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Solution of Newell – Whitehead – Segal Equation of Fractional Order by Using Sumudu Decomposition Method

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Abstract Newell Whitehead Segal (NWS) equation has been used in describing many natural phenomena arising in fluid mechanics and hence acquired more attention. Studies in the past gave importance to obtaining numerical or analytical solutions of this kind of equations by employing methods like Modified Homotopy Analysis Transform method (MHATM), Adomian Decomposition method (ADM), Homotopy Analysis Sumudu Transform method (HASTM), Fractional Complex Transform (FCT) coupled with He's polynomials method (FCT-HPM) and Fractional Residual Power Series method (FRPSM). This research aims to demonstrate an efficient analytical method called the Sumudu Decomposition Method (SDM) for the study of analytical and numerical solutions of the NWS of fractional order. The coupling of Adomian Decomposition method with Sumudu transform method simplifies the calculation. From the numerical results obtained, it is evident that SDM is easy to execute and offers accurate results for the NWS equation than with other methods such as FCT-HPM and FRPSM. Therefore, it is easy to apply the coupling of Adomian Decomposition technique with Sumudu transform method, and when applied to nonlinear differential equations of fractional order, it yields accurate results.

Keywords Newell – Whitehead – Segal Equation, Sumudu Decomposition Method, Caputo Fractional Derivative

1. Introduction

Fractional calculus has played an important role in describing many dynamical phenomena in applied science and engineering fields. Dynamical phenomena are noticed in different type of scientific fields such as physics, chemistry, continuum mechanics [1], chaos theory [2], biotechnology [3], electrodynamics [4], and many other fields [5-7]. This feature of fractional calculus has appealed to many researchers in the past [8-12].

An NWS equation has been used in describing many natural phenomena arising in fluid mechanics and hence acquired more attention. This NWS equation describes the appearance of the stripe pattern in two-dimensional systems. Moreover, it has a lot of applications in fluid dynamics such as traveling wave patterns in binary fluids. Studies in the past gave importance to obtaining numerical or analytical solutions of this kind of equations by employing methods like Modified Homotopy Analysis Transform method (HASTM) [15], Adomian Decomposition method (ADM) [16], Homotopy Analysis Sumudu Transform method (HASTM) [17], Fractional Complex Transform (FCT) coupled with He's polynomials method (FCT-HPM) [18] and Fractional Residual Power Series method (FRPSM) [19].

As shown previously, in this study, we applied the Sumudu decomposition method (SDM) to find an analytical and numerical solution to NWS equations of fractional order. SDM, simple and directly without any restrictive assumption as usual, is going in other methods. In many research papers SDM has also been applied to solve intricate problems in engineering, mathematics and applied science [20-26].

In the present research, we consider the fractional model of Newell–Whitehead–Segal (NWS) equation in the operator form:

$$D_\xi^\upsilon \phi(x,\xi) = b\phi_{xx}(x,\xi) + k\phi(x,\xi) - h\phi^d(x,\xi), \xi \geq 0, 0 < \upsilon \leq 1, \tag{1.1}$$

with initial condition:

$$\phi(x,0) = g(x); \tag{1.2}$$

where $D_\xi^\upsilon \phi(x,\xi) = \dfrac{\partial^\upsilon \phi(x,\xi)}{\partial \xi^\upsilon}$, b, k *and* $h \in \Re$ such that $b, h > 0$, and $d \in \square^{+}$. The term $\phi_\xi^\upsilon(x,\xi)$ represents the variation of $\phi(x,\xi)$ with respect to temporal variable ξ at a set position, $\phi_{xx}(x,\xi) = \dfrac{\partial^2 \phi}{\partial x^2}$ denotes the variation of $\phi(x,\xi)$ with spatial variable x at a specific time and the remaining term $k\phi - h\phi^d$ signifies the effect of the source term.

2. Preliminaries

Definition 1: Let's Riemann-Liouville fractional integral operator [9] of order $\upsilon \geq 0$ for $\phi(\xi) \in \square_\mu, \mu \geq -1$, is:

$$J^\upsilon \phi(\xi) = \begin{cases} \dfrac{1}{\Gamma(\upsilon)} \displaystyle\int_0^\xi (\xi - \tau)^{\upsilon-1} \phi(\tau) d\tau, \upsilon > 0, \xi > 0, \\ \phi(\xi), \quad \upsilon = 0. \end{cases} \tag{2.1}$$

Definition 2: The Caputo fractional derivative operator of order [8] $\upsilon > 0$ for $\phi(\xi) \in \square_{-1}^k, k \in \mathbb{N}$, is defined as:

$$D_\xi^\upsilon \phi(\xi) = \frac{\partial^\upsilon \phi(\xi)}{\partial \xi^\upsilon} = \begin{cases} J^{\upsilon-k}\phi^k(\xi), k-1 < \upsilon \leq k, k \in \mathbb{N}, \\ \dfrac{\partial^k \phi(\xi)}{\partial \xi^k}, \upsilon = k, k \in \mathbb{N}. \end{cases} \tag{2.2}$$

Definition 3: The Sumudu transform [13] of the continuous function $\phi(\xi)$ is defined by:

$$S[\phi(\xi)] = \frac{1}{\rho} \int_0^\infty \phi(\xi) e^{-\frac{\xi}{\rho}} d\xi = \phi(\rho). \tag{2.3}$$

Definition 4: The Sumudu transform of $\dfrac{\partial^\upsilon \phi(x,\xi)}{\partial \xi^\upsilon}$ w.r.t ξ can be calculated as [14]:

$$S\left[\frac{\partial^\upsilon \phi(x,\xi)}{\partial \xi^\upsilon}\right] = \rho^{-\upsilon} S[\phi(x,\xi)] - \sum_{k=0}^{r-1} \phi^k(x,0)\rho^{-\upsilon+k}. \tag{2.4}$$

3. Sumudu Decomposition Method (SDM)

In this section, we will briefly discuss SDM, to solve fractional-order nonlinear (1.1). By applying the Sumudu transform (ST) to (1.1), we get

$$\rho^{-\upsilon} S[\phi(x,\xi)] - \rho^{-\upsilon}\phi(x,0) = S[b\phi_{xx}(x,\xi) + k\phi(x,\xi) - h\phi^d(x,\xi)], \tag{3.1}$$

substituting the (1.2) into (3.1), we obtain

$$S[\phi(x,\xi)] = g(x) + \rho^\upsilon S[b\phi_{xx}(x,\xi) + k\phi(x,\xi) - h\phi^d(x,\xi)]. \tag{3.2}$$

On employing the inverse ST for (3.2), we get

$$\phi(x,\xi) = S^{-1}[g(x)] + S^{-1}\left[\rho^\upsilon S[b\phi_{xx}(x,\xi) + k\phi(x,\xi) - h\phi^d(x,\xi)]\right]. \tag{3.3}$$

The SDM depends of the solution $\phi(x,\xi)$ in a series form as follows:

$$\phi(x,\xi) = \sum_{m=0}^\infty \phi_m(x,\xi), \tag{3.4}$$

the nonlinear term $\phi^d(x,\xi)$ is decomposed as follows:

$$\phi^d(x,\xi) = \sum_{m=0}^\infty Q_m, \tag{3.5}$$

for some Adomian polynomials Q_m, it is defined as

$$Q_m = \frac{1}{m!} \frac{d^m}{d\beta^m} \left[\left(\sum_{m=0}^\infty \beta^m \phi^d(x,\xi)\right)\right]_{\beta=0} \quad m = 0,1,2,.... \tag{3.6}$$

Substitution (3.4) and (3.5) to (3.3), we have

$$\sum_{m=0}^\infty \phi_m(x,\xi) = S^{-1}[g(x)] + S^{-1}\left[\rho^\upsilon S[b(\textstyle\sum_{m=0}^\infty \phi_m(x,\xi))_{xx} + k(\sum_{m=0}^\infty \phi_m(x,\xi)) - h(\sum_{m=0}^\infty Q_m)]\right], \tag{3.7}$$

Using (3.7), we define the following iterative formula [20]:

$$\phi_0(x,\xi) = S^{-1}[g(x)],$$
$$\phi_k(x,\xi) = S^{-1}\left[\rho^\upsilon S[b(\phi_{k-1}(x,\xi))_{xx} + k(\phi_{k-1}(x,\xi)) - h(Q_{k-1})]\right], k \geq 1. \tag{3.8}$$

Having determined these components, substitute it into $\phi(x,\xi) = \sum_{m=0}^\infty \phi_m(x,\xi)$, to obtain the solution in a series form.

4. Elucidative Examples

In this section, we demonstrate the applicability of the previous method by giving examples.

Example1. By substituting $b=1, k=-2, h=0$ in (1.1), to yield

$$\phi_{\xi}^{\upsilon}(x,\xi)=\phi_{xx}(x,\xi)-2\phi(x,\xi), \xi\geq 0, 0<\upsilon\leq 1, \quad (4.1)$$

with initial condition:

$$\phi(x,0)=e^{x}, \quad (4.2)$$

a time-fractional linear NWS equation.

By using SDM, (3.7) becomes

$$\sum_{m=0}^{\infty}\phi_m(x,\xi)= S^{-1}[e^x]+ S^{-1}\left[\rho^{\upsilon}S\left[\left(\sum_{m=0}^{\infty}\phi_m(x,\xi)\right)_{xx}-2\left(\sum_{m=0}^{\infty}\phi_m(x,\xi)\right)\right]\right]. \quad (4.3)$$

The iterative relation can be constructed from (4.3) given by:

$$\phi_0(x,\xi)=S^{-1}[e^x],$$
$$\phi_k(x,\xi)= S^{-1}\left[\rho^{\upsilon}S\left[\left(\phi_{k-1}(x,\xi)\right)_{xx}-2\left(\phi_{k-1}(x,\xi)\right)\right]\right], k\geq 1. \quad (4.4)$$

Therefore, using SDM, the first few iterative solutions are

$$\phi_0(x,\xi)=e^{x},$$
$$\phi_1(x,\xi)=-\frac{e^{x}\xi^{\upsilon}}{\Gamma[1+\upsilon]},$$
$$\phi_2(x,\xi)=\frac{e^{x}\xi^{2\upsilon}}{\Gamma[1+2\upsilon]}, \quad (4.5)$$
$$\phi_3(x,\xi)=-\frac{e^{x}\xi^{3\upsilon}}{\Gamma[1+3\upsilon]},$$
$$\phi_4(x,\xi)=\frac{e^{x}\xi^{4\upsilon}}{\Gamma[1+4\upsilon]}.$$

So, the solution $\phi(x,\xi)$ in series form is given by;

$$\phi(x,\xi)=e^x\left[1-\frac{\xi^{\upsilon}}{\Gamma[1+\upsilon]}+\frac{\xi^{2\upsilon}}{\Gamma[1+2\upsilon]}-\frac{\xi^{3\upsilon}}{\Gamma[1+3\upsilon]}+\frac{\xi^{4\upsilon}}{\Gamma[1+4\upsilon]}\right], \quad (4.6)$$

Putting $\upsilon=1$, we get

$$\phi(x,\xi)=e^{x-\xi}. \quad (4.7)$$

Fig. 1(a) and Fig. 1(b) respectively show the exact solution and approximate solutions yielded by the SDM at υ=1. Furthermore, the analysis of absolute errors is summarized in Table 1.

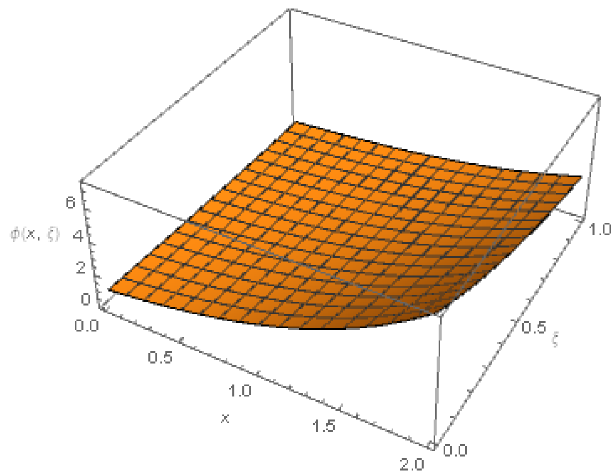

Figure 1(a). Exact solution

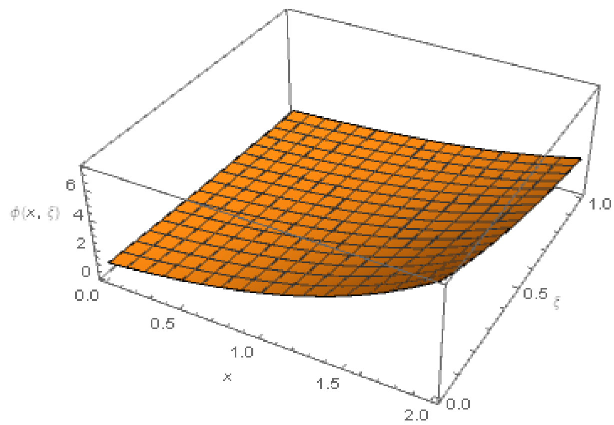

Figure 1(b). SDM

Example 2. By substituting $b=1, k=2, h=-3$ *and* $d=2$ in (1.1),

$$\phi_{\xi}^{\upsilon}(x,\xi)=\phi_{xx}(x,\xi)+2\phi(x,\xi)-3\phi^2(x,\xi), \xi\geq 0, 0<\upsilon\leq 1, \quad (4.8)$$

with initial condition:

$$\phi(x,0)=\gamma, \quad (4.9)$$

a time-fractional non- linear NWS equation.

Using the previous aforesaid in Example 1, we get:

$$\phi_0(x,\xi)=\gamma,$$

$$\phi_1(x,\xi)=\frac{2\xi^{\upsilon}\gamma}{\Gamma[1+\upsilon]}-\frac{3\xi^{\upsilon}\gamma^2}{\Gamma[1+\upsilon]},$$

$$\phi_2(x,\xi)=\frac{4\xi^{2\upsilon}\gamma}{\Gamma[1+2\upsilon]}-\frac{18\xi^{2\upsilon}\gamma^2}{\Gamma[1+2\upsilon]}+\frac{18\xi^{3\upsilon}\gamma^3}{\Gamma[1+3\upsilon]},$$

$$\phi_3(x,\xi)=\frac{8\xi^{3\upsilon}\gamma}{\Gamma[1+3\upsilon]}-\frac{60\xi^{3\upsilon}\gamma^2}{\Gamma[1+3\upsilon]}+\frac{144\xi^{3\upsilon}\gamma^3}{\Gamma[1+3\upsilon]}-\frac{108\xi^{3\upsilon}\gamma^4}{\Gamma[1+4\upsilon]}-$$
$$\frac{12\xi^{3\upsilon}\gamma^2\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+3\upsilon]}+\frac{36\xi^{3\upsilon}\gamma^3\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+3\upsilon]}-\frac{27\xi^{3\upsilon}\gamma^4\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+3\upsilon]}, \tag{4.10}$$

$$\phi_4(x,\xi)=\frac{16\xi^{4\upsilon}\gamma}{\Gamma[1+4\upsilon]}-\frac{168\xi^{4\upsilon}\gamma^2}{\Gamma[1+4\upsilon]}+\frac{648\xi^{4\upsilon}\gamma^3}{\Gamma[1+4\upsilon]}-\frac{1080\xi^{4\upsilon}\gamma^4}{\Gamma[1+4\upsilon]}+$$
$$\frac{648\xi^{4\upsilon}\gamma^5}{\Gamma[1+4\upsilon]}-\frac{24\xi^{4\upsilon}\gamma^2\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+4\upsilon]}+\frac{144\xi^{4\upsilon}\gamma^3\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+4\upsilon]}-$$
$$\frac{270\xi^{4\upsilon}\gamma^4\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+4\upsilon]}+\frac{162\xi^{4\upsilon}\gamma^5\Gamma[1+2\upsilon]}{\Gamma[1+\upsilon]^2\Gamma[1+4\upsilon]}-\frac{48\xi^{4\upsilon}\gamma^2\Gamma[1+3\upsilon]}{\Gamma[1+\upsilon]\Gamma[1+2\upsilon]\Gamma[1+4\upsilon]}$$
$$+\frac{288\xi^{4\upsilon}\gamma^3\Gamma[1+3\upsilon]}{\Gamma[1+\upsilon]\Gamma[1+2\upsilon]\Gamma[1+4\upsilon]}-\frac{540\xi^{4\upsilon}\gamma^4\Gamma[1+3\upsilon]}{\Gamma[1+\upsilon]\Gamma[1+2\upsilon]\Gamma[1+4\upsilon]}+\frac{324\xi^{4\upsilon}\gamma^5\Gamma[1+3\upsilon]}{\Gamma[1+\upsilon]\Gamma[1+2\upsilon]\Gamma[1+4\upsilon]}.$$

Therefore, the solution $\phi(x,\xi)$ in series form is given by;

$$\phi(x,\xi)=\phi_0(x,\xi)+\phi_1(x,\xi)+\phi_2(x,\xi)+\phi_3(x,\xi)+\phi_4(x,\xi)+\ldots, \tag{4.11}$$

putting $\upsilon=1$, we get

$$\phi(x,\xi)=\frac{-\frac{2}{3}\gamma e^{2\xi}}{-\frac{2}{3}+\gamma-\gamma e^{2\xi}}. \tag{4.12}$$

Table 1. Comparison between Exact solution and Approximate solution of Example 1 at $\upsilon=1$.

x	ξ	Exact	Approximation	Absolute Error
0.2	0.16	6.296538261026657	6.296544549363063	6.28834×10^{-6}
	0.32	5.365555971121974	5.365752049033128	1.96078×10^{-4}
	0.48	4.572225195142159	4.573676761867118	1.45157×10^{-3}
	0.64	3.896193301795214	3.90215934335963	5.96604×10^{-3}
	0.80	3.320116922736547	3.33788294175694	1.7766×10^{-2}
	0.96	2.82921701435156	2.872373195690292	4.31562×10^{-2}

Table 2. Comparison of results between SDM and FCT-HPM, at $\gamma=0.001$, and $\upsilon=1$ for Example 2

ξ		SDM		FCT-HPM [18]	
	Exact	Fifth Appr. Sol	Absolute Error	Fifth Appr. Sol	Absolute Error
0.16	0.00137635	0.00137632	2.80827×10^{-8}	0.00137393	2.41742×10^{-6}
0.32	0.00189393	0.00189299	9.47645×10^{-7}	0.00187387	2.00623×10^{-5}
0.48	0.0026054	0.0025978	7.60049×10^{-6}	0.00253329	7.21125×10^{-5}
0.64	0.00358269	0.00354881	3.38792×10^{-5}	0.00339589	1.86797×10^{-4}
0.80	0.00492384	0.00481432	1.0952×10^{-4}	0.00451565	4.08187×10^{-4}
0.96	0.00676192	0.00677288	2.89035×10^{-4}	0.00595679	8.05131×10^{-4}

Table 3. Comparison of results between SDM and FRPSM, at $\gamma = 0.001$, and $\upsilon = 1$ for Example 2

ξ	Exact	SDM		FRPSM [19]	
		Fifth Appr. Sol	Absolute Error	Fifth Appr. Sol	Absolute Error
0.16	0.00137635	0.00137632	2.80827×10^{-8}	0.00137629	6.07525×10^{-8}
0.32	0.00189393	0.00189299	9.47645×10^{-7}	0.00189273	1.209×10^{-6}
0.48	0.0026054	0.0025978	7.60049×10^{-6}	0.00259692	8.48258×10^{-6}
0.64	0.00358269	0.00354881	3.38792×10^{-5}	0.00354672	3.59701×10^{-5}
0.80	0.00492384	0.00481432	1.0952×10^{-4}	0.00481023	1.13604×10^{-4}
0.96	0.00676192	0.00677288	2.89035×10^{-4}	0.00646583	2.96092×10^{-4}

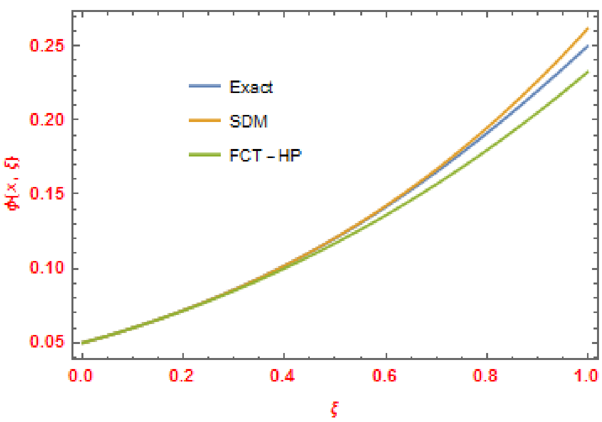

Figure 2. Comparison between exact solution, SDM and FCT- HPM at $\gamma = 0.001$, and $\upsilon = 1$ for Example 2

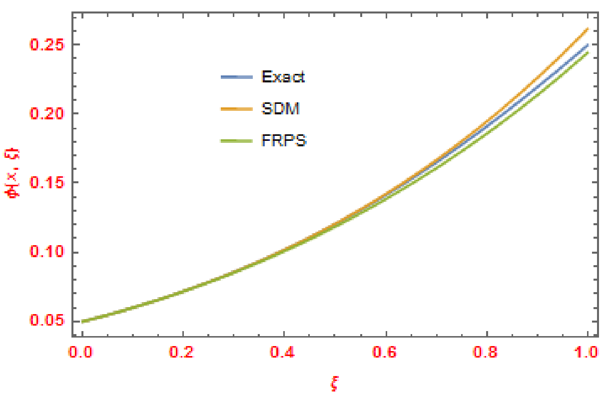

Figure 3. Comparison between exact solution, SDM and FRPSM at $\gamma = 0.001$, and $\upsilon = 1$ for Example 2

The numerical results shown in Tables 2 and 3 and Figs.2 and 3 illustrate that SDM offers accurate results in comparison with FCT-HPM [18] and FRPSM [19].

5. Conclusions

In this paper, SDM had been successfully applied to get approximate solutions of Newell–Whitehead–Segal (NWS) equation of fractional order. It is clearly seen from the numerical results that SDM is easy to execute and offers accurate results for the NWS equation. Hence, SDM is a simple and effective method to obtain approximate and analytical solutions for many differential fractional equations.

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