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Fuzzy Time Series Forecasting Model Based on Intuitionistic Fuzzy Sets via Delegation of Hesitancy Degree to the Major Grade De-i-fuzzification Method

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Abstract Fuzzy time series is a powerful tool to forecast the time series data under uncertainty. Fuzzy time series was first initiated with fuzzy sets and then generalized by intuitionistic fuzzy sets. The intuitionistic fuzzy sets consider the degree of hesitation in which the degree of non-membership is incorporated. In this paper, a fuzzy set time series forecasting model based on intuitionistic fuzzy sets via delegation of hesitancy degree to the major grade de-i-fuzzification approach was developed. The proposed model was implemented on the data of student enrollments at the University of Alabama. The forecasted output was obtained using the fuzzy logical relationships of the output, and the performance of the forecasted output was compared with the fuzzy time series forecasting model based on fuzzy sets using the mean square error, root mean square error, mean absolute error, and mean absolute percentage error. The results showed that the forecasting model based on induced fuzzy sets from intuitionistic fuzzy sets performs better compared to the fuzzy time series forecasting model based on fuzzy sets.

Keywords De-i-fuzzification, Forecasting Enrollment, Fuzzy Time Series, Hesitancy Degree, Induced Fuzzy Set, Intuitionistic Fuzzy Set

1. Introduction

Fuzzy time series was first introduced by Song and Chissom [1] and it was applied to the data of student enrollments at the University of Alabama [2]. After that, many researchers have developed and improved the fuzzy time series forecasting model by modifying the interval lengths, the order of the fuzzy logical relationships, and the rules for calculating the forecasted output [3-10]. Fuzzy forecasting is better than other forecasting methods since it can handle unclear and uncertain data [11].

However, the aforementioned models did not consider the degree of hesitation in their forecasting methods. In 1986, Atanassov [12] first introduced the concept of intuitionistic fuzzy sets, which incorporates the degree of membership and non-membership of the fuzzy sets. Castillo et al. [13] applied the intuitionistic fuzzy sets in the fuzzy time series by applying them to plant monitoring and diagnosis. Since then, many researchers have developed the fuzzy time series forecasting model based on intuitionistic fuzzy sets such as Joshi and Kumar [14], Gangwar and Kumar [15], Kumar and Gangwar [16], Fan et al. [17], Bisht et al. [18] and, Gupta and Kumar [19]. In a study by Joshi and Kumar [14], an algorithm for fuzzy time series forecasting, which includes the hesitation index in establishing a fuzzy logical relationship was introduced. Meanwhile, Kumar and Gangwar [16] used the fuzzy set induced by the intuitionistic fuzzy set to establish the fuzzy time series forecasting model. Bisht et al. [18] then introduced the application of dual hesitant fuzzy sets in intuitionistic fuzzy set forecasting to handle non-determinism that occurs due to multiple valid fuzzification methods for time series data. While Abhishekh et al. [20] proposed the score function for intuitionistic fuzzifying of the historical time series data, Bisht and Kumar [21], recently, have also proposed an intuitionistic fuzzy set with a simplified algorithm in financial prediction.

A process of converting an intuitionistic fuzzy set into a fuzzy set is one of the core procedures in the fuzzy time series forecasting model based on the intuitionistic fuzzy set, which was termed as the de-i-fuzzification process by Attanassova [22] and Ban et al. [23]. During this process, the hesitancy degree is reduced to zero, and the summation of membership and non-membership degree becomes one. Ansari et al. [24] also introduced three different methods of de-i-fuzzification, which are assigning hesitancy to the major grade, equal distribution of hesitancy, and proportionate allocation.

In this paper, we propose the fuzzy time series forecasting model based on fuzzy sets induced from intuitionistic fuzzy sets using the de-i-fuzzification approach by assigning the degree of hesitancy to the major grade. The performance of the proposed method was compared with the fuzzy time series forecasting based on fuzzy sets.

This paper is organized as follows: Section 2 reviews the fuzzy time series and intuitionistic fuzzy sets; Section 3 presents the proposed fuzzy time series model based on induced fuzzy sets; Section 4 implements the proposed model on student enrollments at the University of Alabama; Section 5 presents and discusses the forecasted output; and Section 6 concludes the paper.

2. Review of Fuzzy Time Series and Intuitionistic Fuzzy Sets

In this section, some concepts of fuzzy sets, fuzzy time series, and intuitionistic fuzzy sets are reviewed.

Definition 2.1: If S is a fuzzy set on the universe of discourse, $D=\{d_1,d_2,d_3,...,d_n\}$, then D can be written as

$$D=\mu_S(d_1)/d_1+\mu_S(d_2)/d_2+\mu_S(d_3)/d_3+...+\mu_S(d_n)/d_n \quad (1)$$

where $\mu_S(d_i)$ is the membership grade of the elements $d_i \in D$ for $i=1,2,3,...,n.$

Definition 2.2: If $Y(t),(t=0,1,2,...)$ is the universe of discourse and fuzzy sets $f_i(t),(i=0,1,2,...)$ are defined on $Y(t)$, then the fuzzy time series of $Y(t)$ is a collection of fuzzy sets $f_i(t)$.

Definition 2.3: If only $Y(t)$ affects $F(t)$, then $F(t)=F(t-1)\circ R(t,t-1)$ represents $F(t-1)\rightarrow F(t)$, where $R(t,t-1)$ is a fuzzy relation between $F(t)$ and $F(t-1)$.

Definition 2.4: An intuitionistic fuzzy set I in D can be written in the form

$$I=\{\langle x,\mu_I(x),\nu_I(x)\rangle : x\in D\} \quad (2)$$

where the functions $\mu_I(x):D\rightarrow[0,1]$ and $\nu_I(x):D\rightarrow[0,1]$ are the degree of membership and non-membership of x in D, respectively. For every x in D,

$$0\le \mu_I(x)+\nu_I(x)\le 1. \quad (3)$$

The degree of indeterminacy is defined by $\pi_I(x)=1-\mu_I(x)-\nu_I(x)$. π_I is also known as the degree of hesitancy of x in D.

Definition 2.5: Let $\beta\in\kappa$ where κ is the collection of all fuzzy sets in D. Let $\alpha:D\rightarrow[0,1]$ and $\beta:D\rightarrow[0,1]$. $f:[0,1]^2\times[0,1]\rightarrow L^*$, where $f(x,\alpha,\beta)=(f_\mu(x,\alpha,\beta), f_\nu(x,\alpha,\beta)$ and

$$f_\mu(x,\alpha,\beta)=x(1-\alpha\beta),$$
$$f_\nu(x,\alpha,\beta)=1-x(1-\alpha\beta)-\alpha\beta. \quad (4)$$

2.1. De-i-fuzzification Process by Assigning Hesitancy to Major Grade [24]

Ansari et al. [24] proposed a de-i-fuzzification process by assigning the degree of hesitancy to the major grade of membership or non-membership. The proposed de-i-fuzzification process is given as follows:

Let π be the degree of hesitancy and the intuitionistic fuzzy set is given as $I=\langle x,\mu(x),\nu(x)\rangle$. If the degree of membership μ is greater than the degree of non-membership ν, then the degree of membership of the fuzzy set is $\mu+\pi$. However, if the degree of membership μ is less than the degree of non-membership ν, then the degree of membership of fuzzy set remains as μ.

3. Proposed Fuzzy Time Series Forecasting Model Based on Fuzzy Sets and IFSs

In this section, we will explain the following ten steps of the proposed fuzzy time series forecasting model in detail:

Step 1: Define the universe of discourse, $D=[D_{\min}-D_a, D_{\max}+D_b]$ where $D_{\min}$ and $D_{\max}$ are the minimum and maximum values of the historical data, while D_a and D_b are two suitable positive integers.

Step 2: Partition D into a certain number of intervals using the frequency density-based method [25].

Step 3: Fuzzify the historical data using triangular fuzzy numbers.

Step 4: Convert the fuzzy sets into intuitionistic fuzzy sets using Definition 2.5.

Step 5: De-i-fuzzify the intuitionistic fuzzy sets into fuzzy sets by assigning the degree of hesitancy to major grade [24].

Step 6: Obtain the induced fuzzy sets by considering the newly obtained membership values only.

Step 7: Establish the fuzzy logical relationships from the fuzzy sets obtained in Step 3 and induced fuzzy sets obtained in Step 6. Suppose the fuzzy logical relationships obtained are $\tilde{F}_a \to \tilde{F}_b, \tilde{F}_c \to \tilde{F}_d, ..., \tilde{F}_y \to \tilde{F}_z$, then a matrix R can be obtained by the relation $\tilde{F}_a^T \times \tilde{F}_b \cup \tilde{F}_c^T \times \tilde{F}_d \cup ... \cup \tilde{F}_y^T \times \tilde{F}_z$.

Step 8: Perform the max-min composition operation on the matrix obtained in Step 7 to obtain the fuzzified forecasted data as $A_i = A_{i-1} \circ R$, whereby A_i and A_{i-1} are the forecasted data at year i and $i-1$, respectively, and $\circ$ is the max-min composition operator.

Step 9: From the fuzzified forecasted data, obtain the fuzzy logical relationships.

Step 10: Calculate the forecasted output using the fuzzy logical relationships obtained in Step 9.

The arithmetic rules for calculating the forecasted output are as follows:

1. If the fuzzified data of year n is $\tilde{F}_a$ and there is a unique fuzzy logical relationship, for instance $\tilde{F}_a \to \tilde{F}_b$, where the membership value of $\tilde{F}_b$ occurs in $D_b = [T_{b1}, T_{b2}]$, then the forecasted enrolment of year $n+1$ is given by

$$\frac{T_{b1} + T_{b2}}{2}.$$

2. If the fuzzified data of year n is $\tilde{F}_a$ and there are p fuzzy logical relationships, say $\tilde{F}_a \to \tilde{F}_b$, $\tilde{F}_a \to \tilde{F}_c$, …, $\tilde{F}_a \to \tilde{F}_k$, where the membership values of $\tilde{F}_b, \tilde{F}_c, ..., \tilde{F}_k$ occur in $D_b = [T_{b1}, T_{b2}]$ $D_c = [T_{c1}, T_{c2}]$ …, $D_k = [T_{k1}, T_{k2}]$ respectively, then the forecasted output of year $n+1$ is given by

$$\frac{T_{b1} + T_{b2} + T_{c1} + T_{c2} + ... + T_{k1} + T_{k2}}{2p}.$$

3. If the fuzzified data of year n is $\tilde{F}_a$ whose membership value occurs in $D_a = [T_{a1}, T_{a2}]$ and there are no fuzzy logical relationships, then the forecasted data of year $n+1$ is given by

$$\frac{T_{a1} + T_{a2}}{2}.$$

The above steps are illustrated in the following flowchart:

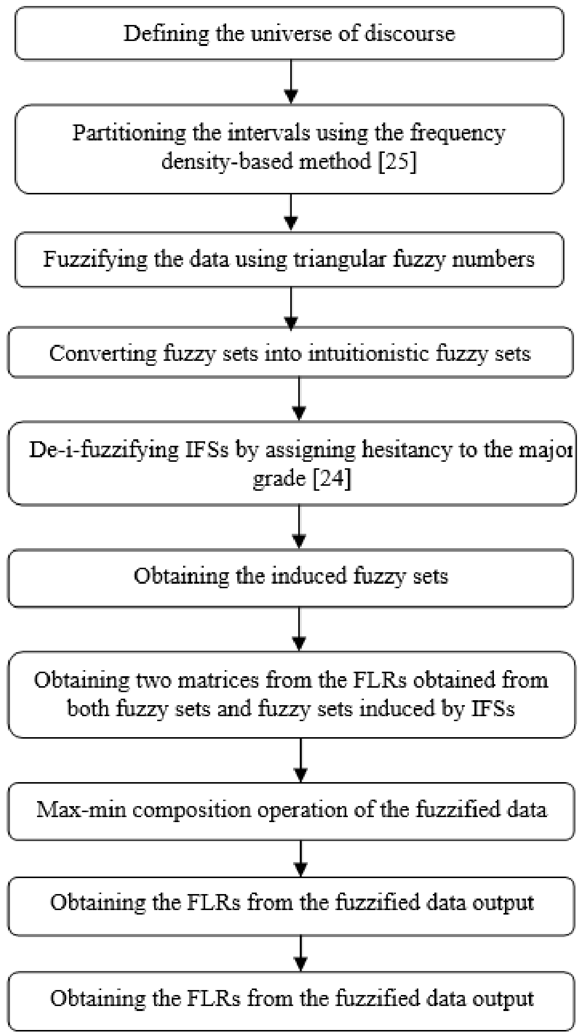

Figure 1. Steps of the proposed model

4. Forecasting Student Enrollments at the University of Alabama

Figure 2 presents the historical data of student enrollments at the University of Alabama from 1971 until 1992 as adopted from Song and Chissom [2-3].

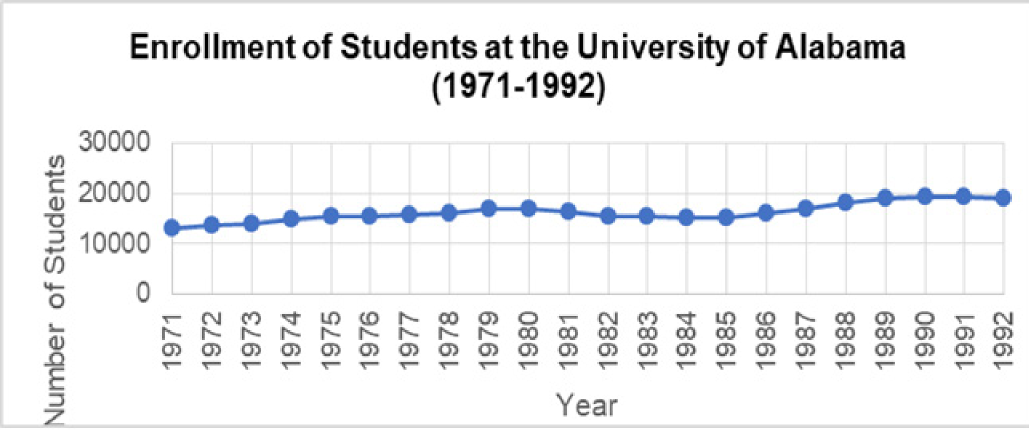

Figure 2. Student Enrollments at the University of Alabama (1971-1992)

The minimum and maximum data extracted based on Figure 2 are 13055 and 19337, respectively. The universe of discourse was defined by $D=[13055-55,19337+663]=[13000,20000]$. Then, D was divided using the frequency density-based method [25] and 14 intervals were obtained. The triangular fuzzy sets corresponding to these intervals are shown in Table 1.

Table 1. Triangular fuzzy sets corresponding to the intervals.

Intervals	Triangular Fuzzy Sets
$D_1=[13000,13500]$	$E_1=(13000,13500,14000)$
$D_2=[13500,14000]$	$E_2=(13500,14000,15000)$
$D_3=[14000,15000]$	$E_3=(14000,15000,15250)$
$D_4=[15000,15250]$	$E_4=(15000,15250,15500)$
$D_5=[15250,15500]$	$E_5=(15250,15500,15750)$
$D_6=[15500,15750]$	$E_6=(15500,15750,16000)$
$D_7=[15750,16000]$	$E_7=(15750,16000,16333)$
$D_8=[16000,16333]$	$E_8=(16000,16333,16667)$
$D_9=[16333,16667]$	$E_9=(16333,16667,17000)$
$D_{10}=[16667,17000]$	$E_{10}=(16667,17000,18000)$
$D_{11}=[17000,18000]$	$E_{11}=(17000,18000,18500)$
$D_{12}=[18000,18500]$	$E_{12}=(18000,18500,19000)$
$D_{13}=[18500,19000]$	$E_{13}=(18500,19000,20000)$
$D_{14}=[19000,20000]$	$E_{14}=(19000,20000,20000)$

The data were then fuzzified to obtain the following fuzzy sets.

$$F_1=0.110/13055+0.874/13563+0.266/13867$$

$$F_2=0.126/13563+0.734/13867+0.304/14696$$

$$F_3=0.696/14696+0.420/15145+0.348/15163$$

$$F_4=0.580/15145+0.652/15163+0.756/15311 +0.268/15433+0.160/15460+0.012/15497$$

$$F_5=0.244/15311+0.732/15433+0.840/15460 +0.988/15497+0.588/15603$$

$$F_6=0.412/15603+0.556/15861+0.064/15984$$

$$F_7=0.444/15861+0.936/15984$$

$$F_8=0.835/16388$$

$$F_9=0.165/16388+0.580/16807+0.423/16859 +0.243/16919$$

$$F_{10}=0.420/16807+0.577/16859+0.757/16919$$

$$F_{11}=0.700/18150$$

$$F_{12}=0.300/18150+0.248/18876+0.060/18970$$

$$F_{13}=0.752/18876+0.940/18970+0.672/19328 +0.663/19337$$

$$F_{14}=0.328/19328+0.337/19337$$

The fuzzy sets were then converted into intuitionistic fuzzy sets using Definition 2.5. The intuitionistic fuzzy sets obtained are as follows:

$$I_1=\{(13055,0.099,0.804),(13563,0.790,0.114), (13867,0.240,0.663)\}$$

$$I_2=\{(13563,0.114,0.793),(13867,0.666,0.241), (14696,0.276,0.632)\}$$

$$I_3=\{(14696,0.527,0.230),(15145,0.318,0.440), (15163,0.264,0.494)\}$$

$$I_4=\{(15145,0.575,0.416),(15163,0.646,0.345), (15311,0.749,0.242),(15433,0.266,0.725) (15460,0.159,0.832),(15497,0.012,0.979)\}$$

$$I_5=\{(15311,0.184,0.574),(15433,0.556,0.203), (15460,0.637,0.121),(15497,0.750,0.009) (15603,0.446,0.313)\}$$

$$I_6 = \{(15603,0.397,0.567),(15861,0.536,0.428),(15984,0.062,0.903)\}$$

$$I_7 = \{(15861,0.259,0.325),(15984,0.547,0.037)\}$$

$$I_8 = \{(16388,0.253,0.050)\}$$

$$I_9 = \{(16388,0.149,0.755),(16807,0.524,0.380),(16859,0.383,0.522),(16919,0.220,0.685)\}$$

$$I_{10} = \{(16807,0.286,0.396),(16859,0.394,0.289),(16919,0.516,0.166)\}$$

$$I_{11} = \{(18150,0.357,0.153)\}$$

$$I_{12} = \{(18150,0.295,0.687),(18876,0.244,0.738),(18970,0.059,0.923)\}$$

$$I_{13} = \{(18876,0.283,0.093),(18970,0.354,0.023),(19328,0.253,0.124),(19337,0.250,0.127)\}$$

$$I_{14} = \{(19328,0.292,0.598),(19337,0.300,0.590)\}$$

The intuitionistic fuzzy sets obtained in the previous step were de-i-fuzzified by assigning the degree of hesitancy to the major grade that is $\pi_I(x)$, which was added to the maximum of $\mu_I(x)$ or $\nu_I(x)$ [24]. The induced fuzzy sets obtained are as follows:

$$\tilde{F}_1 = 0.099/13055 + 0.886/13563 + 0.240/13867$$

$$\tilde{F}_2 = 0.114/13563 + 0.759/13867 + 0.276/14696$$

$$\tilde{F}_3 = 0.770/14696 + 0.318/15145 + 0.264/15163$$

$$\tilde{F}_4 = 0.584/15145 + 0.655/15163 + 0.758/15311 + 0.266/15433 + 0.159/15460 + 0.012/15497$$

$$\tilde{F}_5 = 0.184/15311 + 0.797/15433 + 0.879/15460 + 0.991/15497 + 0.687/15603$$

$$\tilde{F}_6 = 0.397/15603 + 0.572/15861 + 0.062/15984$$

$$\tilde{F}_7 = 0.259/15861 + 0.963/15984$$

$$\tilde{F}_8 = 0.950/16388$$

$$\tilde{F}_9 = 0.149/16388 + 0.620/16807 + 0.383/16859 + 0.220/16919$$

$$\tilde{F}_{10} = 0.286/16807 + 0.711/16859 + 0.834/16919$$

$$\tilde{F}_{11} = 0.847/18150$$

$$\tilde{F}_{12} = 0.295/18150 + 0.244/18876 + 0.059/18970$$

$$\tilde{F}_{13} = 0.907/18876 + 0.977/18970 + 0.876/19328 + 0.873/19337$$

$$\tilde{F}_{14} = 0.292/19328 + 0.300/19337$$

From the induced fuzzy set $\tilde{F}_i$, we listed out the fuzzy logical relationships as $\tilde{F}_1 \to \tilde{F}_1$, $\tilde{F}_1 \to \tilde{F}_2$, $\tilde{F}_2 \to \tilde{F}_3$, $\tilde{F}_3 \to \tilde{F}_5$, $\tilde{F}_5 \to \tilde{F}_4$, $\tilde{F}_4 \to \tilde{F}_5$, $\tilde{F}_5 \to \tilde{F}_6$, $\tilde{F}_6 \to \tilde{F}_9$, $\tilde{F}_9 \to \tilde{F}_{10}$, $\tilde{F}_{10} \to \tilde{F}_8$, $\tilde{F}_8 \to \tilde{F}_5$, $\tilde{F}_5 \to \tilde{F}_5$, $\tilde{F}_4 \to \tilde{F}_4$, $\tilde{F}_4 \to \tilde{F}_7$, $\tilde{F}_7 \to \tilde{F}_{10}$, $\tilde{F}_{10} \to \tilde{F}_{11}$, $\tilde{F}_{11} \to \tilde{F}_{13}$ and $\tilde{F}_{13} \to \tilde{F}_{13}$. Then, the matrix $R = \tilde{F}_1^T \times \tilde{F}_1 \cup \tilde{F}_1^T \times \tilde{F}_2 \cup \ldots \cup \tilde{F}_{11}^T \times \tilde{F}_{13} \cup \tilde{F}_{13}^T \times \tilde{F}_{13}$ was obtained as follows:

$$R = \begin{pmatrix}
0.099 & 0.099 & 0.099 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.099 & 0.886 & 0.759 & 0.318 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.276 & 0.276 & 0.770 & 0.687 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.655 & 0.655 & 0.655 & 0.655 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.655 & 0.991 & 0.687 & 0.758 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.655 & 0.687 & 0.687 & 0.572 & 0 & 0.149 & 0.397 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.149 & 0.834 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.950 & 0.687 & 0 & 0 & 0 & 0.149 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.834 & 0.620 & 0 & 0.834 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.847 & 0.847 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.977 & 0.876 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.876 & 0.876
\end{pmatrix}.$$

Next, we performed the max-min composition of the fuzzified enrollments for each year with R to obtain the fuzzified forecasted enrollments as $A_i = A_{i-1} \circ R$.

Table 2. The fuzzified forecasted enrollments after max-min composition with R

Year	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}	D_{11}	D_{12}	D_{13}	D_{14}
1972	0.099	0.886	0.759	0.318	0	0	0	0	0	0	0	0	0	0
1973	0.099	0.886	0.759	0.318	0	0	0	0	0	0	0	0	0	0
1974	0.099	0.759	0.759	0.318	0.276	0.276	0	0	0	0	0	0	0	0
1975	0	0	0.276	0.318	0.77	0.687	0.318	0	0	0	0	0	0	0
1976	0	0	0	0	0.655	0.687	0.758	0	0.149	0.397	0	0	0	0
1977	0	0	0	0.655	0.758	0.687	0.758	0	0	0	0	0	0	0
1978	0	0	0	0	0.655	0.687	0.758	0	0.149	0.397	0	0	0	0
1979	0	0	0	0.397	0.397	0.397	0.397	0	0.149	0.572	0	0	0	0
1980	0	0	0	0	0.149	0.149	0	0	0.62	0.62	0	0.62	0	0
1981	0	0	0	0	0	0	0	0	0.834	0.62	0	0.834	0	0
1982	0	0	0	0	0.95	0.687	0	0	0	0.149	0	0	0	0
1983	0	0	0	0	0.655	0.687	0.758	0	0.149	0.397	0	0	0	0
1984	0	0	0	0	0.655	0.687	0.758	0	0.149	0.397	0	0	0	0
1985	0	0	0	0.655	0.758	0.687	0.758	0	0	0	0	0	0	0
1986	0	0	0	0.655	0.758	0.687	0.758	0	0	0	0	0	0	0
1987	0	0	0	0	0	0	0	0	0.149	0.834	0	0	0	0
1988	0	0	0	0	0	0	0	0	0.834	0.62	0	0.834	0	0
1989	0	0	0	0	0	0	0	0	0	0	0	0	0.847	0.847
1990	0	0	0	0	0	0	0	0	0	0	0	0	0.977	0.876
1991	0	0	0	0	0	0	0	0	0	0	0	0	0.977	0.876
1992	0	0	0	0	0	0	0	0	0	0	0	0	0.977	0.876
1993	0	0	0	0	0	0	0	0	0	0	0	0	0.977	0.876

By choosing the fuzzified enrollments with the highest membership for each year from Table 2, the results as shown in Table 3 were produced.

Table 3. The Fuzzified Enrollment Output as in Table 2

Year	Fuzzified Output	Year	Fuzzified Output
1972	$\tilde{F}_2$	1983	$\tilde{F}_7$
1973	$\tilde{F}_2$	1984	$\tilde{F}_7$
1974	$\tilde{F}_2$	1985	$\tilde{F}_5$
1975	$\tilde{F}_5$	1986	$\tilde{F}_5$
1976	$\tilde{F}_7$	1987	$\tilde{F}_{10}$
1977	$\tilde{F}_5$	1988	$\tilde{F}_9$
1978	$\tilde{F}_7$	1989	$\tilde{F}_{13}$
1979	$\tilde{F}_{10}$	1990	$\tilde{F}_{13}$
1980	$\tilde{F}_{10}$	1991	$\tilde{F}_{13}$
1981	$\tilde{F}_9$	1992	$\tilde{F}_{13}$
1982	$\tilde{F}_5$	1993	$\tilde{F}_{13}$

After obtaining the fuzzified enrollment output, we formed another set of fuzzy logical relationships and grouped the FLRs accordingly (see Table 4).

Table 4. The FLRs obtained from fuzzy sets and induced fuzzy sets

FLRs from induced fuzzy sets
$\tilde{F}_2 \rightarrow \tilde{F}_2, \tilde{F}_2 \rightarrow \tilde{F}_5$
$\tilde{F}_5 \rightarrow \tilde{F}_5, \tilde{F}_5 \rightarrow \tilde{F}_7, \tilde{F}_5 \rightarrow \tilde{F}_{10}$
$\tilde{F}_7 \rightarrow \tilde{F}_5, \tilde{F}_7 \rightarrow \tilde{F}_7, \tilde{F}_7 \rightarrow \tilde{F}_{10}$
$\tilde{F}_9 \rightarrow \tilde{F}_5, \tilde{F}_9 \rightarrow \tilde{F}_{13}$
$\tilde{F}_{10} \rightarrow \tilde{F}_9, \tilde{F}_{10} \rightarrow \tilde{F}_{10}$
$\tilde{F}_{13} \rightarrow \tilde{F}_{13}$

By using the arithmetic rules, the forecasted enrollments could be obtained.

5. Results and Discussion

In this section, we present the forecasted enrollments using the proposed model and compare them with the fuzzy time series forecasting model based on fuzzy sets. The forecasted enrollments are given in Table 5.

Table 5. Forecasted enrollments at the University of Alabama

Year	Actual Enrollment	Forecasted Enrollments Based on Fuzzy Sets	Forecasted Enrollments Based on Induced Fuzzy Sets
1972	13563	-	-
1973	13867	14562.5	14562.5
1974	14696	14562.5	14562.5
1975	15460	14562.5	14562.5
1976	15311	16104.25	16027.83
1977	15603	16104.25	16027.83
1978	15861	16104.25	16027.83
1979	16807	16104.25	16027.83
1980	16919	16500	16666.75
1981	16388	16500	16666.75
1982	15422	17062.5	17062.5
1983	15497	16104.25	16027.83
1984	15145	16104.25	16027.83
1985	15163	16104.25	16027.83
1986	15984	16104.25	16027.83
1987	16859	16104.25	16027.83
1988	18150	16500	16666.75
1989	18970	17062.5	17062.5
1990	19328	18750	18750
1991	19337	18750	18750
1992	18876	18750	18750
1993		18750	18750

To evaluate the performance of the forecasting model, we calculated the mean square error (MSE), root mean square error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE).

Table 6. The MSE, RMSE, MAE, and MAPE for the forecasted enrollments

Types of Error	Forecasted Enrollments Based on Fuzzy Sets	Forecasted Enrollment Based on Induced Fuzzy Sets
MSE	769925.3406	723395.4344
RMSE	877.4538966	850.526563
MAE	717.9375	690.4958333
MAPE	0.04354246	0.041827848

In reference to all error values calculated in Table 6, the forecasted enrollments based on fuzzy sets induced by intuitionistic fuzzy sets showed better performance compared to the forecasted enrollments based on fuzzy sets alone. This indicates that the intuitionistic fuzzy sets play a better role in forecasting time series data rather than the classical fuzzy sets.

6. Conclusion

Overall, by comparing the MSE, RMSE, MAE, and MAPE for each model, it can be deduced that the forecasting model based on the induced fuzzy sets obtained from intuitionistic fuzzy sets performs better than the model that is solely based on fuzzy sets.

Since the intuitionistic fuzzy sets consider both the degree of membership and non-membership, the model that uses intuitionistic fuzzy sets as a basis could manage to consider the degree of hesitation in forecasting. This is consistent with the result obtained in [26], which states that the utilization of intuitionistic fuzzy sets is more useful and able to give a good decision. This supports the fact that an intuitionistic fuzzy set is indeed a generalization of a fuzzy set.

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