Universal Journal of Educational Research 8(12A): 7479-7486, 2020 http://www.hrpub.org
DOI: 10.13189/ujer.2020.082531

Modeling Teachers' Collective Knowledge of Rational Numbers through Dialectic between Questions and Answers: A Case of Denmark and Indonesia

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Received September 22, 2020; Revised November 4, 2020; Accepted November 29, 2020

Cite This Paper in the following Citation Styles

(a): *[1] Zetra Hainul Putra , "Modeling Teachers' Collective Knowledge of Rational Numbers through Dialectic between Questions and Answers: A Case of Denmark and Indonesia," Universal Journal of Educational Research, Vol. 8, No. 12A, pp. 7479 - 7486, 2020. DOI: 10.13189/ujer.2020.082531.*

(b): *Zetra Hainul Putra (2020). Modeling Teachers' Collective Knowledge of Rational Numbers through Dialectic between Questions and Answers: A Case of Denmark and Indonesia. Universal Journal of Educational Research, 8(12A), 7479 - 7486. DOI: 10.13189/ujer.2020.082531.*

Abstract This paper aims at presenting a method for comparing teachers' mathematical and didactical knowledge of rational numbers. The method is developed based on the anthropological theory of the didactic (ATD) using a specific notion of dialectic between questions and answers. The initial questions and the pupils' answers are integrated into hypothetical teacher tasks (HTTs) which have been tested to Indonesian and Danish pre-service teachers (PsTs) who mostly work in pairs. In this particular study, we focus the comparative analysis on a specific case of two pairs, working on the addition and subtraction of fractions, to present the potential of the method to study teachers' collective knowledge. The findings show that the analysis through the dialectic between questions and answers provides a general picture of how the two pairs differ from formulating collective mathematical and didactical knowledge in teaching addition and subtraction of fractions. The analysis shows what directions their discussions take and how the two pairs link between their mathematical and didactical knowledge. The Indonesian pair focused their discussion exclusively on the standard procedures for adding and subtracting fractions. In contrast, the Danish pair discussed both the reasons for the pupils' error and tried to find a visual representation for teaching addition and subtraction of fraction. In addition, the findings also show that neither the Danish nor Indonesian pair considers to change fractions to decimals or to use an instrumental technique such as using a calculator to support pupils' learning of adding and subtracting fractions. An implication from this study is that the method gives a detailed picture of how the mathematical and didactical knowledge is shared and developed between two PsTs during their collective work.

Keywords Anthropological Theory of the Didactic, Dialectic between Question and Answers, Hypothetical Teacher Tasks, Rational Numbers

1. Introduction

Massive studies on pre-service and in-service teachers' mathematical knowledge lead us to some questions; to what extent do we perceive teachers' knowledge? And how could a method, a model, or an approach be used to investigate teachers' knowledge? Some answers have been presented to both questions, and most common studies are to present what mathematical knowledge teachers have and how they teach pupils about that knowledge [1], [2], under a practice-based theory of content knowledge and pedagogical content knowledge for teaching [3]. Teachers' knowledge is mostly defined as an individual competence that is measured through written tests.

Mathematical knowledge for teaching (MKT), developed based on Shulman's seminal work of content and pedagogical knowledge [3], has become the main interest of many previous studies [2], [4], [5]. MKT has been used to study teachers' knowledge, and it provides a general picture of teachers' mathematical and pedagogical knowledge. However, this model does not highlight teachers' knowledge as a collective construction that teachers should build together in a learning and teaching situation [6]. Therefore, it is needed to develop a model that could provide a detailed picture of how teachers build their mathematical and didactical knowledge during their interaction on a particular mathematical problem. We seek this answer from the perspective of the anthropological theory of the didactic (ATD).

The anthropological theory of the didactic (ATD) considers knowledge, including teachers' knowledge, as shared knowledge that is developed in an institution [7]–[9]. That knowledge can be analysed as a praxeology that can be studied from dialectic between questions and answers during teachers' interaction or collaboration [10]. In this paper, we are interested in studying and comparing Danish and Indonesian pre-service teachers' (PsTs) collective development of mathematical and didactical knowledge in interacting with hypothetical teacher tasks (HTTs) of rational numbers (For more detail about HTTs, see [6], [11]–[14]). A motivation to do this comparative study is to search for possible causes of a gap between Danish and Indonesian pupils' mathematics achievement in PISA and TIMSS [15], [16]. The Danish pupils scored above average, whereas the Indonesian pupils scored far below the average. Besides, we could question current teaching practices and identify best practices in teaching [17]. Thus, the research questions of this study are: "How can PsTs' collective mathematical and didactical knowledge of rational numbers be analysed in terms of dialectic between questions and answers? What this method can contribute to a comparative study of PsTs' mathematical and didactical praxeologies?"

In the following section, we first present the theoretical framework of modeling teachers' collective knowledge through dialectic between questions and answers. Then, the methodology section discusses how this study carried out with two pairs of PsTs from Denmark and Indonesia. After that, the result section presents the praxeological analysis of PsTs' knowledge from the dialectic between questions and answers. In this point, we also describe a path from PsTs' collective work. Finally, the section about discussion and concluding remarks presents the answers to the research questions, some implications and limitations of this study.

2. Modeling Teachers' Collective Knowledge: Dialectic between Questions and Answers

The anthropological theory of the didactic (ATD) investigates the phenomena of school mathematics related to how knowledge is introduced and reconstructed at schools or institutions [7], [18]. A body of knowledge mostly produced by scholars is needed to be transposed to taught and learnt knowledge in a given educational institution. The didactic transposition process of knowledge involves relations among knowledge, learners, and also institutions [19]. In general, this process can be modeled as a relation R between learners (X) and an object of knowledge (O) that occurs in institutions I, and it can be denoted $R_I(X, O)$. In social interaction, learners X get some help from a person Y, mostly a teacher y, to study the object of knowledge (O), and this process is known as a didactical situation that can be modeled into a didactical system of the type $S(X, Y, O)$ [10], [20], [21].

The object of knowledge to be learnt is made of praxeological components; types of tasks, techniques, a technology and a theory [7], [20],[21]–[24]. A mathematical type is needed to be solved through some techniques. In a case study of teachers' knowledge, there is not a single technique for teaching a mathematical praxeology. In many cases, pre-service or in-service teachers provide various mathematical and didactical praxeologies during their collective work. The process of constructing knowledge in this study can be influenced by the interaction between pre-service teachers (PsTs) and HTTs during their collective work [12], [13].

PsTs engage in an initial question Q_0 situated in HTTs, and they bring this question into a pair discussion as dialectic between questions and answers. An existing answer to a mathematical question Q_0 presented in HTTs as a pupil's answer can play as a pre-established answer $A\lozenge$. Other questions Q_k, involving mathematical and didactical questions, are derived from the initial question Q_0, and interaction between two PsTs leads to some possible answers A_k. In the end of this process is expected to reach a final answer $A\heartsuit$, involving mathematical and didactical praxeologies. A sequence of linked questions and answers in a broader environment, for instance classroom activity, is called a study and research path (SRP) [10].

3. Methodology

The study started by designing five HTTs about rational numbers [6], [14]. All five HTTs have been solved by 32 (16 pairs) Indonesian PsTs from one Elementary School Teacher Training study program, Indonesia (to teach pupils from grade 1 to grade 6) and 31 (14 pairs and one group of 3 PsTs) Danish PsTs from four different Teacher Training Colleges (to teach pupils from grade 1 to 6 or grade 4 to 9). All Indonesian PsTs have already completed all courses in (didactic of) mathematics, e.g., mathematics education for the upper grades of elementary school. Most of the Danish PsTs were taking a course on didactics of mathematics, e.g. learning mathematics, numbers, and arithmetic/algebra.

This particular paper focuses on the two cases (one from each country) of PsTs working on an HTT about adding and subtracting of fractions (figure 1). Through this specific case, we would like to show some possible differences in how they discuss and propose mathematical and didactical praxeologies. We use dialectic between questions and answers as a tool to present the result from their discussion.

Three initial questions with one given pupils' answer are presented in the HTT (Figure 1). From those questions, one can hypothesise some paths of questions as well as answers that might occur during the discussion (Figure 2).

In figure 2, we try to illustrate some possible paths starting from the initial question Q_0 and pupils' answer A_0 stated in the HTT. The first path is that a pair focuses their discussion on a mathematical technique to solve the tasks. They first confirm that the pupils' answer is incorrect and then show a correct mathematical technique, an algorithm of fractions or an algorithm of decimals after conversion. In the end, they instruct the pupils to apply it to a similar task. The second path is based on technological discourse (TD) where PsTs focus on the meaning of fractions such as a part-whole relationship that could lead them to propose a didactical technique based on a diagram representation, and in the end, they might link it to explain how such an algorithm can work. For PsTs who discuss a measurement model, it will support pupils to have more flexible and coherent praxeologies not only for addition and subtraction but also for multiplication and division of fractions. There is also a possible path through first proving that the pupils' answers are incorrect, and then directly explain a correct mathematical technique or provide technological reasoning for developing a better didactical technique (including the meaning of fractions) for teaching addition and subtraction of fractions (indicated by dashed lines).

You ask sixth-grade pupils to solve $\frac{2}{3}+\frac{1}{2}=\cdots$, and $\frac{4}{7}-\frac{1}{3}=\cdots$

a. How do you solve these problems? (*to be solved individually within 3 minutes*)

You find that many pupils add and subtract fractions in the following way : $\frac{2}{3}+\frac{1}{2}=\frac{3}{5}$, and $\frac{4}{7}-\frac{1}{3}=\frac{3}{4}$.

b. How do you interpret the pupils' methods? (*to be solved individually within 3 minutes*)

c. What strategies can you propose to teach these pupils? (*to be discussed and solved in pair, 5 minutes*)

Figure 1. HTT about addition and subtraction of fractions

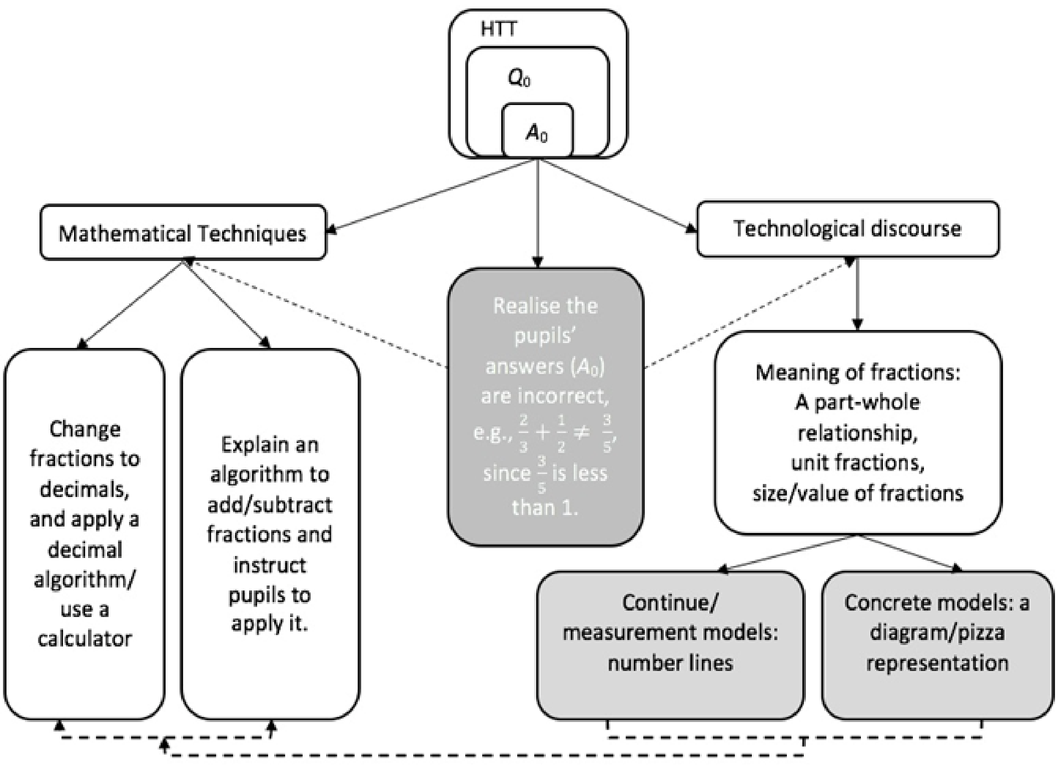

Figure 2. General structure of paths derived from the HTT about addition and subtraction of fractions

4. Results

4.1. The Case of Danish PsTs

A pair of third-year Danish PsTs from group 6 is selected with two considerations. First, it is to have the data to be more comparable to the Indonesian counterparts because they are prepared to teach pupils from grade 1 to 6, and secondly, they have already completed the course of learning mathematics, numbers, and arithmetic.

The two Danish PsTs did not get any difficulty to solve the addition and subtraction tasks individually. They applied an algorithm of addition and subtraction of fractions: change each fraction into a common denominator and add numerators. To the second task about their interpretation of pupils' answers, both of them considered that the pupils add and subtract two fractions based on their position, adding a numerator to a numerator and a denominator to a denominator. Then the pupils need to find a common denominator to solve both tasks. In addition, one of them, DS_1 also conjectured that pupils perhaps apply a mathematical technique for multiplication of two fractions. After that, they elaborated their answers during the discussion of question c, and the first discussion focused on the interpretation of pupils' answers as follows:

DS_1: Ok. Should we first talk about what is wrong, or what they have answered?

DS_2: Yes. There are two things that are wrong, according to what I can see. First, there is an issue of common denominator, and then in both addition and subtraction there is the problem that they operate both in numerators and denominators. Here, they subtract numerators and denominators from each other, and here they add numerators and denominators. If we had a common denominator, then it would suddenly become twelfths, we would be working with…You have probably made this one into sixths. (points the addition task)

…

DS_1: Yes. I have a theory on that. I totally agree with you, but I also have a theory that they perhaps have taken a strategy from multiplication of fraction, where you multiply a numerator with a numerator and a denominator with a denominator, and then [they] try to use it here. Perhaps they have learned that one, and they try to use it here.

The first path that both Danish PsTs created from their discussion is to interpret possible TD for the pupils' answers. They gave two main interpretations behind the pupils' mistakes: on the one hand, pupils must think a fraction consists of two independent natural numbers (TD1), and, on the other hand, pupils seem to interchange the technique for multiplication and for addition/subtraction of fractions (TD2). In addition, there is dialectic when DS_2 said "they would get seven twelfths". It means that even though the pupils know how to find a common denominator, they still need to realise that both denominators must not be added. The pupils need to grasp a technological discourse underlying the algorithm of fractions. The Danish PsTs did not discuss that further, and DS_2 only said "*...they are not to be added".*

Both Danish PsTs continued their discussion on how they teach the pupils.

DS_1: Which strategies would we suggest to teach those pupils? Again I think, it might be good to represent fractions in a different way, perhaps. Perhaps with a pizza. Because the fractions only make sense when you have a common denominator. And that can be expressed by, if you have a pizza, all the pieces are equally big. This wouldn't work (points uneven sized sectors of a circle that he drew). Then we would have... well, which denominator should we put on that one? Because they are mixing pieces of different seizes.

DS_2: And then again, the visual. You know, make some… make a little model (Figure 3), again, as we talked about before. Which is simply called?

…

DS_2 drew the model as follows:

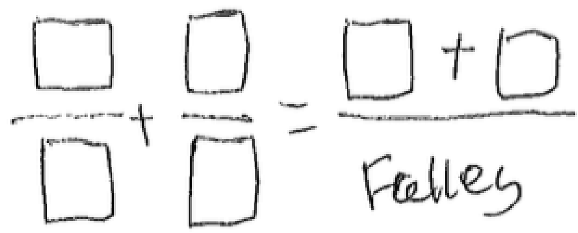

Figure 3. A visual representation for addition of fractions (*fælles* means a common denominator)

DS_2: Yes. And then do the same for subtraction, multiplication and… do the same all the way down, so you would have something visual, oh, now we have to add them, now we have to do this.

DS_1: Yes.

DS_2: Given that you cannot use a collection of formulas, [and] it is very much learning by rote if one has to, well, could one have something in the process so you don't have to refer back all the time to see [inaudible].

From the discussion, it is evident that both Danish PsTs constructed the second path focusing on didactical techniques through concretising the mathematical tasks. No fundamental questions appear during the discussion that can lead them to discuss more on mathematical and didactical technologies or theories. For instance, DS_1 suggested to represent a fraction into a pizza diagram, but he did not provide a further explanation of how to do it and what technological discourse could support it. His partner,

DS_2, also did not provide questions in that respect, but he proposed an idea to concretise the algorithm (Figure 3). We do not see that the Danish PsTs try to explain how representations can help pupils perceive an algorithm for addition and subtractions of fractions, although in the end, DS_2 realised that rote learning, such as memorising the algorithm, might be needed. The interaction of the two Danish PsTs can be summarised in the following path (Figure 4).

4.2. The Case of Indonesian PsTs

A pair of Indonesian PsTs from group 13 is selected to be analysed. The reason to choose this pair is that they represent a common picture of Indonesian PsTs' collective knowledge.

The two Indonesian PsTs also did not get any difficulty to solve the addition and subtraction task individually and applied an algorithm of addition and subtraction of fractions. To the second task about their interpretation of pupils' methods (Figure 1), only IS_1 considered that the pupils added and subtracted two fractions based on their position, and then both of them wrote that the pupils first needed to find a common denominator to add and subtract fractions. In addition, IS_2 also added a statement that the pupils did not need to have a common denominator for multiplication and division of fractions.

Through a similar analysis to the Danish case, we summarise the collective work of the Indonesian pair into three different paths (figure 5). The first one leads to TD for interpreting pupils' answers. Both of them agreed that the pupils' improper technique is due to a lack of "understanding" the concept of fractions. It means that the pupils confuse a mathematical technique for addition and subtraction with that for multiplication and division of fractions. For instance, IS_2 said "*the pupils solved the task mostly using the method for multiplication and division of fractions*" (TD2).

Secondly, both PsTs agreed on applying the standard algorithm for addition and subtraction of fractions. The pupils need to find the least common multiple (LCM) of the denominators in order to master the algorithm. The discussion was dominated by IS_2 who started explaining what fractions mean. He defined as a fraction consisting of a numerator and a denominator, and he expected pupils to grasp this meaning without giving a further technological discourse underlying it. A dialectic between questions and answers was mostly around the algorithm of addition and subtraction of fractions, e.g., how can pupils find LCM of the denominators? And have they learned about LCM before? The answer given by IS_2 "*before pupils learn fractions, they have already learned how to find LCMs. One subject relates to the other"* indicates that there is a link among several punctual or local praxeologies, and pupils' preliminary praxeology is a kind of support for developing a new praxeology. Nevertheless, instructing pupils to work with the algorithm seems to be the only didactical technique that they propose to help the pupils to solve the addition and subtraction of fractions.

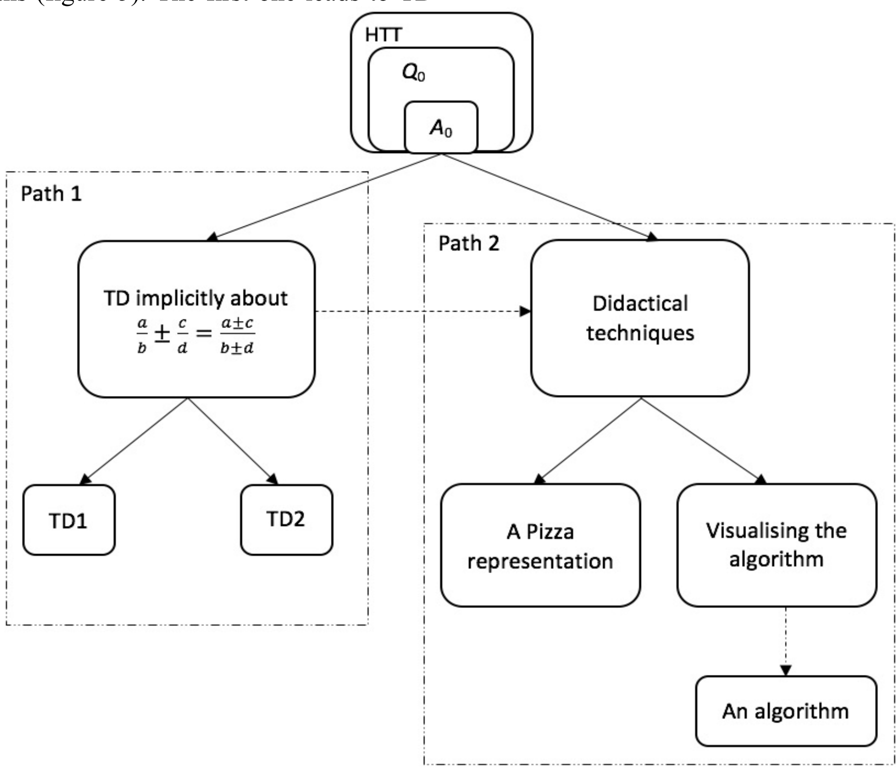

Figure 4. A path derived from the Danish pairs' collective work

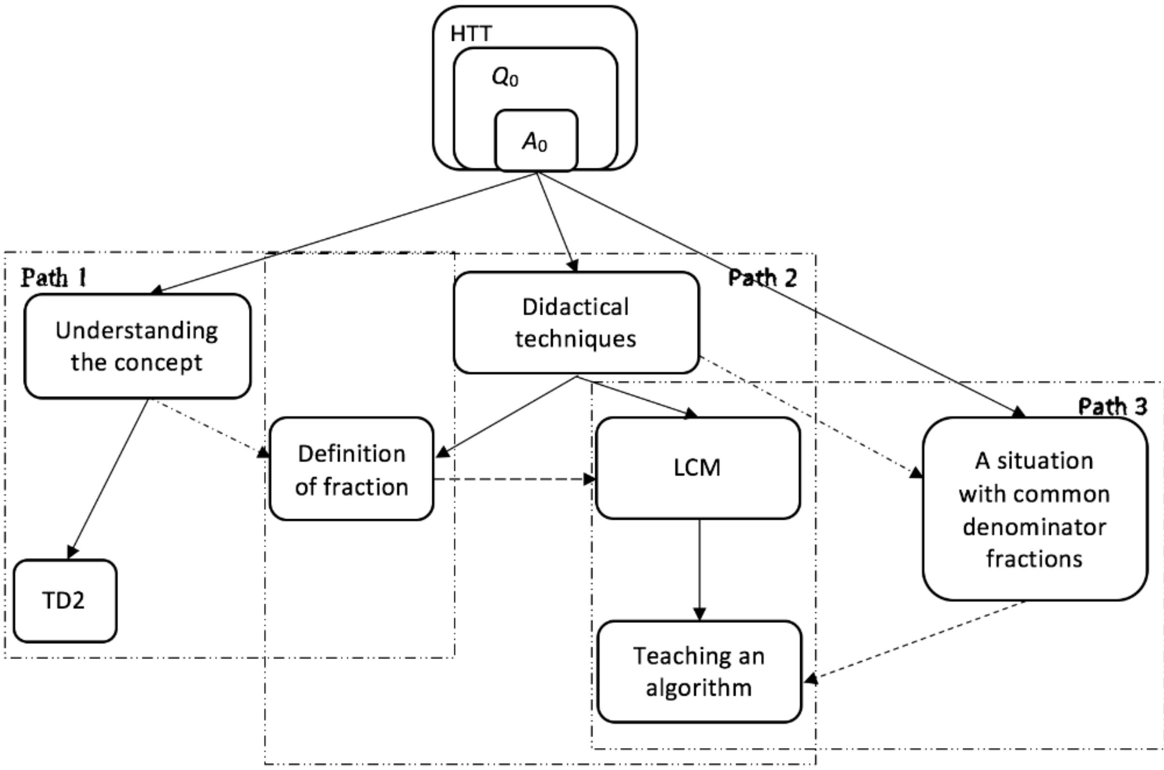

Figure 5. A path derived from the Indonesian pair's collective work

The third path is to provide the pupils with a situation of adding two fractions with a common denominator. They gave an example of mathematical tasks, 2/3 + 1/3, but they continued to focus on the discussion on the algorithm of adding fractions, just adding the numerators. None of them tried to elaborate on this situation in order to prove that the algorithm applied by pupils is incorrect. For instance, when IS_1 assumed that the pupils might give an answer 3/6, a teacher could provoke pupils to prove that it is not possible because 3/6 is equalled to a half, and it is less than 2/3; but this is not proposed.

5. Discussion and Concluding Remarks

First, we discuss some common principles found in the results that show similarities of PsTs' mathematical and didactical knowledge of rational numbers, specifically on addition and subtraction of fractions. Both pairs successfully solved the mathematical task presented in question a. They applied the algorithm for addition and subtraction of fractions. So, we cannot expect them to give more diverse answers since the question directly instructs them to solve the task.

The answers both pairs given to question b. are also similar. They provide an interpretation of the pupils' wrong answers: the pupils add and subtract both numbers based on their positions; they consider that a fraction consists of two independent natural numbers. The word "interpret", stated in the task, gives birth to the discussion that leads them to explain further their position. They suggest that pupils need to know how to find a common denominator, and the Indonesian pair focuses on presenting the algorithm for how to add and subtract two fractions, and this situation commonly appeared in some previous studies on Indonesian PsTs' collaborative work on fractions [6], [12].

The last similarity that we observed during their discussions is that both pairs first discussed TD2 underlying the pupils' methods and then related to finding a common denominator. The Indonesian pair considered teaching an algorithm as a didactical technique for addition and subtraction of fractions. By contrast, the Danish PsTs did not apply it as a didactical technique. It becomes the main difference of didactical knowledge presented by both groups, and it is also what has been found in Putra's previous study [12].

The Indonesian pair tried to present an alternative praxeology to teach pupils through giving them a task of adding two fractions with a common denominator, but they still focused the discussion on how the algorithm works to that task (Path 2 in figure 5). In contrast, the Danish PsTs completely switched their ideas from applying the algorithm to the mathematical task into constructing visual representations (Showed by a dashed line from path 1 to path 2 in figure 4). However, the Danish pair did not show in detail how to construct and visualise the mathematical tasks or unclear use of a contextual situation. We may doubt that they have sufficient mathematical and didactical knowledge to concretise the mathematical task.

In addition, the data show that neither of the Danish nor Indonesian pair considers to change fractions to decimals or to use an instrumental technique such as using a calculator (Figure 2). There might be another potential didactical technique to help pupils realise that two different representations, fractions and decimals, have the same value. At the same time, we also observed considerable challenges for PsTs when they try to change fractions to decimals, e.g., 2/3=0.66…, and a posteriori when it comes to developing ideas for teaching pupils about "repeating decimals".

The analysis through the dialectic between questions and answers presented in diagrams (Figure 4 and 5) provides a general picture of how the two pairs differ in formulating collective mathematical and didactical praxeologies in teaching addition and subtraction of fractions. The paths show what directions their discussion takes and how they link their mathematical and didactical praxeologies [25]. For instance, the Indonesian pair focused their discussion exclusively on the algorithm (a mathematical technique) for addition and subtraction of fractions, and only implicitly on didactic techniques for teaching these unique mathematical techniques. By contrast, the Danish pair discussed both the reasons (a mathematical and didactic technology) for the pupils' error and tried to find a visual representation for teaching (a didactic technique).

In conclusion, the method gives a picture of how the mathematical and didactical knowledge is shared and developed between two PsTs. This becomes a specific feature of the study that contrasts with other studies of mathematical knowledge for teaching that mostly focuses on teachers' individual knowledge (e.g., [1]). In addition, we would like to address some limitations from this study. First limitation is related to the subject of this study in which we only analyse the data from two pairs, so the results of this study could not represent a broader picture of the differences between the two countries, Denmark and Indonesia. Secondly, the study only focuses on an HTT, so the result could not represent the picture of PsTs' knowledge of rational numbers. Further study has to address these limitations by expanding the number of participants and by providing different types of mathematical and didactical tasks that could contribute to rich mathematical and didactical praxeology. In addition, the integration of technology could be the issue needed to be address for a further study because the learning and teaching are faced on the use of technology in the classroom [4], [26].

Acknowledgements

The author thanks the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia for funding this research under the grant no.102.7/E4.4/2015. The preliminary results of this study have been presented at 6th International Congress of Anthropological Theory of Didactics (CITAD 6) at Grenoble, France, 22 – 26 January 2018.

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