

Application of Mathematical Function to Estimate the Compaction Characteristics of Unsaturated Soils

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Abstract The study aims to propose a mathematical approach to determine the optimum moisture content (W_{OP}) and the corresponding maximum dry unit weight (γ_{dmax}) of unsaturated fine-grained clay soils with accuracy. Laboratory tests such as grain size distribution, Atterberg limits, specific gravity, Proctor compaction test, and soil suction measurement are conducted to assess soil properties. The W_{OP} and γ_{dmax} are determined using the mathematical approach based on differential function (∂) and the graphical method. The differences in optimum moisture content values between the mathematical approach and the graphical method (ΔW_{OP}) values are 0.43 %, 0.36 %, 0.42 %, 0.24 %, respectively for soils PES, BFS, WIS, BES, and induced differences in total suction of 179.17 kPa, 144.00 kPa, 175.00 kPa, 96.00 kPa, respectively for soils PES, BFS, WIS, BES. Moreover, the differences in matric suction are 148.27 kPa, 116.13 kPa, 144.83 kPa, 80.00 kPa, respectively for soils PES, BFS, WIS, BES. ΔW_{OP} and $\Delta \gamma_{dmax}$ values are smaller than 0.5 % and marginal in the context of saturated soil mechanics. However, the total suction and matric suction values induced by ΔW_{OP} values are significant for unsaturated soils. An accurate estimation of γ_{dmax} and W_{OP} can be performed on unsaturated compacted soils using the mathematical approach.

Keywords Mathematical Approach, Graphical Method, Compaction Parameters, Soil Suction

1. Introduction

In 1933 Proctor developed a compaction test in the lab to assess the maximum dry unit weight (γ_{dmax}) of compacted soils used for field compaction requirements [1]. Compaction is the densification and rearrangement of soil particles by removing air void using mechanical equipment such as a compaction machine. Soil compaction with carefully controlled moisture content improves the shear stress, slope stability of embankment, bearing limit of soil in the construction of buildings, roads, and other engineering structures. Also, compaction diminishes the quantity of unwanted settlement of constructions and the compressibility of soil support. The compaction parameters of soil are obtained from the lab compaction test. A steady value of applied energy to a specific soil specimen at W_{OP} induces a γ_{dmax} . The W_{OP} is required to provide the best path to enter energy into the soil and compact it. The W_{OP} and γ_{dmax} are fundamental indexes in the analysis of compacted soil behaviour. Researchers like [2,3,4,5] have described in detail the various methods for obtaining W_{OP} and γ_{dmax} of clayey soils from proctor compaction. However, in the compaction test method, the W_{OP} and corresponding γ_{dmax} are determined under the prerequisite of a given compaction work and adopted according to standard [6]. In practice, W_{OP} and γ_{dmax} are commonly estimated graphically from the compaction curve. References [7,8,9,10,11] proposed equations to predict the compaction characteristics. Nevertheless, these equations do not consider the effect of soil suction on compaction parameters. Moreover, [12]

proposed a technique to assess the W_{OP} of fine-grained soils utilizing a non-linear ultrasonic. Reference [13] proposed an arithmetic approach to assess the W_{OP} . Nonetheless, the non-linear ultrasonic technique and the arithmetic approach cannot estimate the γ_{dmax} . In general, the precedent research works related to the determination of the W_{OP} and γ_{dmax} are based on saturated soils and do not consider the effect of suction on the estimation of the W_{OP} and γ_{dmax} . The primary objective of this current research work is to propose a mathematical approach to accurately determine the W_{OP} and the corresponding γ_{dmax} of unsaturated fine-grained clay soils.

2. Material and Testing Program

2.1. Material

The type of soil plays an essential role in the compaction parameter values. In this current study, soil samples are collected from the site by digging. Soil samples are selected in a manner to obtain different gradations and consistency limit values so that to encompass a wide range of soil suction and display the effect of suction on the determination of compaction characteristics. Table 1 shows the material properties of soil samples.

2.2. Testing Program

The assessment of the soil's physical and hydro-mechanical properties is performed according to the laid down protocols and standards found in the literature. Sieve analysis [14], hydrometer analysis [15], Atterberg limits [16], compaction test [6], soil suction measurement using the filter paper method [17].

2.2.1. Soil suction measurement

The calibration curve Equation (1) derives from the Whatman No42 filter paper calibration using a salt solution. Equation (2) is utilized to determine the moisture content inside the filter paper (W_f). The substitution of the moisture content value (W_f) in Equation (1) gives the soil suction. Compacted cylindrical soil specimens are split into two parts with a width of 75 mm and a deepness of 35 mm so that the soil specimen can be placed and withdrawn from the glass container easily. See Figure 1. Soil suction is estimated utilizing the Whatman No 42 filter paper (Ashless circles 70 mm diameter Cat No 1442-070). Three filter papers: two protectives and one for suction evaluation are set between the two surfaces utilizing tweezers to estimate the matric suction. The two specimen parts are joined by electrical tape and inserted into the glass container. A plastic ring is put on the specimen, and the filter paper is set on the plastic ring to

gauge the total suction. The glass containers are sealed, named, and put into a temperature regulatory equipment at $25\pm 1^\circ\text{C}$ for an equilibrium period of one month. Filter papers are oven-dried to expel moisture and guarantee that a similar wetting way is followed for each situation to prevent hysteresis effect (Swarbrick, 1995). Moisture cans are oven-dried at 105°C overnight. The filter paper moisture content is estimated utilizing a 0.0001g readable balance. The mass of moisture in the filter paper is (M_w). The mass of filter paper is (M_f). The relation (1) below describes the soil suction (ψ) in kPa.

$$\log(\psi) = -0.0791 \times W_f + 5.313 \quad (1)$$

$$W_f = \frac{M_w}{M_f} \times 100 \quad (2)$$

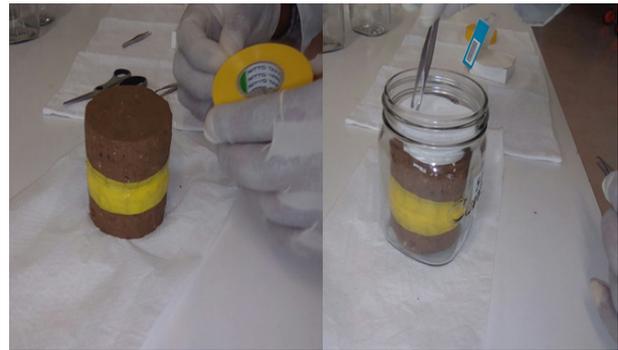


Figure 1. Soil suction measurement test

2.2.2. Proctor compaction test

The test is performed using a mould with separable collar, a compacting base plate with a thick spacer plate as illustrated in Figure 2. Samples are sieved using a 4750-micron sieve. Roughly 35 kg of the specimen is oven-dried at 105°C for 16 to 24 hours and divided into five bowls of similar material. The compaction blows are distributed evenly over the total layer. After tamping the first layer, the depth of the surface of the tamped material is measured without the collar. The tamping of four additional layers is identical. After compaction, a typical specimen is taken from the material to assess the moisture content. A wet soil specimen is weighed with accuracy to the nearest 0.1 gram and dried in the oven at 105°C . The other points of compaction curves are determined using the same process. The dry unit weight (γ_d) is obtained for each moisture content. (γ) is the bulk dry unit weight, (W) is the moisture content, and (γ_d) is described by Equation (3).

$$\gamma_d = \frac{\gamma}{1 + \frac{W}{100}} \quad (3)$$



Figure 2. Proctor compaction test

2.3. Mathematical Approach to Estimate the Maximum Dry Unit Weight and the Optimum Moisture Content

Data obtained from the compaction test are utilized to plot the compaction curve graph using the EXCEL Program. The fitting equations such as linear, logarithmic, inverse, quadratic, cubic, power, compound, growth, exponential are analyzed. The cubic equation exhibits the best-fitting equation. Besides, the compaction curve graph uses two parameters: the dry unit weight (γ_d) and moisture content (W). Therefore, in the mathematical concept, γ_d can be expressed as a function of W as follows: $\gamma_d = f(W)$. Further, the bulk dry unit weight (γ) is the ratio of compacted soil weight and the volume of the mould at each specific W in the compacted specimen. The calculation of the γ is required before obtaining γ_d . Therefore, the mathematical approach function considers both W and γ . Moreover, the third-degree mathematical function $\gamma_d = f(W)$ describes the best fitting compaction curve with a determination coefficient $R^2 \approx 1$. Therefore, the mathematical function takes the form of a third-order Equation (4) with constant real number values (a, b, c, d).

$$\gamma_d = a \times W^3 + b \times W^2 + c \times W + d, \quad R^2 \approx 1 \quad (4)$$

$$a, b, c, d \in \mathbb{R}$$

Equation (4) of the compaction curve is differentiated by the moisture content (W) using a partial differential function (∂) as follows:

$$\frac{\partial(\gamma_d)}{\partial W} = \frac{\partial}{\partial W} (a \times W^3 + b \times W^2 + c \times W + d) \quad (5)$$

$$\frac{\partial(\gamma_d)}{\partial W} = \frac{\partial(a \times W^3)}{\partial W} + \frac{\partial(b \times W^2)}{\partial W} + \frac{\partial(c \times W)}{\partial W} + \frac{\partial(d)}{\partial W} \quad (6)$$

$$\frac{\partial(\gamma_d)}{\partial W} = a \times \frac{\partial(W^3)}{\partial W} + b \times \frac{\partial(W^2)}{\partial W} + c \times \frac{\partial(W)}{\partial W} \quad (7)$$

$$\frac{\partial(\gamma_d)}{\partial W} = 3a \times W^2 + 2b \times W + c \quad (8)$$

The optimum moisture content (W_{OP}) is determined by solving Equation (9) below.

$$\frac{\partial(\gamma_d)}{\partial W} = 0 \quad (9)$$

$$3a \times W_{OP}^2 + 2b \times W_{OP} + c = 0 \quad (10)$$

The W_{OP} is selected as the workable value among the two solutions of Equation (11). Besides, the γ_{dmax} is obtained by substituting the W_{OP} in the compaction curve Equation (4) as follows:

$$\gamma_{dmax} = a \times W_{OP}^3 + b \times W_{OP}^2 + c \times W_{OP} + d \quad (11)$$

3. Results and Discussions

3.1 Material Properties

The material properties of the soil samples are summarized in Table 1. PES, BFS, WIS are fine-grained soil, more than 50 % passing sieve No 200 (0.075mm). Also, the liquid limit values of BLS, WBS, and WKS are greater than 50% above the A-line of the plasticity chart. Thus, these soils exhibit high plasticity and classified (CH). Besides, for BES soil, 49.50% passing sieve No 200 (0.075mm). Nonetheless, BES contained more fine-grained soils than sand and gravel. The liquid limit value is less than 50% above the A-line of the plasticity chart. Then, BES displays low plasticity and classified (CL).

Table 1. Material properties

Soil designation	Liquid limit, (LL) %	Plasticity index (PI) %	Specific gravity (Gs)	Clay %	Fine %	Sand %	Gravel %	USCS
PES	54.83	34.87	2.66	29.85	58.45	27.69	13.86	CH
BFS	61.27	38.25	2.68	32.20	61.82	28.49	9.69	CH
WIS	66.22	44.10	2.76	36.50	69.10	28.20	2.40	CH
BES	48.37	23.09	2.63	20.00	49.50	44.00	6.30	CL

3.2. Hydro-mechanical Properties Analysis

3.2.1. Proctor compaction test

The W_{OP} and γ_{dmax} are determined from compaction curves using the mathematical approach and graphical method for soils PES, BFS, WIS, BES presented in Figures 3, 4, 5 & 6. As a result, WIS exhibits smaller γ_{dmax} values and higher W_{OP} values. Further, BES soil displays higher γ_{dmax} values and smaller W_{OP} values. PES and BFS soil exhibit γ_{dmax} and W_{OP} mean values. The results based on the mathematical approach and the graphical method are shown in Table 2. The results are justified by the fact that when the fine-grained (Clay+Silt) content increases, the γ_{dmax} decreases, and the W_{OP} increases upon the same compaction energy. Likewise, WIS consists of a higher fine-grained fraction of 69.10 % when BES consists of a smaller fine-grained fraction of 49.50 %. BFS and PES consist of a mean fine-grained fraction respective of 61.82 %, 58.54%. Therefore, the γ_{dmax} and W_{OP} are a function of fine-grained soils. Moreover, the differences in W_{OP} value between mathematical approach and graphical method denoted by (ΔW_{OP}) are 0.43 %, 0.36 %, 0.42 %, 0.24 % respectively for soils PES, BFS, WIS, BES, which are less than 0.50 % and marginal in the context of saturated soil mechanics. Besides, the differences in γ_{dmax} values between mathematical approach and graphical method ($\Delta \gamma_{dmax}$) are 0.02 kN.m^{-3} , 0.11 kN.m^{-3} , 0.41 kN.m^{-3} , 0.14 kN.m^{-3} , respectively for soils PES, BFS, WIS, BES are less than 0.50% and marginal. By and large, the compaction parameters (W_{OP} , γ_{dmax}) determined by the mathematical approach and graphical method are nearly similar. Nevertheless, section 3.3 describes the assessment of the impact of the difference in W_{OP} between the mathematical and graphical technique (ΔW_{OP}) on total suction and matric suction in unsaturated soils.

Table 2. Compaction test results

Soil designation	Mathematical approach		Graphical method	
	W_{opt} %	γ_{dmax} kN.m^{-3}	W'_{opt} %	γ'_{dmax} kN.m^{-3}
PES	20.38	18.00	19.95	17.98
BFS	22.61	17.16	22.25	17.27
WIS	24.58	16.71	25.00	16.73
BES	18.24	18.76	18.00	18.90

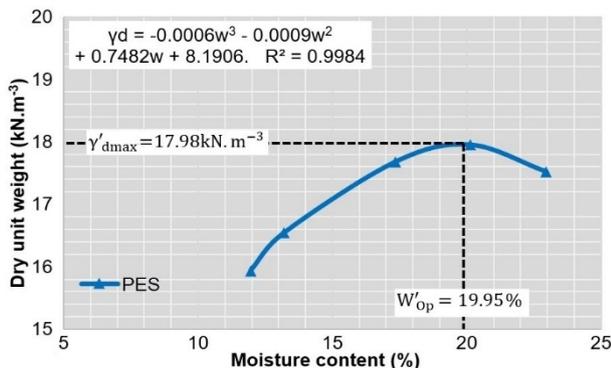


Figure 3. Compaction curve graph, $\gamma_d=f(W)$, (PES)

Compaction characteristic values (γ_{dmax} , W_{OP}) of PES fine-grained soils are obtained using the graphical method.

$$W'_{OP} = 19.95 \% \quad (12)$$

$$\gamma'_{dmax} = 17.98 \text{ kN.m}^{-3} \quad (13)$$

Compaction characteristic values (γ_{dmax} , W_{OP}) of PES fine-grained soils are obtained using the mathematical approach.

$$\gamma_d = -0.0006W^3 - 0.0009W^2 + 0.7482W + 8.1906 \quad (14)$$

$$\frac{\partial(\gamma_d)}{\partial w} = -0.0018W^2 - 0.0018W + 0.7492 \quad (15)$$

$$\frac{\partial(\gamma_d)}{\partial w} = 0 \text{ say } -0.0018W_{OP}^2 - 0.0018W_{OP} + 0.7482 = 0 \quad (16)$$

$$W_{OP} = 20.38 \% \quad (17)$$

$$\gamma_{dmax} = -0.0006W_{OP}^3 - 0.0009W_{OP}^2 + 0.7482W_{OP} + 8.1906 \quad (18)$$

$$\gamma_{dmax} = -0.0006(20.38^3) - 0.0009(20.38^2) + 0.7482(20.38) + 8.1906 \quad (19)$$

$$\gamma_{dmax} = 18.00 \text{ kN.m}^{-3} \quad (20)$$

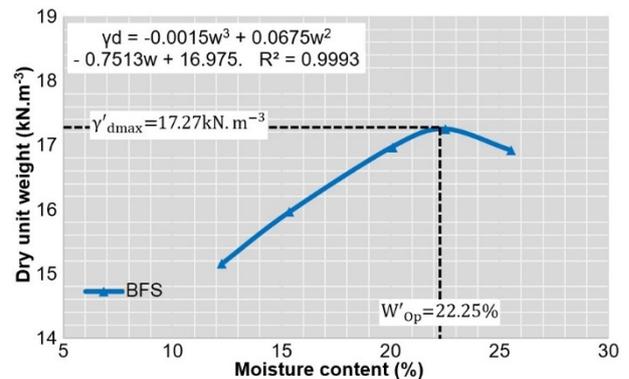


Figure 4. Compaction curve graph, $\gamma_d=f(W)$, (BFS)

Compaction characteristic values (γ_{dmax} , W_{OP}) of BFS fine-grained soils are obtained using the graphical method.

$$W'_{OP} = 22.25 \% \quad (21)$$

$$\gamma'_{dmax} = 17.27 \text{ kN.m}^{-3} \quad (22)$$

Compaction characteristic values (γ_{dmax} , W_{OP}) of BFS fine-grained soils are obtained using the mathematical approach.

$$\gamma_d = -0.0015W^3 + 0.0675W^2 - 0.7513W + 16.975 \quad (23)$$

$$\frac{\partial(\gamma_d)}{\partial w} = -0.0045W^2 + 0.135W - 0.7513 \quad (24)$$

$$\frac{\partial(\gamma_d)}{\partial w} = 0 \text{ say } -0.0045W_{OP}^2 + 0.135W_{OP} - 0.7513 = 0 \quad (25)$$

$$W_{OP} = 22.61 \% \quad (26)$$

$$\gamma_{dmax} = -0.0015W_{OP}^3 + 0.0675W_{OP}^2 - 0.7513W_{OP} + 16.975 \quad (27)$$

$$\gamma_{dmax} = -0.0015(22.61^3) + 0.0675(22.61^2) - 0.7513(22.61) + 16.975 \quad (28)$$

$$\gamma_{dmax} = 17.16 \text{ kN.m}^{-3} \quad (29)$$

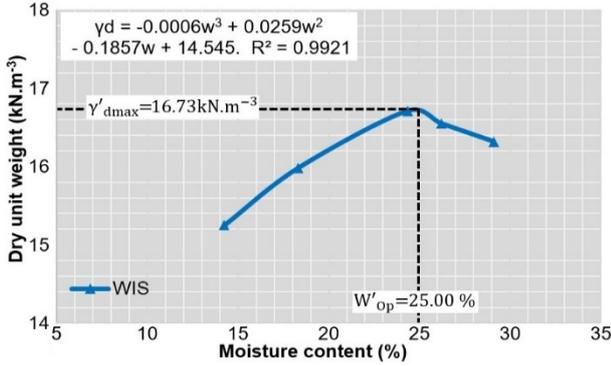


Figure 5. Compaction curve graph, $\gamma_d=f(W)$, (WIS)

Compaction characteristic values (γ_{dmax} , W_{OP}) of WIS fine-grained soils are obtained using the graphical method.

$$W'_{OP} = 25.00 \% \quad (30)$$

$$\gamma'_{dmax} = 16.73 \text{ kN.m}^{-3} \quad (31)$$

Compaction characteristic values (γ_{dmax} , W_{OP}) of WIS fine-grained soils are obtained using the mathematical approach.

$$\gamma_d = -0.0006W^3 + 0.0259W^2 - 0.1857W + 14.545 \quad (32)$$

$$\frac{\partial(\gamma_d)}{\partial w} = -0.0018W^2 + 0.0518W - 0.1857 \quad (33)$$

$$\frac{\partial(\gamma_d)}{\partial w} = 0 \text{ say } -0.0018W_{OP}^2 + 0.0518W_{OP} - 0.1857 = 0 \quad (34)$$

$$W_{OP} = 24.58 \% \quad (35)$$

$$\gamma_{dmax} = -0.0006W_{OP}^3 + 0.0259W_{OP}^2 - 0.1857W_{OP} + 14.545 \quad (36)$$

$$\gamma_{dmax} = -0.0006(24.58^3) + 0.0259(24.58^2) - 0.1857(24.58) + 14.545 \quad (37)$$

$$\gamma_{dmax} = 16.71 \text{ kN.m}^{-3} \quad (38)$$

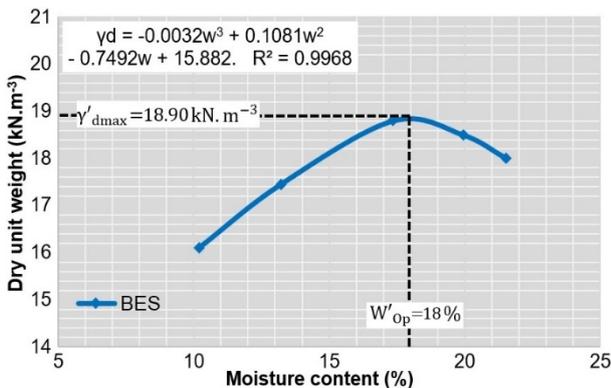


Figure 6. Compaction curve graph, $\gamma_d=f(W)$, (BES)

Compaction characteristic values (γ_{dmax} , W_{OP}) of BES fine-grained soils are obtained using the graphical method.

$$W'_{OP} = 18 \% \quad (39)$$

$$\gamma'_{dmax} = 18.90 \text{ kN.m}^{-3} \quad (40)$$

Compaction characteristic values (γ_{dmax} , W_{OP}) of BES fine-grained soils are obtained using the mathematical approach.

$$\gamma_d = -0.0032W^3 + 0.1081W^2 - 0.7492W + 15.882 \quad (41)$$

$$\frac{\partial(\gamma_d)}{\partial w} = -0.0096W^2 + 0.2162W - 0.7492 \quad (42)$$

$$\frac{\partial(\gamma_d)}{\partial w} = 0 \text{ say } -0.0096W_{OP}^2 + 0.2162W_{OP} - 0.7492 = 0 \quad (43)$$

$$W_{OP} = 18.24 \% \quad (44)$$

$$\gamma_{dmax} = -0.0032(W_{OP}^3) + 0.1081(W_{OP}^2) - 0.7492(W_{OP}) + 15.882 \quad (45)$$

$$\gamma_{dmax} = -0.0032(18.24^3) + 0.1081(18.24^2) - 0.7492(18.24) + 15.882 \quad (46)$$

$$\gamma_{dmax} = 18.76 \text{ kN.m}^{-3} \quad (47)$$

3.2.2. Soil suction and water content relationship

The relationship between soil suction and water content for each soil specimen is studied. Figures 7, 8, 9 & 10 show a variation in total suction, matric suction, and osmotic suction related to water content respectively for soils PES, BFS, WIS, BES. The change in total suction is fundamentally equivalent to a variation in matric suction and vice versa. The curve of total suction is located above the matric suction curve and is very similar in shape. Besides, the shape of the osmotic suction curve is different from the one of matric and total suctions. The matric suction contribution to the total suction is higher than the osmotic suction contribution. Further, WIS exhibits higher suction values, BES displays smaller suction values, and BFS, PES exhibit mean suction values. That is justified by the fact that WIS contained a higher clay fraction of 36.50%, BES contained a smaller clay fraction of 20.00%. BFS and PES contained respective 32.20%, 29.85% clay fraction. Moreover, the results are in line with the research work performed by Fredlund and Xing (1994) on the SWCC of various types of soils. They reported that soil specimen with a higher clay fraction displays a higher soil suction at the same water content than soil specimen with a smaller clay fraction. On the other hand, the soil suction equation of the soil samples is as follows: PES suction trend line equations are $\Psi_t = (23.147 - W)/0.0024$, $\Psi_m = (23.05 - W)/0.0029$ with respective determination coefficients of 89.01% and 90.00%. BFS trend line equations are $\Psi_t = (26.69 - W)/0.0025$, $\Psi_m = (26.31 - W)/0.0031$ with respective determination

coefficients of 93.55 % and 92.03 %. WIS suction trend line equations are $\Psi_t = (28.512 - W)/0.0024$, $\Psi_m = (26.008 - W)/0.0029$ with respective determination coefficients of 97.75 % and 97.24 %. BES suction trend line equations are $\Psi_t = (19.193 - W)/0.0025$, $\Psi_m = (18.953 - W)/0.0030$ with respective determination coefficients of 93.05 % and 90.74%. In general, there is a strong correlation between the soil suctions (total suction, matric suction) and the water content. Therefore, these equations can describe precisely the total and matric suction at the W_{OP} using the mathematical approach and graphical method.

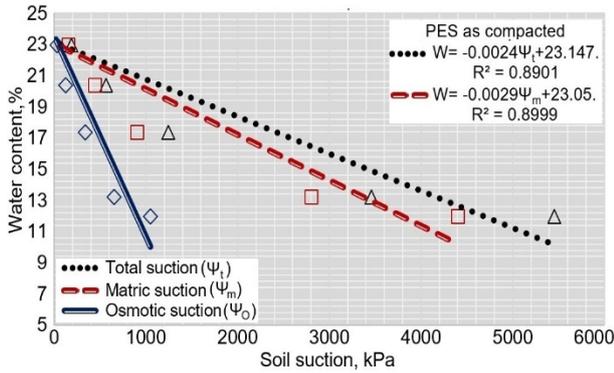


Figure 7. Suction vs water content, $\Psi = f(W)$, (PES)

$$\Psi_t = \frac{23.147 - W}{0.0024} \quad (48)$$

$$\Psi_m = \frac{23.050 - W}{0.0029} \quad (49)$$

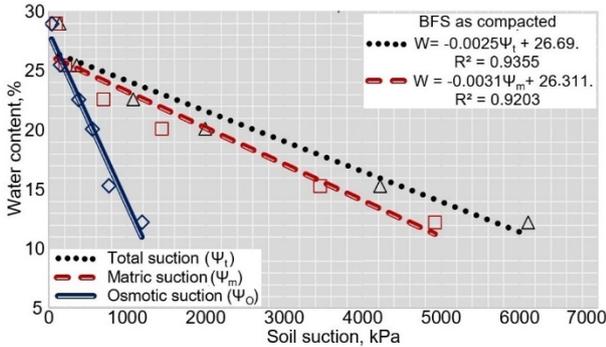


Figure 8. Suction vs water content, $\Psi = f(W)$, (BFS)

$$\Psi_t = \frac{26.69 - W}{0.0025} \quad (50)$$

$$\Psi_m = \frac{26.311 - W}{0.0031} \quad (51)$$

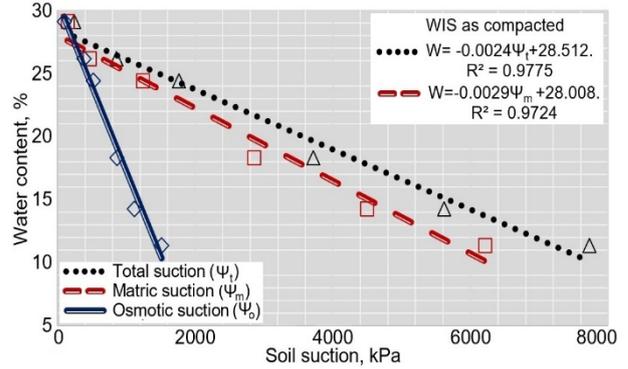


Figure 9. Suction vs water content, $\Psi = f(W)$, (WIS)

$$\Psi_t = \frac{28.512 - W}{0.0024} \quad (52)$$

$$\Psi_m = \frac{23.008 - W}{0.0029} \quad (53)$$

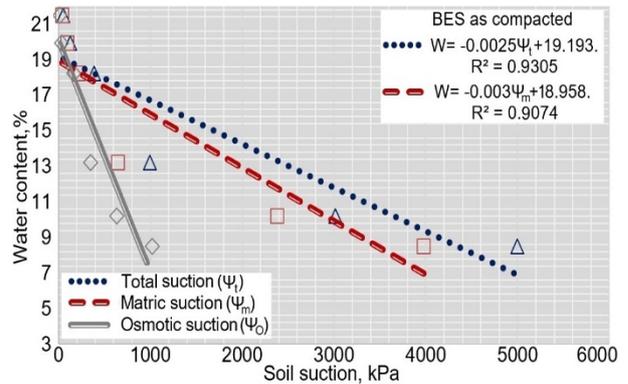


Figure 10. Suction vs water content, $\Psi = f(W)$, (BES)

$$\Psi_t = \frac{19.193 - W}{0.0025} \quad (54)$$

$$\Psi_m = \frac{18.953 - W}{0.0030} \quad (55)$$

3.3. Assessment of the Impact of the Difference between the Mathematical Approach and Graphical Method on Soil Suction at W_{OP}

The assessment of the impact of the difference at WOP between the mathematical approach and the graphical method on the total suction and matric suction is summarized respectively in Tables 3 & 4. The differences in W_{OP} value between graphical method and mathematical approach (ΔW_{OP}) are 0.43 %, 0.36 %, 0.42 %, 0.24 %, respective for soils PES, BFS, WIS, BES. In Table 3, Equations 48, 50, 52, 54 are utilized to calculate the total suction values at W_{OP} denoted by $\Psi_t(W_{OP})$ and $\Psi_t(W'_{OP})$. W_{OP} and W'_{OP} are the optimum moisture content given respectively for the mathematical approach and graphical method. $\Psi_t(W_{OP})$ values are 1152.92 kPa, 1632.00 kPa, 1638.33 kPa, 381.20 kPa, respectively for PES, BFS, WIS, BES. Also, $\Psi_t(W'_{OP})$ values are 1332.08 kPa, 1776.00 kPa, 1463.33 kPa, 477.20 kPa respectively for PES, BFS, WIS, BES. The difference in total suction at the W_{OP} between mathematical approach value $\Psi_t(W_{OP})$ and graphical method value $\Psi_t(W'_{OP})$ is denoted by $\Delta\Psi_t(W_{OP})$ and calculated using Equation 56.

$$\Delta\Psi_t(W_{OP}) = |\Psi_t(W_{OP}) - \Psi_t(W'_{OP})| \quad (56)$$

In Table 4, Equations 49, 51, 53, 55 are used to calculate the matric suction at W_{OP} denoted by $\Psi_m(W_{OP})$ and $\Psi_m(W'_{OP})$.

(W'_{OP}). W_{OP} and W'_{OP} are the optimum moisture content given respectively for the mathematical approach and graphical method. $\Psi_m(W_{OP})$ values are 920.69 kPa, 1193.87 kPa, 1182.07 kPa, 237.67 kPa respectively for PES, BFS, WIS, BES. Moreover, $\Psi_m(W'_{OP})$ values are 1068.97 kPa, 1310.00 kPa, 1037.24 kPa, 317.67 kPa, respectively for PES, BFS, WIS, BES. The difference in matric suction at the W_{OP} between mathematical approach value $\Psi_m(W_{OP})$ and graphical method value $\Psi_m(W'_{OP})$ is denoted by $\Delta\Psi_m(W_{OP})$ and determine from Equation 57.

$$\Delta\Psi_m(W_{OP}) = |\Psi_m(W_{OP}) - \Psi_m(W'_{OP})| \quad (57)$$

The differences in optimum moisture content between the mathematical approach values and graphical method values (ΔW_{OP}) are 0.43 %, 0.36 %, 0.42 %, 0.24 % respectively for soils PES, BFS, WIS, BES and induce a difference in total suction of 179.17 kPa, 144.00 kPa, 175.00 kPa, 96.00 kPa, and a difference in matric suction of 148.27 kPa, 116.13 kPa, 144.83 kPa, 80.00 kPa, respectively for soils PES, BFS, WIS, BES. Although ΔW_{OP} values are marginal, the total suction and matric suction values induced by ΔW_{OP} are significant in the context of unsaturated soil mechanics. Therefore, the mathematical approach is more accurate and efficient than the graphical method in assessing the compaction characteristics in unsaturated soils.

Table 3. Differences in OMC and Total suction

Soil designation	OMC using mathematical approach %	OMC using graphical Method, %	Difference in OMC, %	Total suction Ψ_t in kPa @ OMC using W_{OP}	Total suction in kPa @ OMC using W'_{OP}	Difference total suction in kPa @ OMC
	W_{OP}	W'_{OP}	$\Delta W_{OP} = W_{OP} - W'_{OP} $	$\Psi_t(W_{OP})$	$\Psi_t(W'_{OP})$	$\Delta\Psi_t(W_{OP}) = \Psi_t(W_{OP}) - \Psi_t(W'_{OP}) $
PES	20.38	19.95	0.43	1152.92	1332.08	179.17
BFS	22.61	22.25	0.36	1632.00	1776.00	144.00
WIS	24.58	25.00	0.42	1638.33	1463.33	175.00
BES	18.24	18.00	0.24	381.20	477.20	96.00

Table 4. Differences in OMC and Matric suction

Soil designation	OMC using mathematical approach %	OMC using graphical Method, %	Difference in OMC, %	Matric suction in kPa @ OMC using W_{OP}	Matric suction in kPa @ OMC using W'_{OP}	Difference matric suction in kPa @ OMC
	W_{OP}	W'_{OP}	$\Delta W_{OP} = W_{OP} - W'_{OP} $	$\Psi_m(W_{OP})$	$\Psi_m(W'_{OP})$	$\Delta\Psi_m(W_{OP}) = \Psi_m(W_{OP}) - \Psi_m(W'_{OP}) $
PES	20.38	19.95	0.43	920.69	1068.97	148.27
BFS	22.61	22.25	0.36	1193.87	1310	116.13
WIS	24.58	25.00	0.42	1182.07	1037.24	144.83
BES	18.24	18.00	0.24	237.67	317.67	80.00

4. Concluding Remarks

The objective of the research work is to propose a mathematical approach to assess the compaction characteristics of unsaturated fine-grained clay soils. Soil properties are determined using lab tests such as grain size distribution, Atterberg limits, specific gravity, Proctor compaction test, and soil suction measurement. A mathematical approach and graphical method are utilized to determine the compaction characteristics. The ΔW_{OP} values are 0.43 %, 0.36 %, 0.42 %, 0.24 % respectively for soils PES, BFS, WIS, BES, and induce differences in total suction of 179.17 kPa, 144.00 kPa, 175.00 kPa, 96.00 kPa for respective soils PES, BFS, WIS, BES. Moreover, the differences in matric suction are 148.27 kPa, 116.13 kPa, 144.83 kPa, 80.00 kPa respectively for soil PES, BFS, WIS, BES. The ΔW_{OP} and $\Delta \gamma_{dmax}$ values are smaller than 0.5 % and marginal in saturated soil mechanics. However, the total suction and the matric suction values induced by ΔW_{OP} are significant for unsaturated soils. Finally, the mathematical approach leads to an accurate estimation of the compaction characteristics of unsaturated fine-grained clay soils.

REFERENCES

- [1] Olufemi Ogunsanwo, "Geotechnical Properties of Undisturbed and Compacted Amphibolite Derived Laterite Soil," *Bulletin of the International Association of Engineering Geology*, vol. 42, no. 1, pp. 67-73, 1990. DOI: 10.1007/BF02592621.
- [2] Jack W. Hilf, "A Rapid Method of Construction Control for Embankments of Cohesive Soil," *Engineering Monograph*, no 26, pp. 1-29, 1956.
- [3] Georges W. Ring, John R. Sallberg, Webster H. Collins, "Correlation of Compaction and Classification Test Data," *Highway Research Bulletin*, No. 325, pp. 55-75, 1962.
- [4] BK. Ramaiah, V. Viswanath, HV. Krishnamurthy, "Interrelationship of Compaction and Index Properties," *The Second South East Asian Conference on Soil Engineering*, Singapore, 1970, pp. 577-587.
- [5] Mian C. Wang, Chin-Chuan Huang, "Soil Compaction and Permeability Prediction Models," *Journal of Environmental Engineering*, vol. 110, no. 6, pp. 1063-1083, 1984. DOI: 10.1061/(ASCE)0733-9372(1984)110:6(1063).
- [6] SABS, "Civil Engineering Test Methods, Determination of the Maximum Dry Density and Optimum Moisture Content," SANS 3001, Part GR30, South Africa Bureau of Standards. Pretoria, 2016.
- [7] Ksenija Đoković, Dragoslav Rakić, Milenko Ljubojev, "Estimation of Soil Compaction Parameters Based on the Atterberg Limits," *Mining and Metallurgy Engineering Bor*, no. 4, pp. 1-16, 2013. DOI: 10.5937/mmeb1304001D.
- [8] Kok Shien Ng, Yee Ming Chew, Mohammad Hafeez Osman, SK. Mohamad Ghazali, "Estimating Maximum Dry Density and Optimum Moisture Content of Compacted Soils," *The International Conference on Advances in Civil and Environmental Engineering*, Pulau Pinang, Malaysia, 2015. <https://www.researchgate.net/publication/280734291> (accessed Sept. 05, 2020).
- [9] K. Hannah Jyothirmayi, Taminneri. Gnanananda, Kommo Suresh, "Prediction of Compaction Characteristics of Soil Using Plastic Limit," *International Journal of Research in Engineering and Technology*, vol. 4, no. 1, pp. 253-256, 2015. <https://ijret.org/volumes/2015v04/i06/IJRET20150406042.pdf> (accessed Sept. 06, 2020).
- [10] Binu Sharma, Asuri Sridharan, Pran Hari Talukdar, "Static Method to Determine Compaction Characteristics of Fine-Grained Soils," *Geotechnical Testing Journal*, vol. 39, no. 6, pp.1048-1055, 2016. DOI: 10.1520/GTJ20150221.
- [11] Worku Firomsa Kabeta, Emer Tucay Quezon, "Parametric Modelling on the Relationships Between Atterberg Limits and Compaction Characteristics of Fine-grained Soils," *International Journal of Advanced Research in Engineering and Applied Sciences*, vol. 8, no. 7, pp. 1-20, 2019. <https://www.researchgate.net/publication/334737749> (accessed Sept. 05, 2020).
- [12] Jun Chen, Hao Wang, Yangping Yao, "Determination of Optimum Moisture Content of a Clay Using Nonlinear Ultrasonic Technique," *Japanese Geotechnical Society Special Publication*, vol. 1, no. 2, pp. 1-5, 2015. DOI: 10.3208/jgssp.CPN-02.
- [13] Azhar Sadiq Yasun, Jamal N. Al Abbasi, "A Proposed Approach for Evaluating Soils Optimum Moisture Content Arithmetically and Use Statistical Functions for Checking Method," *International Journal of Engineering & Technology*, vol. 7, no 4, pp. 287-292, 2018. DOI: 10.14419/ijet.v7i4.20.25941.
- [14] ASTM, "Standard Test Method for Particle-Size Distribution (Gradation) of Soils Using Sieve Analysis," D 6913; 04. 09, American Society for Testing and Materials, PA, USA, 2009. DOI: 10.1520/d6913-04r09.
- [15] ASTM, "Standard Test Method for Particle-size Distribution (Gradation) of Fine-Grained Soils Using the Sedimentation (Hydrometer) Analysis," D7928; 04. 09, American Society for Testing and Materials, PA, USA, 2016. DOI: 10.1520/d7928-16e01.
- [16] ASTM, "Standard Test Method for Liquid Limit, Plastic Limit, and Plasticity Index of Soils," D 4318; 04. 08, American Society for Testing and Materials, PA, USA, 2005. DOI: 10.1520/d4318-17e01.
- [17] ASTM, "Standard Test Method for Measurement of Soil potential (Suction) using filter paper," D 5298; 04. 08, American Society for Testing and Materials, PA, USA, 2016. DOI: 10.1520/d5298-16.