

# Capturing Prospective Teachers' Beliefs about Mathematical Problem Solving

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**Abstract** Beliefs about problem solving are very important because they form the basis for learning mathematics. This research describes beliefs of prospective mathematics teachers toward mathematical problem solving. Participants of this research consisted of 157 prospective mathematics teachers at a private University in Semarang. Participants have sufficient background in the field of mathematics and pedagogy study in learning. Data were collected through semi-structured interviews and beliefs questionnaire on problem solving with answer choices. Questionnaire we used was to investigate the beliefs of prospective teachers about mathematical problem solving, while the interview process that we used was to obtain qualitative data. Based on the results of the questionnaire, each one of the prospective teacher students who had instrumentalist, platonist and constructivist beliefs was interviewed for their beliefs in mathematical problems, problem solving and problem solving processes. The results of the research show that most of prospective teacher tended to have platonist beliefs in every statement of mathematical problems and mathematical problem solving. But on the statement of the problem solving process, the most of prospective teachers tended to have constructivist beliefs. The belief perspective is built to help teachers understand and develop mathematical knowledge needed in solving mathematical problems.

**Keywords** Beliefs, Mathematical Problems Solving, Prospective Teachers

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## 1. Introduction

Problem solving has become an important issue that is discussed deeply in mathematics education. NCTM (2000) recommends that problem solving can be the focus of mathematics instruction as the basis for developing

thinking skills implemented in the 2013 curriculum. Simple problem solving is the process of accepting problems as a challenge to solve them. Polya (1973) defines problem solving as an attempt to find a way out of a difficulty, to achieve a goal that is not immediately achievable. A problem is a problem for someone at a time, but it is not a problem in the next time (Dossey, McCrone, O'Sullivan & Gonzales, 2006; Nizaruddin, Muhtarom, & Zuhri, 2019; Pathuddin, Budayasa, & Lukito, 2018; 2019), if one already knows the way or the process of getting the problem solving. Through mathematical problem solving, students are directed to develop their skills such as building new mathematical knowledge, solving problems in various contexts related to mathematics, applying the necessary strategies and reflecting the process of mathematical solution (Anderson, White and Sullivan, 2005; Chapman, 2015; Muhtarom, Juniati, & Siswono, 2017a; Nizaruddin, Muhtarom, & Murtianto, 2017). Greer, Verschaffel & Corte (2002), state that one of the most effective ways of generating beliefs in problem solving is through teachers, textbooks, learning strategies, and the main one is the use of problems around students for learning activities. During math lessons, students learn not only mathematical concepts and procedures, but also how to interact in the classroom, they learn about a set of beliefs, and they learn how to behave in math lessons. Students' beliefs about mathematics, mathematics learning and their skills for solving mathematical problems can be understood through their beliefs in problem solving (Garofalo, 1989; Kloosterman & Stage, 1992; Mason, 2003). Garofalo (1989) shows different types of beliefs about mathematical problem solving that affect mathematical achievement, for example: the difficulty degree of the problem, the operations to be performed and the student's decision to check what has been done.

Obviously, beliefs to mathematical problems solving is an important thing that must be instilled in children since

the early days because belief can be the basis for disposition, the basis for action, the basis for change and the basis for learning (Chapman, 2015). Pehkonen & Pietila (2003) do not place beliefs in the human affective domain but somewhere between the cognitive domain and the affective domain, called the "twilight zone". Beliefs as subjective knowledge and perception can be interchangeable (Presmeg, 2002) in the context of mathematics. Beliefs as a cognitive and affective construct is essential for the problem-solving learning process (Bal, 2015; Callejo & Vila, 2009; Ozturk & Guven, 2015; Schoenfeld, 1992; Thompson, 1992). Prospective teachers can successfully solve the problem of belief in the importance of mathematics and the need to understand the problem; while students who believe in problem solving take a short time and they can handle it by memorizing the rules, believing that problem solving is difficult (Schoenfeld, 1992).

Before discussing the beliefs of problem solving that becomes the focus of this article, we need to first describe the concept of beliefs (e.g. Anderson, White & Sullivan, 2005; Beswick, 2005; 2012; Boz, 2008; Cheng et al., 2009; Thompson, 1992), with categories: instrumentalist, platonist, and problem solving. Table 1 summarizes some of the expert views on these three categories of beliefs (Beswick, 2012).

The research of beliefs in problem solving has been carried out by several researchers (e.g. Duell, & Hutter, 2005; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Schommer-Aikins., Mkomange & Ajagbe, 2012). For example, Schommer-Aikins., Duell, & Hutter (2005) find that beliefs in mathematics have a direct influence on the students' mathematical problem-solving performance. Mkomange & Ajagbe (2012) conclude that most mathematics prospective teachers have positive beliefs about the importance of understanding mathematical problems, problems with some completion ways and the type of mathematical learning emphasized by contemporary principles. Most of prospective teachers do not have enough beliefs in solving problems, especially about mathematical skills, place of mathematics and problem solving beliefs sub-dimensions (Memnun, Hart & Akkaya, 2012). Other researchers provide an overview of beliefs in problem solving by looking at aspects of the

problem-solving process (Ozturk & Guven, 2015; Muhtarom, Juniati, & Siswono, 2017b), problem solving content knowledge and problem solving pedagogical knowledge (Chapman, 2015; Siswono, Kohar, & Hartono, 2017). Teachers' beliefs have a strong relationship with the teacher's knowledge of problem solving. In particular, the instrumentalist's beliefs are consistent with the inadequate knowledge of problem solving, while platonist and constructivist teacher beliefs are consistent with their knowledge of problem solving (Muhtarom et al., 2017b; Siswono et al., 2017). This indicates that the development of prospective teachers' beliefs in problem solving is a necessity. Based on the background, our research aimed to express the beliefs of prospective teachers toward mathematical problem solving.

## 2. Methods

### 2.1. Participants

Participants of this research consisted of 157 prospective mathematics teachers at a private University in Semarang. Participants have sufficient background in the field of mathematics and pedagogy study in learning.

### 2.2. Instruments and Procedures

The research data were collected through questionnaires of belief about problem solving compiled by researchers primarily by utilizing previously developed instruments (e.g. Chapman, 2015; Callejo & Vila, 2009; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Siswono et al., 2017), with some modifications and additions to the choice of answers that cited the categories of beliefs: instrumentalist, platonist and constructivist. Before being used, a questionnaire of belief about problem solving has been performed expert validation and tested internal consistency with reliability coefficient 0.651. Questionnaire consisted of 14 statements divided into three parts, namely beliefs to math problems (4 statements), beliefs in problem solving (6 statements), and beliefs in problem-solving process (4 statements), as an example instrument presented in Table 2.

**Table 1.** Related Beliefs

Beliefs about nature mathematics	Beliefs about mathematics teaching	Beliefs about mathematics learning
Instrumentalist / as tool	Content focused with an emphasis on performance	Skill Mastery, passive reception of knowledge
Platonist / body static	Content focused with an emphasis on understanding	Active construction of understanding
Problem solving / constructivist	Learner focused	Autonomous exploration of own interest

**Table 2.** Samples of Questionnaire Statements Belief about Mathematical Problem Solving

Category	Sub Category	Examples of statements
Mathematical Problems	The subject's belief about 'who' should make math problems	Mathematical problems should be made by ... a. Teachers themselves 'spontaneously' in accordance with textbooks b. Specially designed by the teachers in accordance with the material needs and problems created by the students themselves. c. Specially designed by the teachers in accordance with the needs of the subject matter in accordance with the textbook
Problem Solving	The subject's belief on how to help students become good problem solvers, when and how to engage as long as students solve problems.	To improve students' misunderstanding of the material, I teach during doing my learning activities ... a. I will straighten out the misconceptions of the students by giving a more detailed explanation, especially on the misunderstood part b. I will involve students more actively and more carefully carrying out the remaining learning activities as I have previously planned c. I will give students the opportunity to discuss by comparing their ideas to determine the best way to improve their misconceptions
Problem Solving Process	The subjects' beliefs about the truth of the answer in solving the problem	What is done in checking by implementing the completion steps to get the correct answer is ... a. Every time I do a re-check of the completion steps, calculations and concepts that have been implemented so that I understand why an answer is true according to the context of the problem b. After getting an answer, I then check the completion steps, calculations and concepts that I have applied so that I understood why an answer is correct according to the context of the problem c. After I get the answer, I re-check the completion steps, calculations and concepts that have been applied so I get the correct answer

### 2.3. Data Analysis

Scores varied to show the belief level on mathematical problem solving from 1.00 (instrumentalist) to 3.00 (constructivist). Scores were given to each sample on each statement based on the following formula: average score = scores obtained: number of statements. Based on the average score then each participant's beliefs are grouped into three categories: instrumentalist, platonist, and constructivist (see Table 3) (Siswono, Kohar, Kurniasari, & Astuti, 2016).

**Table 3.** Beliefs Categories Grouping

Score	Categories of Beliefs
Average Score <1.67	Instrumentalist
$1.67 \leq \text{Average Score} \leq 2.33$	Platonist
Average Score > 2.33	Constructivist

These data were also supported by qualitative data to provide a description of prospective teachers' beliefs in mathematical problem solving by using a set of interview guidelines. Prospective teachers were selected to be interviewed individually about their beliefs about problem solving, including problem knowledge, problem solving knowledge and knowledge of problem solving process. Data analysis was done by firstly reducing data, displaying data, and finally drawing conclusion and verification (Miles & Huberman, 1992). The conclusion was there was

a description of the prospective teachers' beliefs about mathematical problem solving.

## 3. Results and Discussion

There were fourteen statements to measure the beliefs of prospective teachers on mathematical problem solving. Table 4 obviously shows that the distribution of the number of prospective teachers choosing each answer option for each statement. For example, the statement of 'mathematical problem maker' indicated that 25.4% of prospective teachers believed that mathematical problems should be made by teachers 'spontaneously' in accordance with textbooks, 51% of prospective teachers believed that mathematical problems should be specially designed by teachers in accordance with material needs lessons that were appropriate to textbooks, and 23.6% of prospective teachers believed that mathematical problems should be specially designed by the teachers according to the material needs and problems that were made by the students themselves. Furthermore, the prospective teachers believed that the purposes of the problems were: the students were able to complete according to the procedure according to the teacher or book (10.8%), the students were able to understand the relation among mathematical ideas, concepts, and mathematical procedures to be able to solve the problems (67.5%) and

students were actively involved in performing mathematical tasks in exploring and formulating ideas to be able to complete the problems (21.7%). Furthermore, 31 prospective teachers improved the misunderstanding of students by providing a more detailed explanation, especially in the wrong part, 74 prospective teachers improved the misunderstanding of students by involving students more actively and more carefully in carrying out the learning activities and 52 prospective teachers improved the misunderstanding of students by allowing students to discuss by comparing ideas and deciding the best way to improve their misconceptions.

As described in Table 4, we can note that the most of

prospective teachers tended to have platonist beliefs about every statement of mathematical problems and mathematical problem solving. However, in the statement of the problem-solving process, the most of prospective teachers tended to have constructivist beliefs. Specifically, the results of the research showed that 55.4% of prospective teachers had platonist beliefs about mathematical problem solving, 40.1% of prospective teachers had constructivist-oriented beliefs about mathematical problem solving, and the rest had beliefs in the solving of instrumentalist-oriented mathematical problem solving.

**Table 4.** Results Description of Each Statement

Category	Statement	The answer choice of Instrumentalist		The answer Choice of Platonist		The answer Choice of Constructivist	
		n n	% %	n n	% %	n n	% %
Mathematical problems	The type of problem that must be solved by the students	23 23	14.6 14.6	79 79	50.4 50.4	55 55	35.0 35.0
	Mathematical problems maker	40 40	25.4 25.4	80 80	51.0 51.0	37 37	23.6 23.6
	The purpose of giving problems in learning	17 17	10.8 10.8	106 106	67.5 67.5	34 34	21.7 21.7
	The way of students in learning problem-solving	31 31	19.7 19.7	72 72	45.9 45.9	54 54	34.4 34.4
Problem Solving	Problem solving as a learning objective of mathematics	16 16	10.2 10.2	91 91	58.0 58.0	50 50	31.8 31.8
	Problem solving as a process	24 24	15.3 15.3	76 76	48.4 48.4	57 57	36.3 36.3
	Problem solving as a basic skill	43 43	27.4 27.4	45 45	28.7 28.7	69 69	43.9 43.9
	Improving students' misunderstanding	31 31	19.7 19.7	74 74	47.2 47.2	52 52	33.1 33.1
	Students who have difficulty	13 13	8.3 8.3	82 82	52.2 52.2	62 62	39.5 39.5
	How to train students to solve math problems	43 43	27.4 27.4	48 48	30.6 30.6	66 66	42.0 42.0
Problem Solving Process	Understanding math problems	20 20	12.7 12.7	40 40	25.5 25.5	97 97	61.8 61.8
	Planning for solving math problems	18 18	11.5 11.5	84 84	53.5 53.5	55 55	35.0 35.0
	Implementing the planning in a math problem solving	30 30	19.1 19.1	27 27	17.2 17.2	100 100	63.7 63.7
	Looking back the completion	27 27	17.2 17.2	86 86	54.8 54.8	44 44	28.0 28.0

Furthermore, we presented the result of interview with Mayya (woman and pseudo name) as based on the result of belief questionnaire in solving problem, the participant seemed to illustrate her belief in the instrumentalist view. Mayya had belief that to be able to solve the problem, one had to remember the way ever taught by the teacher then imitate the way of completion. Mayya had sufficient understanding in the process of solving the problem. Her idea was guiding students to solve math problems such as, asking students to read the problem, understanding the concepts of the given problem, and understanding that in planning the problem solving, one should make completion procedures, then use the procedures that had been made to get the solution of the problem. The truth of the answer was believed to be based on the process of completion stage and correctness of the calculation process undertaken. We can see the interview quote as follows.

Mayya: What students should do first is that students must understand concepts in mathematics. Then students should be more active reading, learning then completing some math questions. When they understand the exact concept, the students can solve the problem in various ways. Mathematics does not have only one way of solving, and there are many ways and answers that are true and appropriate.

Mayya: To plan the problem solving it means we understand the problem first. After that we make the completion procedures. Later we know how the process to get the answer.

Mayya: We know the stages then continue to count so that the answer is met. So do it based on the problems and the stages.

Mayya: We are back to problem solving. If our procedures counting is appropriate, the answer is correct.

Subjek Insiatun (nama perempuan dan pseudo) tampak memiliki keyakinan terhadap pemecahan masalah dalam pandangan platonis. Participant Insiatun (woman and pseudo name) seemed to have belief in solving problems in platonist view. Insiatun understood that the purpose of giving problems in learning activities was that students were able to understand the relationship between mathematical concepts and procedures. Therefore, students should have tried to master the mathematical concepts associated with the problem first, then solved the math problem. Participant also stated that the role of teachers was to train students many times to master the concept and make connections between mathematical concepts. Mathematical problems had to be made by the teacher in accordance with the needs of the subject matter in accordance with the textbook and they had some solutions. To be able to solve mathematical problems, the first step that was believed by Insiatun was to know the available information, what was asked and to find out enough information available to answer the problem. The next was making a connection between the mathematical concepts

used and how it related to other material concepts and then made the completion steps, solved the problem according to plan by checking every step of completion, calculation and concepts that had been applied so that they understood why an answer was true according to the context of the problem.

Septin (nama perempuan dan pseudo) tampak memiliki keyakinan terhadap pemecahan masalah dalam pandangan konstruktivis. Participant Septin (woman and pseudo name) seemed to have belief in solving problems in the constructivist view. Subject understood that the purpose of giving problem in learning activities was to make students actively involved in carrying out mathematical tasks in exploring ideas. Therefore, they tried to solve the problem in their own way based on the knowledge and experience they had. Septin also stated that the role of teachers was to guide students to try to solve problems in their own way based on knowledge and experience. Mathematical problems had to be made by the teacher in accordance with the material needs and problems created by the students themselves and they had some solutions. To be able to solve math problems, the first step that Septin believed was to know whether the information was available enough to answer the problem, separate the main information from the problem and represent the problem. The next step was making connections between the mathematical concepts used and how they related to other material concepts and then arranging the completion steps, completing the problems based on the strategy-oriented plan and checking every step at a time and whether the steps were correct so they understood why an answer was true according to the context of the problem. Here is a brief excerpt of the interview.

Septin: The purpose of giving problems to students is of course to entice the student to think creatively, so that they can solve the problem not only from one point of view, but from another point of view. Thus, it was enticing the students to think creatively.

Septin: So teachers and students should be able to make math problems. So later the students do the questions from the books of which the problems were made by the students themselves. By doing so the students could be more developed because they are able to make questions-based problems.

Septin: Mathematical problem solving is a strategy used by students to discover why the answer can be true.

Most mathematics prospective teachers had platonist and constructivist beliefs about mathematical problem solving. It means their beliefs were in line with the current reformation movement in mathematics education in Indonesia. For example, to improve students' misconceptions, they would involve students more actively and more carefully in performing learning activities (platonist), and allow students to discuss by comparing their ideas to determine the best way to correct misconceptions (constructivist). The prospective teachers

believed that the most important thing a person had to do to make the students able to solve the math problem was to ask the students to try to solve the problem in their own ways based on their knowledge and experience. Basically they chose problems requiring mathematical thinking and reasoning. However, at the same time they preferred a problem that did not spend too much time. It was clear that they still held some instrumentalist beliefs toward mathematical problems solving. For example, some prospective teachers tended to make mathematical problems which were appropriate to textbooks or student worksheets that had one way of completion, students tended to see the problem solving as a reason for practicing computing skills, following a predetermined sequence of steps while solving problems and only oriented on the correct answer. Understanding the beliefs of prospective teachers on mathematical problem solving is very important. Further research is needed to understand the practice of learning-oriented development of beliefs about problem solving and how to involve

teachers in learning it (Chapman, 2015, Muhtarom et al., 2017b; 2018; 2019; Siswono et al., 2017).

In this research, we obtained the data using a set of questionnaires and interview guidelines that the reliability had been customized and tested. This is different from the commonly used method of exploring belief in mathematical problem solving (e.g. Duell, & Hutter, 2005; Kloosterman & Stage, 1992; Memnun, Hart & Akkaya, 2012; Schommer-Aikins., Mkomange & Ajagbe, 2012). The research could also be the basis for studying specifically about the role of belief in mathematical problem solving. We proposed extending this category to include belief in mathematical problem solving (see Table 5), so the research on belief not only focused on beliefs about: mathematics nature, mathematics teaching and mathematics learning (Anderson, White & Sullivan, 2005; Beswick, 2005; 2012; Boz, 2008; Cheng et al., 2009, Thompson, 1992). However, another thing we need to be aware of is that one's belief is not necessarily consistent in all categories.

**Table 5.** Extension of Beliefs Categories

Beliefs about nature mathematics	Beliefs about mathematics teaching	Beliefs about mathematics learning	Beliefs about mathematics problem solving
Instrumentalist / as tool	Content focused with an emphasis on performance	Skill Mastery, passive reception of knowledge	Remembering the way of completion that has been taught and emphasizing the step sequence performance and orienting to the right answer
Platonist / body static	Content focused with an emphasis on understanding	Active construction of understanding	Making a connection between mathematical concepts and understanding why an answer is true
Problem solving / constructivist	Learner focused	Autonomous exploration of own interest	Orienting to the development of problem-solving strategy and understanding why an answer is true

## 4. Conclusions

Our results show that the most of prospective teachers tend to have platonist beliefs about every statement of mathematical problems and mathematical problem solving. However, in the statement of the problem-solving process, the most of prospective teachers tend to have constructivist beliefs. Prospective teacher students must realize the importance of problem solving beliefs because belief can be the basis for disposition, the basis for action, the basis for change and the basis for learning. The perspective beliefs presented in this article can be constructed to provide a framework of beliefs to help teachers understand and develop the mathematical knowledge needed in mathematical problem solving.

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