text{and} \quad \frac{\partial \Omega(p^*, r^*, T)}{\partial T} = 0 \quad (18)$$

Using  $\frac{\partial \Omega(p^*, r^*, T)}{\partial T} = 0$ ,  $\frac{\partial \Omega(p^*, r^*, T)}{\partial T} = 0$ , the optimal length cycle which is given as

$$T^* = \sqrt{\frac{\sum_{i=1}^N \mu_i}{\sum_{i=1}^N \left[ \frac{(u_i - v_i p_i) d_i}{(\lambda_i + 1)} \left( (x_i + y_i)(1 - z_i^*)^{\lambda_i} + \lambda_i x_i z_i^* - \left( \frac{y_i}{\beta_i^{\lambda_i}} + x_i \right) \right) \right]}} \quad (19)$$

where  $z_i$  is a new variable defined as

$$z_i = \frac{-r_i}{(u_i - v_i p_i) d_i T}, \quad \text{and} \quad (1 - z_i)^{\lambda_i} - \frac{z_i^{\lambda_i}}{(\beta_i - 1)^{\lambda_i}} - \frac{x_i}{(x_i + y_i)} = 0 \quad (20)$$

Also, the optimal lot quantity ( $r_i^*$ ) and the optimal re-order point ( $Q_i^*$ ) are given as follows:

$$r_i^* = -z_i^*(u_i - v_i p_i) d_i T^* \quad (21)$$

$$Q_i^* = (u_i - v_i p_i) d_i T^* \quad (22)$$

Now the condition to optimize the TPF (total profit function)

i.e.,  $\Omega(\bar{p}, \bar{r}^*, T^*)$  for any  $r_1^*, r_2^*, \dots, r_N^*, T^*$  is given as

$$\frac{\partial \Omega(\bar{p}, \bar{r}^*, T^*)}{\partial p_i} = -u_i d_i - 2v_i p_i d_i + v_i c_i d_i - \frac{v_i d_i T^*}{(\lambda_i + 1)} \left( \frac{y_i}{\beta_i^{\lambda_i}} + x_i \right) (x_i + y_i) \frac{v_i}{(\lambda_i + 1) d_i^{\lambda_i} T^{*\lambda_i}} \left[ \frac{(r_i (u_i - v_i p_i) d_i T^*)^{\lambda_i} (u_i - v_i p_i) d_i T^* - \lambda_i r_i}{(u_i - v_i p_i)^{\lambda_i + 1}} - \frac{\lambda_i (-r_i)^{\lambda_i + 1}}{(\beta_i - 1)^{\lambda_i} (u_i - v_i p_i)^{\lambda_i + 1}} \right] \quad (23)$$

### 4. Process of Finding the Optimal Values

It is difficult to solve the equation (23) as it is complex and time consuming also. So, an alternate process is used to obtain the optimal values of  $\bar{p}, \bar{r}, T^* \geq T^*, \bar{p}, \bar{r}, T$  and  $\Omega$ .

**Step 1:** For every item *i*,  $p_i = c_i$ , where  $c_i < \frac{u_i}{v_i}$ .

If  $T^* \geq T^* \geq 0$

$$\text{i.e., } \sum_{i=1}^N \left[ \frac{\mu_i}{\left( \frac{(u_i - v_i p_i) d_i}{(\lambda_i + 1)} \right) \left( (x_i + y_i)(1 - z_i^*)^{\lambda_i} + \lambda_i x_i z_i^* - \left( \frac{y_i}{\beta_i^{\lambda_i}} + x_i \right) \right)} \right] \geq 0$$

determine  $T^*$  and then find out  $\bar{r}^*$  and  $\Omega(\bar{p}, \bar{r}^*, T^*)$ , otherwise there is no required solution.

**Step 2:** For every item *i* ( $i = 1, 2, \dots, N$ ), it is assumed that  $p_i = p_i + e$  and  $p_j = p_j$  for each  $i \neq j$ . Then determine best economic period of scheduling ( $\bar{r}_i^*, T_i^*$ ) (say) and the function of total profit  $\Omega_i(\bar{p}, \bar{r}_i^*, T_i^*)$  (say).

**Step 3:** Select the item *k* with the following conditions

- a). It is possible to determine  $T_k^*$ .
- b).  $p_k < \frac{u_k}{v_k}$   $p_k < \frac{u_k}{v_k}$
- c).  $\Omega_k(\bar{p}, \bar{r}_k^*, T_k^*) > \Omega(\bar{p}, \bar{r}^*, T^*)$  and  $\Omega_k(\bar{p}, \bar{r}_k^*, T_k^*) > \Omega_j(\bar{p}, \bar{r}_j^*, T_j^*)$  for each  $j \neq k$ .

If these conditions work properly then the best solutions are  $\bar{r}^*, T^*, \Omega(\bar{p}, \bar{r}^*, T^*)$  and reach to the step 5.

**Step 4:** If  $p_k = p_k + e$  and  $p_j = p_j$  for each  $j \neq k$  then  $T^* = T_k^*$  and  $\Omega(\bar{p}, \bar{r}^*, T^*) = \Omega_k(\bar{p}, \bar{r}_k^*, T_k^*)$ , thus

we return to the step 2.

**Step 5:** End.

### 5. Numerical Verification of the Study

Here the present study is verified by a numerical example.

Taking an inventory model for a unique item with the following input values of the parameters as:

$$\mu = 102, x = 4.8, y = 4.2, d = 1180, u = 106, v = 2, c = 11, \beta = 1.5, \text{ and the power pattern index } \lambda = 3.$$

The results with these parameters are as follows:

$$z^* = 0.16542, T^* = 0.7435, Q^* = 3372.5, r^* = -568.215, p^* = 29, \text{ and } TP^* = 949845.$$

\*Optimal schemes of the present model considering many combinations of the parameters  $\beta$  and  $c$  are presented in Table 1.

\*Optimal schemes of the present model considering many combinations of the parameters  $\lambda$  and  $c$  presented in Table 2.

\*Optimal schemes of the present model considering the distinct value of  $u$  presented in Table 3.

**Table 1.** Optimal schemes of the present model considering many combinations of the parameters  $\beta$  and  $c$ .

Rate of production	Cost of production	$z^*$	$T^*$	$r^*$	$Q^*$	$p^*$	$\Omega^*$
$\beta = 1.2$	C=12	0.065324	0.1602	-475.7654	7271.4	30	957430
	C=16		0.1765	-452.8765	6674.2	34	755674
	C=20		0.1834	-418.6373	6245.5	36	545262
	C=24		0.1940	-378.8675	5688.2	38	395423
	C=28		0.2542	-365.7654	5188.1	40	251674
$\beta = 1.4$	C=12	0.143562	0.0932	-578.8758	4274.2	30	947430
	C=16		0.0985	-562.6277	3986.3	34	745674
	C=20		0.1123	-512.6546	3693.2	36	542677
	C=24		0.1176	-478.8075	3286.6	38	385423
	C=28		0.1256	-437.8754	2784.6	40	241674
$\beta = 1.6$	C=12	0.163865	0.0712	-582.4562	3486.3	30	942654
	C=16		0.0784	-534.6578	3024.5	34	742897
	C=20		0.0832	-485.7688	2865.6	36	537654
	C=24		0.0965	-446.7634	2576.8	38	378964
	C=28		0.1426	-4234.765	2254.8	40	237644
$\beta = 1.8$	C=12	0.180544	0.0675	-564.7654	3215.4	30	937430
	C=16		0.0712	-512.8727	2856.2	34	735674
	C=20		0.0782	-475.3772	2675.8	36	532677
	C=24		0.0864	-421.6543	2365.4	38	375423
	C=28		0.0975		2163.6	40	241674
$\beta = 2.0$	C=12	0.182345	0.0642	-543.8764	2898.0	30	927430
	C=16		0.0674	-498.8756	2765.8	34	725674
	C=20		0.0732	-470.6543	2610.5	36	512677
	C=24		0.0812	-410.2345	2286.0	38	367423
	C=28		0.0876	-385.3565	2046.6	40	232474

**Table 2.** Optimal schemes of the present model considering many combinations of the parameters  $\lambda$  and  $c$ .

Rate of production	Cost of production	$z^*$	$T^*$	$r^*$	$Q^*$	$p^*$	$\Omega^*$
$\lambda = 0.6$	$c = 15$	0.096542	0.0745	-310.542	3486.3	35	967415
	$c = 25$		0.0834	-287.234	3024.5	40	543276
$\lambda = 1.5$	$c = 15$	0.176542	0.0756	-576.345	3565.6	35	967425
	$c = 25$		0.0854	-498.456	2576.8	40	543286
$\lambda = 2.5$	$c = 15$	0.187642	0.0768	-724.865	3698.6	35	967445
	$c = 25$		0.0878	-645.342	3286.0	40	543290
$\lambda = 3.5$	$c = 15$	0.176554	0.0785	-590.342	3576.6	35	967455
	$c = 25$		0.0886	-512.452	3065.8	40	543298

**Table 3.** Optimal schemes of the present model considering the distinct value of  $u$ .

$u$	$z^*$	$T^*$	$r^*$	$Q^*$	$p^*$	$\Omega^*$
110	0.187342	0.0744	-554.123	3217.2	32	968072
210		0.0456	-865.345	4849.5	32	4966724
310		0.0385	-1123.24	6028.2	32	11965000

## 6. Conclusions and Future Scope

In the present study an inventory policy in a manufacturing system with time varying power pattern demand is presented. It is assumed that the rate of production varies with the rate of demand and the requirement of customers is bounded by price linearity. Multiple items are considered in the system of inventory. The shortages are permitted and completely backordered. Optimization methods are used to obtain the maximum profit and numerical verification is presented to illustrate the study. If the rate of production is a fixed parameter than an increment in unit production cost results decrement of total profit as well as economic lot whereas there is an increment in the optimal price and optimal length of inventory cycle. Similarly, if the powers demand index is fixed then an increment in unit production cost results a decrement in total profit as well as economic lot whereas the optimal price and optimal length of inventory cycle increase. It is also observed that the increment in rate of production results a slight decrement in total profit and decrement in both optimal lot size and scheduling period.

The present study can be extended by using it for obtaining products and shortages with partial backorder in an inventory system.

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