

Problem-Based Approach to Develop Creative Thinking in Students Majoring in Mathematics at Teacher Training Universities

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Abstract This paper looks into the issue of implementing problem-based approach to developing creative thinking in students majoring in Mathematics at teacher training universities. The object of the study is the process of developing creative thinking in students, majoring in Mathematics at teacher training Universities, through task-based approach. The methods used in the present study are theoretical analysis of research papers and resources, as well as a survey, conducted among students and lecturers. This survey is aimed at determining distinctive features of the problems in Elementary Mathematics, which can help to develop creative thinking in students. The aim of this study is to introduce the syllabus of the online-course “Creative Thinking through Teaching Elementary Mathematics” and to design a system of Open-ended and Integrative Problems in order to develop the components of creative thinking in students, majoring in Mathematics at teacher training universities. The tasks of the present study: to determine what key components of creative thinking in students majoring in Mathematics can be developed, when teaching Elementary Mathematics to them; to determine distinctive features of the Rich Tasks that will contribute to developing creative thinking in students in the course of teaching Elementary Mathematics to them; to introduce the syllabus of the online-course “Creative Thinking through Teaching Elementary Mathematics”; to design a system of rich tasks for arranging the learning process.

Keywords Creative Thinking, Elementary Mathematics, Mathematics Students, Rich Tasks, Open-ended Problems, Integrative Problems, An On-line Course, A Syllabus

1. Introduction

1.1. Problem Statement and Justification of Its Relevance

The issue of developing and enhancing creative thinking in students of different ages has always been of scientific and practical interest for both, researchers and academic staff.

As it is stated in Standards for preparing teachers of Mathematics, developed by the Association of Mathematics Teacher Educators [1] mathematics teachers must contribute to the development of creative thinking in students (Indicator C.3.1. Anticipate and Attend to Students’ Thinking About Mathematics Content). The developers of Standards (Indicator P.3.1. Address Deep and Meaningful Mathematics Content Knowledge) refer mathematical problems to key tools for the development of such type of thinking and state, that it is crucial to do a research into the efficiency of selecting a system of tasks, into how they comply with the current trends in mathematics education, as well as into how positive students’ perception of certain tasks is in course of studying Mathematics.

This idea is in line with the conclusions by Allen et al. [2] and Sullivan [36], who highlight the importance of selecting and systematizing the math problems, used in the learning process, given that students’ opinions and expectations are taken into account. The researchers in the aforementioned papers consider that developing creative thinking in students must start in the first year of study at university, when teaching Mathematics: on the one hand, a mathematical problem must be used as a key object of studying and on the other – as a means of studying.

Elementary Mathematics is one of the most important branches of Mathematics, and is the foundation for training mathematics students at teacher training universities, which is substantiated by Vlasenko et al. [4], stating that it is this subject that ensures revision, generalization and broadening students' knowledge of mathematical definitions and facts, which students were taught at school. The objective of the subject is also to develop would-be teachers' abilities and skills and teach them to apply those skills to practice. The above mentioned justifies timeliness and relevance of the issues of systematizing math problems, solving which will contribute to developing creative thinking in students when teaching Elementary Mathematics.

This article is aimed at undertaking a theoretical analysis of the resources focusing on the development of creative thinking in mathematics students at teacher training universities, as well as on papers devoted to the use of the problem-based approach to teaching Elementary Mathematics; developing and introducing the syllabus of the online-course "Creative Thinking through Teaching Elementary Mathematics" and a system of Rich tasks for this course.

1.2. Analysis of the Latest Researches and Publications

A considerable volume of literature has been published on solving problems in teaching Mathematics. As it was emphasized by Dickman [5], there are more than 100 different definitions of creativity in various research papers. The author investigated the interconnection between creativity and wording of mathematical tasks in primary school.

Silver [6] also mentioned that there is a direct connection between students' creativity and the problems which they solve in the course of learning. The similar research concerning links between mathematical problems and creative thinking was conducted by Van Harpen and Sriraman [7] and Maharani [8], who considered the key components of creative thinking and provided guidelines on how to develop them with the help of the search of geometrical problems. Kajander et al. [9] carried out a fundamental research into the issue of developing mathematical creativity of teachers and students through problems that are built around creativity. Scientists focused on the critical role of problems in developing creative thinking in students.

The authors in this paper turned to the analysis of those publications for two reasons: firstly, in order to identify the ways to develop creative thinking in students, when teaching Mathematics and the role of a math problem in this process.

Secondly, it was a question of selecting and systematizing the math problems in the course of designing a syllabus for mathematics students. Renkl [10] considered the role of corrective tasks and exercises in

order to deal with the students' mistakes and improve their level of mathematical competencies. Breen and O'Shea [11] looked into the process of systematizing and selecting mathematical problems for Master Students, which would aim at developing their mathematical thinking.

When seeking a method of research, the authors of the present study turned to a paper by Yeo [12], who offered a classification of mathematical problems depending on the learning objectives and an assessment method to deal with learning activity of students.

The project "Types of Problems in Teaching Mathematics" by Clarke and Roche [13] was also of the scientific and practical interest. Within the framework of this project a survey concerning the selection of types of problems, depending on learning objectives was undertaken among teachers. Sullivan [3] made a significant contribution by conducting surveys among students, targeted determining types of problems. He found out what kind of problems is interesting for students. A wider coverage of problems in the research paper by Allen et al. [2], dedicated to students' attitudes towards various methods of teaching Mathematics, among which there were mathematical problems, drew the attention of the authors of the present study as well.

Most scientists share the opinion that the usage and selection of problems when teaching Mathematics at secondary school or Higher Mathematics at University, involves the preliminary analysis and systematization of certain types of problems, depending on the goal which a teacher sets. Such an approach, as the experts (Allen et al. [2], Sullivan [3], Yeo [12]) think, helps to understand what types of problems will be more efficient for developing creativity and creative thinking in students, their ability to introduce and apply innovations to their work. Added to that, according to the findings by Vlasenko et al. [14], using such an approach would be appropriate for developing an online course on Elementary Mathematics.

During the international conference MATTEX 2016 [15] the idea to systematize and introduce special types of problems in Elementary Mathematics, aimed at developing creative thinking in students was presented by the authors of this study and discussed by the scholars and practitioners, who attended that conference.

2. Materials and Methods

The platform "Higher School Mathematics Teacher" [42] was used to conduct a survey among lecturers of Mathematics at Berdiansk State Teacher Training University, Kryvyi Rih State Teacher Training University, Donbass State Machine Building Academy, Donbas National Academy of Civil Engineering and Architecture, Volodymyr Dahl East-Ukrainian National University, Pavlo Tychyna Uman State Teacher Training University,

Cherkassy State Technological University.

It was focused on using various types of mathematical problems when teaching Elementary Mathematics in order to develop creative thinking in students. Another survey was conducted among mathematics students to find out their opinions and expectations from various types of problems in teaching Elementary Mathematics. The analysis of the responses allowed the authors of this article to start the research into the respondents' current preferences. Thus, they found out the respondents' opinions on creative thinking and its key components, on the term "mathematical problem" and criteria for systematizing types of problems, which contribute to developing creative thinking in students. The existing teaching experience in using integrative and open-ended problems was also analysed.

The Deductive Approach to Content Analysis was applied to understand if it is possible to involve the task-based approach in order to develop creative thinking in students. While selecting the resources, the preference was given to the ones, which present the development of practical aspects of creative thinking, the usage of various types of problems and tools that enhance inherent creativity of students (Table 1). The work undertaken resulted in designing the online syllabus for teaching the discipline.

Table 1. The analysis of the resources, that present on-line courses, aimed at developing creative thinking.

On-line courses	Using tasks
Udemy, 50 Drawing Exercises: Improve Creativity & Creative Thinking [16]	<ul style="list-style-type: none"> • case studies; • open-ended problems
Creative Thinking: Innovative Solutions to Complex Challenges [17]	<ul style="list-style-type: none"> • case studies; • research problems; • problems aimed at integrating knowledge
European Schoolnet Academy, Developing Creative Thinking Skills in Practice [18]	<ul style="list-style-type: none"> • case studies; • problems aimed at integrating knowledge
Australian Curriculum, Critical and Creative Thinking [19]	<ul style="list-style-type: none"> • open-ended problems • problems aimed at integrating knowledge
Universal class, Creative Thinking [20]	<ul style="list-style-type: none"> • open-ended problems • research problems; • tasks aimed at integrating knowledge
School Education Gate Way, Creativity for the future: promoting Critical Thinking and Problem-Solving in the classroom [21]	<ul style="list-style-type: none"> • research problems; • open-ended problems
Coursera, Creative-Thinking-Techniques-and-tools-for-success [22]	<ul style="list-style-type: none"> • case studies; • research problems

By using task-based approach to developing creative thinking in students, the authors of the above mentioned courses are targeted at solving various types of research problems: case studies, open-ended problems, research

problems, problems that integrate knowledge in natural science subjects, different branches of Economics, etc. The analysis of the recommendations, developed by the experts (Table 1), contributed to working out the syllabus sections and the system of open-ended and integrative problems to develop the components of creative thinking in students majoring in Mathematics. Responses by students and lecturers helped the authors of this article to define a name for the on-line course "Creative Thinking through Teaching Elementary Mathematics"; theoretical analysis of problems in Elementary Mathematics and the respondents' answers added to the content of the sections of the course.

2.1. The Syllabus

To prepare the summary of the course, the authors of this article have considered the scientists' viewpoints on the notion "creative thinking". Silver [6], Dickman [5], Kajander et al. [9] regard it as a type of thinking, aimed at generating non-conventional ideas, digressing from conventional algorithms, solving problems faster. So, the conclusion is made by Maharani [8] that creative thinking promotes conscientious and active involvement of students into transforming and improving the learning environment, revealing and refining inner resources.

The analysis of the research, conducted by Harpen and Sriraman [7], Kajander et al. [9], Dickman [5], Treffinger et al. [23], Karasel et al. [24], as well as taking account of the responses by lecturers and students made it possible to formulate the course objective, which is to develop the components of creative thinking in mathematics students at teacher training universities:

- the ability to identify and state problems;
- the ability to generate numerous ideas;
- the flexibility – ability to generate various ideas;
- the novelty – ability to act in a non-conventional way;
- the ability to improve the object, to add details.

When arranging the learning process within the framework of the course, the emphasis is put on solving Elementary Mathematics problems aimed at developing creative thinking in students. In order to determine the type of tasks, researchers follow the position of Yeo [12], who divided the problems into Mathematically-Rich Tasks and Non-Mathematically-Rich Tasks, depending on the impact they have on the development of personal, creative features of students.

Problems, which give students an opportunity to "discover" new mathematical rules (definitions, patterns), to act in a non-conventional way, to develop creative thinking are known as Rich Tasks in Elementary Mathematics. Open-ended and integrative problems are considered to be the main types of Rich Tasks.

Summarising the findings by Frobisher [25], Orton and Frobisher [26], Becker and Shimada [27], the authors of

this paper consider that an open-ended problem is a problem, which does not have a predefined provision (having excessive data or their lack) or the one, which has several solutions depending on how you interpret the provisions.

The course “Creative Thinking through Teaching Elementary Mathematics” means to use case studies, open-ended and research problems. When solving a case study, students have to select mathematical apparatus and to apply it properly to find a solution to a certain problem. Solving open-ended problems, students have to add a certain provision (provisions) to the existing ones, formulate the problem and solve it. Using research problems involves students singling out methods for solving the problem and applying the methods properly.

When choosing the tasks that will ensure the proper arrangement of the learning process within the framework of the course, some aspects of integrating mathematical knowledge into knowledge of other subjects were clarified. Following the findings by Starr [28], Schleigh et al. [29], An et al. [30], Williams et al. [31], who considered integrative teaching of Mathematics and other subjects, when running integrative lessons and designing integrative syllabi.

The opinion of Wraga [32], who believes that most issues that secondary school students have, result from their lack of understanding of connections between the subjects on the school curricula added value to the work on the present paper.

University students, majoring in Mathematics, tend to face similar issues. They don't always understand the importance of unity of knowledge in different branches of Mathematics and the role which Mathematics plays in other areas of knowledge.

Thus, the authors of this paper consider that one of the ways to cope with the problem is to use integrative problems. By an integrative problem, we mean a problem, which promotes integration of the students' knowledge of different branches of secondary school Mathematics

and/or Mathematics, taught at University. Using such problems is believed to provide integration of knowledge of Mathematics into other subjects.

To design the course, three types of integrative problems were singled out:

1. Problems, solving which students understand the connections between content and definitions of separate branches of Mathematics, taught at school (for instance, between Algebra and Geometry).

2. Problems, solving which students understand the connections between Elementary Mathematics and branches of Higher Mathematics, which are studied in parallel or will be studied in future.

3. Problems, generated beyond the scope of Mathematics, but which require applying mathematical methods to solve.

Taking into consideration the types of the problems used in the course, the main requirement to the course users is knowledge of Elementary Mathematics, Algebra, Geometry, Mathematical Logic, and Mathematical Analysis. Therefore, the course is designed for the first-year university students, majoring in Mathematics.

2.2. The System of Open-ended and Integrative Problems to Develop the Components of Creative Thinking

Here are some examples of a developed system of Rich tasks (Table 2), which will be used in the on-line course “Creative Thinking through Teaching Elementary Mathematics”. Open-ended and integrative problems are structured according to three levels of complexity. Thus, the designers of the system of tasks take into account different levels of mathematical competencies of students. Added to that, transition from solving easier problems to more complex ones contributes to developing several components of creative thinking, which was also considered.

Table 2. The examples of a system of Open-ended and Integrative Problems to Develop the Components of Creative Thinking

Components of creative thinking, which are developed in the course of solving the problems		Problems
Open-ended problems		
Flexibility	The 1 st level	<i>Case study.</i> You have a rope with knots, the distance between them is 12, 5 and 13 metres. How to build a right angle with the help of this rope?
The ability to improve the subject, to add details		<i>Open-ended problem.</i> To fill the motorboat, the fishermen used 50 litres of fuel. On the 1 st day they used 20 litres, on the second day.... Complement the provision with a fraction, make up two problems and solve them.
The ability to identify and state the problem		<i>Research problem.</i> Based on the analysis of the manuals in Elementary Mathematics and school textbooks, determine the methods for solving rational equations, using these methods. Draft a table, which consists of two columns (method, example of the equation) and fill it in with the examples, designed by you and with the determined methods.
Flexibility, novelty	The 2 nd level	<i>Case study.</i> You have a land plot, rectangular in shape, 130 m long and 80 m wide. How to plant trees on this plot?
The ability to improve the subject, to add details; flexibility		<i>Open-ended problem.</i> The lateral sides of a trapezoid equal 13 m and 5 m respectively. Draft 3 tasks of different levels of complexity, using these provisions (no more than 1 provision can be added).
The ability to identify and state the problem, the ability to improve the subject, to add details		<i>Research problem.</i> Based on the analysis of the manuals in Elementary Mathematics and school textbooks, determine the methods for solving trigonometric equations and approximate basics for the activity, using these methods.
Flexibility, the ability to generate a number of ideas, novelty	The 3 rd level	<i>Case study.</i> Can an equilateral triangle be cut into equal pentagons? Justify your answer.
Flexibility		<i>Open-ended problem.</i> A circle is inscribed into a right-angle prism, the sides of which equal 8, 9 and 10 m. Draft as many problems of different levels of complexity as possible, using these provisions (no more than 2 provisions can be added).
The ability to identify and state the problem, flexibility, the ability to improve the subject, to add details		<i>Research problem.</i> Classify the types of problems in topic «Trigonometry», determine the key methods (ways of solving 2 of the types of problems, which you offer).
Integrative problems		
The ability to identify and state the problem	The 1 st level	<i>Problems, solving which enhances the students' understanding of the content and conceptual connections between certain sections in Mathematics, taught at secondary school.</i> 2 semidiametres of a circle divide it into arcs. The lengths of the arcs correlate as 2:4:5. Find the length of the bigger arc, if the circumference is 44π cm.
		<i>Problems, solving which enhances the students' understanding of the connections between Elementary Mathematics and sections of Higher Mathematics.</i> Compare the numbers $\sqrt[1001]{1001}$ i $\sqrt[2018]{2018}$
		<i>Problems, generated beyond the scope of Mathematics, but solving which requires applying mathematical methods.</i> After how many half-life periods there will be 8 g of radioactive substance left out of 64 g?
The ability to identify and state the problem	The 2 nd level	<i>Problems, solving which enhances the students' understanding of the content and conceptual connections between certain sections in Mathematics, taught at secondary school.</i> To calculate the meaning of the mathematical formula $\log_{14}\cos 675^\circ$.
		<i>Problems, solving which enhances the students' understanding of the connections between Elementary Mathematics and sections of Higher Mathematics.</i> At what maximum positive value of the parameter the length of vector $\vec{n}(\dot{a}; 3; -1)$ will be less than 5?
The ability to identify and state the problem, flexibility		<i>Problems, generated beyond the scope of Mathematics, but solving which requires applying mathematical methods.</i> A ball with a mass of m , suspended on a string, is executing periodic motion. When the ball is in an equilibrium state, the tension force of the string equals $T=2mg$. What is the maximum angle α of the deflection of the ball from the vertical position?
The 3 rd level		
The ability to identify and state the problem, flexibility, novelty	The 3 rd level	<i>Problems, solving which enhances the students' understanding of the content and conceptual connections between certain sections in Mathematics, taught at secondary school.</i> To calculate the meaning of the mathematical formula $(\sqrt{21+8\sqrt{5}}+\sqrt{21-8\sqrt{5}})^{\log_{21}2025}$
		<i>Problems, solving which enhances the students' understanding of the connections between Elementary Mathematics and sections of Higher Mathematics.</i> Solve the equation $\sqrt{x-1}+\sqrt{9-x}=x^2-10x+29$.
The ability to identify and state the problem, flexibility		<i>Problems, generated beyond the scope of Mathematics, but solving which requires applying mathematical methods.</i> A bus is moving along a straight motorway at the speed of 16 m per sec. There is a person in the field, 60 m away from the motorway and 400 m away from the bus. The person can move at the speed of 4 m per sec. What direction should this person choose in order to catch the bus?

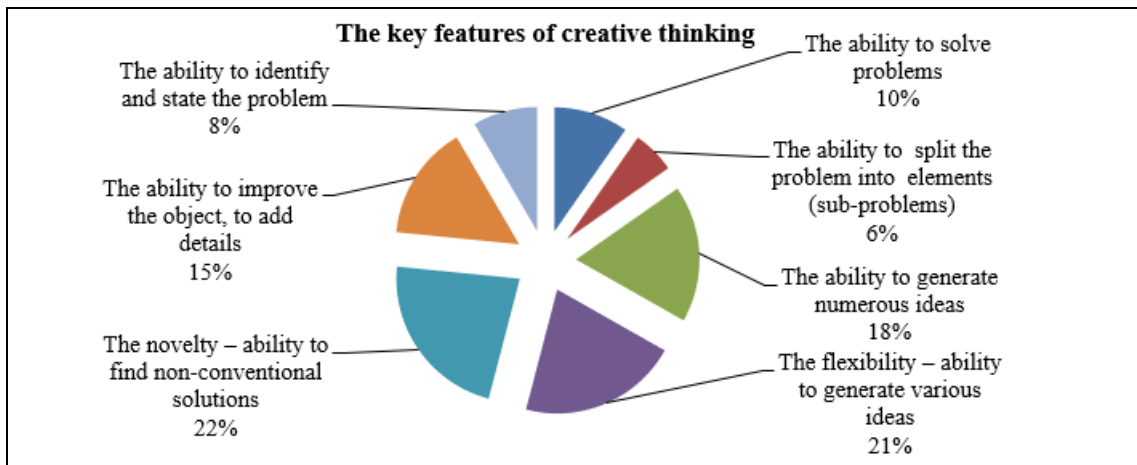


Figure 1. The key features of creative thinking according to the students' responses

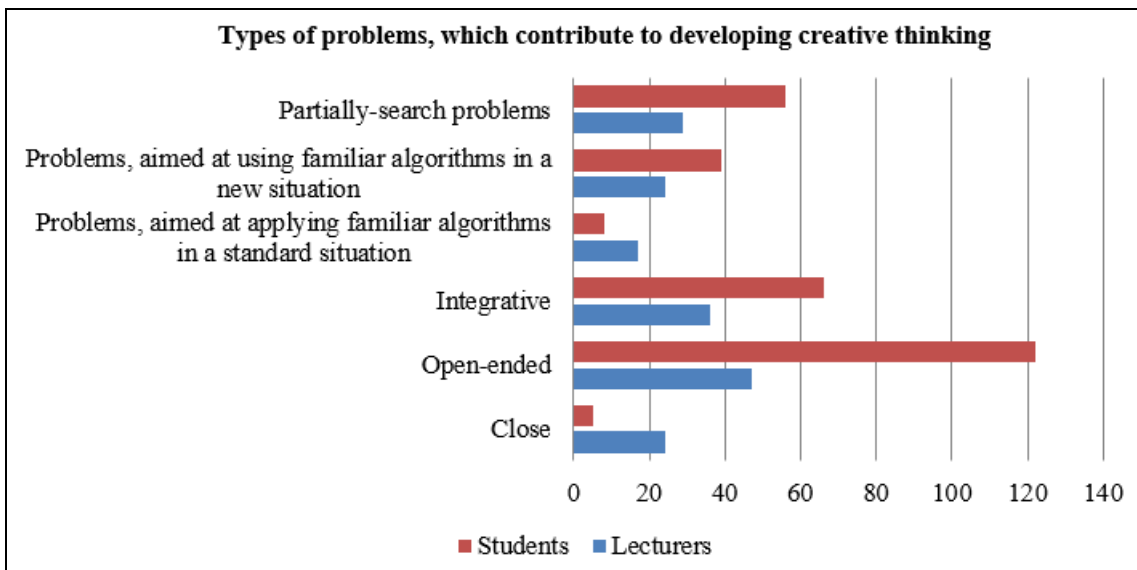


Figure 2. Types of problems, which contribute to developing creative thinking according to the opinions of lecturers and students

3. Results

To conduct the analysis, defined by the objective of the present research a survey on the platform “Higher School Mathematics Teacher” [42] was conducted. University lecturers and students were asked to express their opinions on applying task-based approach to developing creative thinking in students.

Another survey was conducted among mathematics students at teacher training universities in order to define the key features of creative thinking; types of problems in Elementary Mathematics which, in their opinion, will help enhance their creative thinking. In total 65 lecturers and 157 students at different universities were surveyed.

According to the students' responses (Figure 1), the key features of creative thinking are: the ability to generate a significant number of ideas, flexibility, novelty and ability to improve the object.

When systematizing the tasks for the arrangement of the learning process, researchers learnt from the students and the lecturers what types of problems (Figure 2), in their opinion, contribute to developing creative thinking.

Thus, the respondents' answers confirmed the expediency of using open-ended and integrative problems for arranging on-line learning. When systematizing the types of open-ended (Figure 3) and integrative (Figure 4) problems, researchers again focused on the opinion of the respondents – students and lecturers.

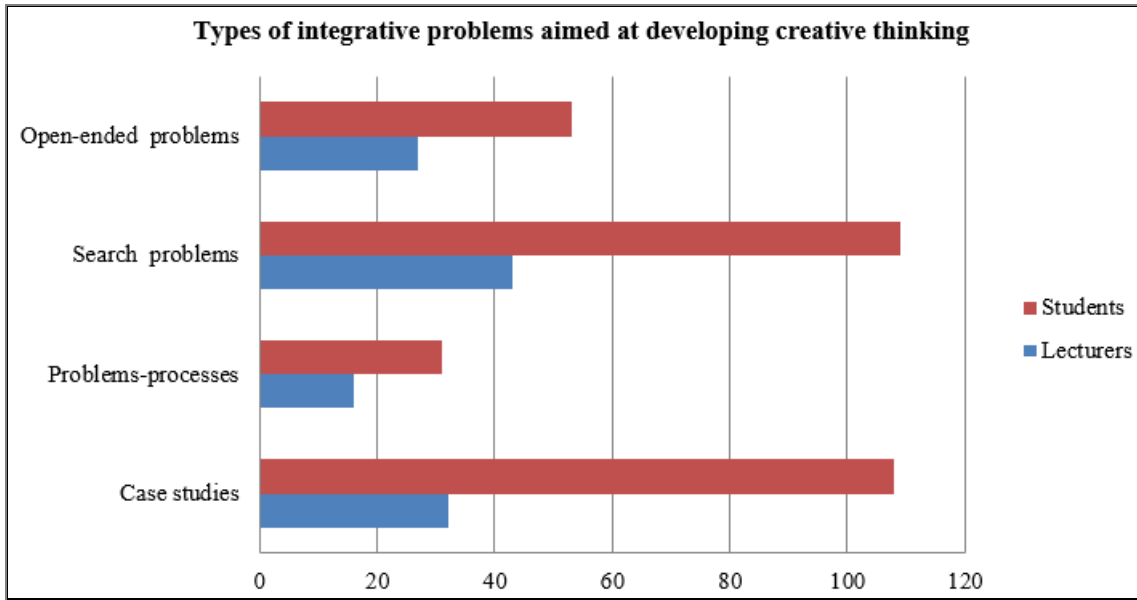


Figure 3. Types of integrative problems aimed at developing creative thinking

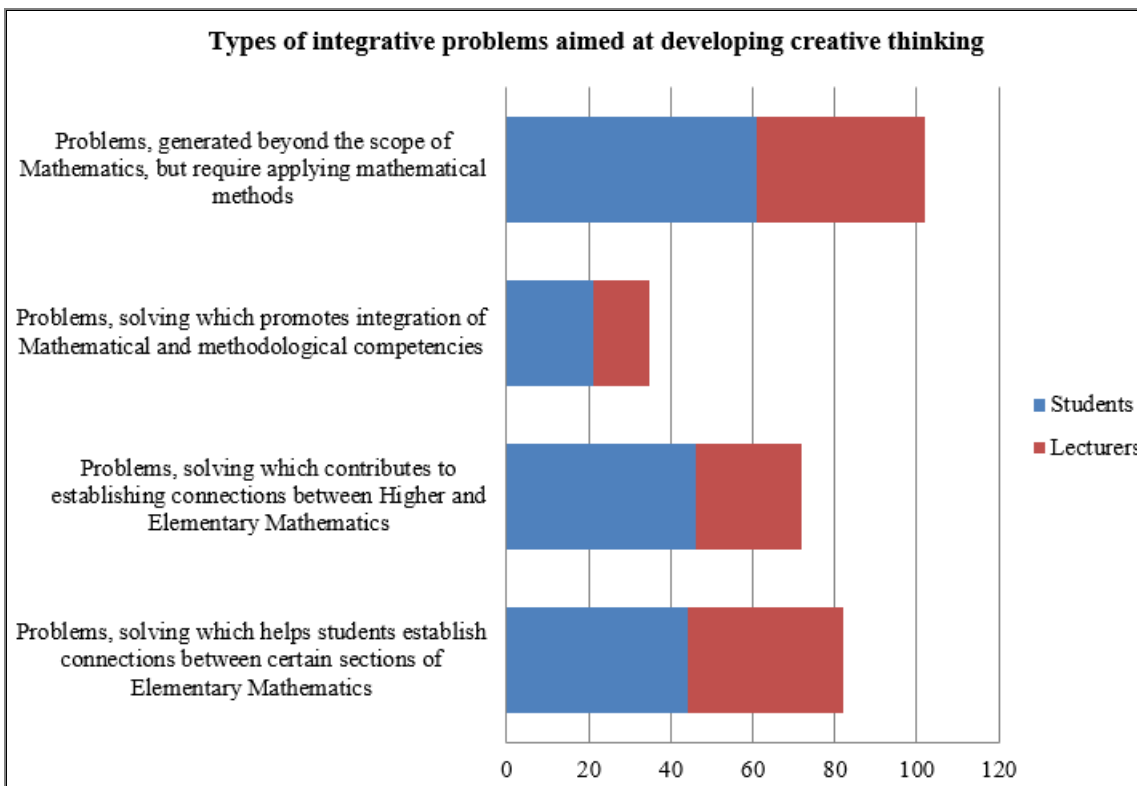


Figure 4. Types of integrative problems aimed at developing creative thinking

Thus, the respondents' answers confirmed the expediency of using open-ended problems, search problems, case studies. The most popular types of integrative tasks among the respondents were: problems, generated beyond the scope of Mathematics, but require applying mathematical methods, problems, solving thereof contributes to establishing connections between Higher and Elementary Mathematics or between sections

of Elementary Mathematics.

Through being proactive and showing interest in the course, both the lecturers and the students helped to define the topics of the course "Creative Thinking through Teaching Elementary Mathematics", the content for the Syllabus sections, and the types of Rich tasks, which should be chosen to systematize.

4. Discussion

In this article the authors share the opinion of Maharani [8], Papadakis et al. [33], Tezer et al. [34], who state that the role of creative thinking in the modern globalized world is becoming more important, as the competition is getting more tough and the problems which future mathematicians are to solve are becoming more sophisticated. Findings of the researchers (Dickman [5], Silver [6], Harpen and Sriraman [7], Kajander et al. [9]) prove the topicality and timeliness of introducing a problem-orientated approach to developing creative thinking in students. They also emphasize that to develop creative thinking is possible with the help of specially selected types of tasks.

Students must be given an opportunity to select types of problems, which they want to learn about while studying Elementary Mathematics, as Sullivan [3], Vlasenko et al. [35] emphasize. The researchers consider it feasible to design an on-line course, aimed at using selected types of problems for developing creative thinking in Mathematics students in teacher training Universities.

The idea to design the on-line course also complies with the conclusions, made by Zaranis et al. [36], Lin and Ying-Wei Wu [37], Hjalmarson [38], Perikos et al. [39], Lovianova et al. [40], that prove the efficiency of on-line education in order to develop creativity in students while teaching Mathematics. Henson et al. [41] consider Syllabi as a course map for any online course, so this statement was taken into account for the purpose of the present research.

All the information (findings, estimates, actions, and content), that is needed to design the course, was collected and evaluated to prepare a complete syllabus for online students before the start of the course. Students are also offered to have Elementary Mathematics training that helps them discover new possibilities and inner resources to create innovations, to develop their abilities to improve the object adding details, find non-conventional solutions, identify and state the problem, split the problem into sub-problems, generate numerous ideas.

Focusing on the research papers and taking account of the expectations of the future course users as to the arrangement of the learning process, open-ended and integrative problems were used by the authors of this paper, who strongly believe, that work with open-ended problems will contribute to developing key components of creative thinking (ability to generate ideas, offer non-conventional methods for solving the tasks, to refine their own proposal – a task, an idea, a product). Solving integrative problems will help students see Mathematics in the world around them, to find connections between different phenomena, using various methods when solving professional tasks and problems.

5. Conclusions

Nowadays, when information is most often shared via the Internet, informal education through on-line courses becomes more widespread. This fact determined and contributed to developing the system of Rich tasks in Elementary Mathematics and designing the syllabus of the online-course “Creative Thinking through Teaching Elementary Mathematics”.

Analysis of academic papers and resources proved the relevance of researching into the issue of developing creative thinking in students who major in Mathematics in teacher training Universities. The Deductive Approach to Content Analysis of the resources made it possible to involve a problem-based approach to selecting topics, various types of tasks and tools that contribute to enhancing inherent human creativity and can be of practical interest for students.

The analysis done by the authors of the present paper helped determine the course syllabus sections and the methodology for providing the content for those sections. When developing the summary of the course, the experience of scholars and course experts must be a guideline, but in order to live up to students’ expectations, it is advisable to conduct a survey among Mathematics students and lecturers. Therefore, when formulating the course objectives and selecting the tasks for the arrangement of the learning process, the respondents’ opinions are to be taken into account.

The interpretation of the respondents’ answers confirmed the importance of developing an online course and their willingness to learn about two types of Rich tasks in Elementary Mathematics: open-ended problems and integrative problems. The requirements for systematizing problems that are used within the framework of the course are to be clarified. These should be the problems that will help students generate new ideas, approach problem-solving in a non-conventional way, analyze all the possible options of the course of action, improve own proposals, establish connections between objects of different nature, see and apply Mathematics and its methods in the world around them, as well as in practical activity.

Given that solving such problems requires knowledge of the elements of Algebra, Geometry, Mathematical Logic, Mathematical Analysis, the course is designed for Mathematics students. It can also be used by those who want to find out how new ideas are generated and non-conventional paths of action are taken with the help of Elementary Mathematics. When systematizing the tasks, it is advisable to select the problems of different levels of complexity, in order to make the course accessible for the students, who have no proper mathematical background. Uploading materials of the on-line course “Creative Thinking through Teaching Elementary Mathematics” to the on-line educational platform “Higher School

Mathematics Teacher” [42] is the focus of further research for the authors of this study. The direction for further research implies describing the implementation of the on-line course into practical training of the students majoring in Mathematics in teacher training universities.

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