

The Semi Analytics Iterative Method for Solving Newell-Whitehead-Segel Equation

Busyra Latif *, Mat Salim Selamat, Ainnur Nasreen Rosli, Alifah Ilyana Yusoff, Nur Munirah Hasan

Faculty of Computer and Mathematical Sciences, University Teknologi MARA Negeri Sembilan, Seremban Campus, 70300 Seremban, Negeri Sembilan, Malaysia

Received June 29, 2019; Revised November 4, 2019; Accepted November 17, 2019

Copyright©2020 by authors, all rights reserved. Authors agree that this article remains permanently open access under the terms of the Creative Commons Attribution License 4.0 International License

Abstract Newell-Whitehead-Segel (NWS) equation is a nonlinear partial differential equation used in modeling various phenomena arising in fluid mechanics. In recent years, various methods have been used to solve the NWS equation such as Adomian Decomposition method (ADM), Homotopy Perturbation method (HPM), New Iterative method (NIM), Laplace Adomian Decomposition method (LADM) and Reduced Differential Transform method (RDTM). In this study, the NWS equation is solved approximately using the Semi Analytical Iterative method (SAIM) to determine the accuracy and effectiveness of this method. Comparisons of the results obtained by SAIM with the exact solution and other existing results obtained by other methods such as ADM, LADM, NIM and RDTM reveal the accuracy and effectiveness of the method. The solution obtained by SAIM is close to the exact solution and the error function is close to zero compared to the other methods mentioned above. The results have been executed using Maple 17. For future use, SAIM is accurate, reliable, and easier in solving the nonlinear problems since this method is simple, straightforward, and derivative free and does not require calculating multiple integrals and demands less computational work.

Keywords Exact Solution, Newell-Whitehead-Segel equation, Semi analytic Iterative Method

1. Introduction

A NWS equation is a nonlinear Partial Differential equation (PDE) and it is used in modeling various phenomena arising in fluid mechanics. This equation is used for some problems in various systems, for example, Faraday instability, biological systems, nonlinear optics, Rayleigh-Benard convection, and chemical reactions. The equation acquired by Newell, Whitehead, and Segel is as follows [1-2]:

$$u_t(x,t) = \alpha u_{xx}(x,t) + bu(x,t) - cu^m(x,t) \quad (1)$$

$$u(x,0) = f(x) \quad (2)$$

where b, c are real numbers and α, m are positive integers.

In recent years, different methods have been used to solve the NWS equation. Ezzati and Shakibi [3] and Manaa [4] employ the Adomian Decomposition method (ADM) to obtain the numerical approximations of NWS. Saravanan and Magesh [5] conducted a comparative study in solving the NWS equation between Reduced Differential Transform Method (RDTM) and ADM. Nourazar et al. [6] and Jassim [7] solved the NWS equation using the Homotopy Perturbation method (HPM) and HPM using Laplace transform respectively. Also, Patade and Bhalekar [8] presented the application of the New Iterative method (NIM) to solve NWS equation. Prakash and Kumar in [9] presented the application of the Variational Iteration method (VIM) while Pue-on [10] applied the Laplace Adomian Decomposition method (LADM) to solve the NWS equation.

The Semi Analytical Iterative method (SAIM) was proposed by Temimi and Ansari to solve linear and nonlinear problems [11-12]. Al-Jawary et al. [13] solved the Fokker-Planck Equations (FPE) using SAIM to assemble the exact solution for the one dimension, two dimensions and three dimensions FPE. The method shows a very accurate and high order of convergence, reliable and effectual and also optional in any restrictive assumptions for non-linear terms. This can be supported in [14-16].

LADM was implemented to solve the approximate solution of nonlinear differential equations by Khuri [17]. Kiyamaz [18] has used LADM to solve initial value problems. Fadaei applied LADM to the linear and nonlinear system of PDEs [19]. Khan et al. [20] proposed LADM in solving Nonlinear Coupled Partial Differential Equation. This method was also successfully employed by Naghipour and Manafian [21] to solve the Burgers' equation.

ADM was introduced by Adomian in 1988 [22]. Wazwaz employed ADM to determine the new exact and new

approximate solutions to the generalized Emden–Fowler equation [23] and solved Bratu-type equations [24]. ADM also was used by Jafari and Daftardar-Gejji in [25-28].

Daftardar-Gejji and Jafari proposed NIM to solve nonlinear functional equations [29]. Bhalekar and Daftardar-Gejji applied NIM in [30-32] while Hemeda applied NIM to solve fractional physical differential equations [33]. In 2009, RDTM was proposed by Keskin [33]. This method got the attention of other researchers [35-37].

There are several advantages of SAIM over the existing methods. SAIM is a very simple and easy method to be implemented. SAIM avoids the calculation of Adomian polynomials for a nonlinear term in ADM and thus demands less computational work. SAIM produces better approximate solutions among the other methods mentioned above.

The purpose of this paper is to employ SAIM to obtain the approximate solution to NWS equation. Illustrative examples in this paper will be solved numerically and the results obtained are compared with the exact solution and other existing results obtained by other methods such as LADM, NIM, RDTM and ADM to reveal the reliability and accuracy of the method.

2. Semi-Analytical Iterative Method (SAIM)

The basic idea of SAIM is presented in this section. Consider the general form of the differential equation as follows [16]

$$L(u(x,t)) + N(u(x,t)) + f(x,t) = 0, \quad B\left(u, \frac{\partial u}{\partial t}\right) = 0 \quad (3)$$

where t is an independent variable, $u(x,t)$ is an unknown function, $f(x,t)$ is a known function, L is a linear operator, N is a nonlinear operator, and B is a boundary operator.

With SAIM, we first assume the initial problem as $u_0(x,t)$ and the work is as follows:

$$L(u_0(x,t)) + f(x,t) = 0, \quad B\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0 \quad (4)$$

Consider the next iteration for the solution as can be computed by solving the following equation

$$\begin{aligned} L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) &= 0, \\ u_1(x,0) &= f(x) \end{aligned} \quad (5)$$

This leads to the general equation of this method in the form as follows

$$\begin{aligned} (L(u_{n+1}(x,t)) + N(u_n(x,t)) + f(x,t)) &= 0, \\ B\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}\right) &= 0 \end{aligned} \quad (6)$$

Each $u_n(x,t)$ is considered alone a solution for (3). This method is direct and straightforward. Continuing these steps will give a good approximate solution.

3. Applications

In this section, the Newell-Whitehead-Segel equation will be solved using SAIM. The Newell-Whitehead-Segel equation is given as below [1-2]

$$u_t(x,t) = \alpha u_{xx}(x,t) + bu(x,t) - cu^m(x,t) \quad (7)$$

with the initial condition,

$$u(x,0) = f(x) \quad (8)$$

Where b, c are real numbers and α, m are positive integers. By using SAIM, (6) will become

$$u_t(x,t) - \alpha u_{xx}(x,t) - bu(x,t) + cu^m(x,t) = 0 \quad (9)$$

Comparing (9) with (3) gives the following

$$L(u(x,t)) = u_t(x,t) \quad (10)$$

$$N(u(x,t)) = -\alpha u_{xx}(x,t) - bu(x,t) + cu^m(x,t) \quad (11)$$

$$f(x,t) = 0 \quad (12)$$

The initial problem with initial condition that needs to be solved is

$$L(u_0(x,t)) = 0, \quad u_0(x,0) = f(x) \quad (13)$$

Where

$$\int_0^t u_{0t}(x,t) dt = 0 \quad (14)$$

Thus we obtained

$$u_0(x,t) = f(x) \quad (15)$$

For the second iterative, we need to solve the following equation

$$\begin{aligned} L(u_1(x,t)) + N(u_0(x,t)) + f(x,t) &= 0, \\ u_1(x,0) &= f(x) \end{aligned} \quad (16)$$

where

$$\int_0^t u_{tt}(x,t) dt = \int_0^t \alpha(u_0)_{xx}(x,t) dt + \int_0^t b(u_0)(x,t) dt - \int_0^t c(u_0)^m(x,t) dt \tag{17}$$

The general form of iterative function to the solution that needs to be solved is given by

$$L(u_{(n+1)}(x,t)) + N(u_n(x,t)) + f(x,t) = 0, \tag{18}$$

$$u_{n+1}(x,0) = f(x)$$

Where

$$\int_0^t u_{(n+1)t}(x,t) dt = \int_0^t \alpha(u_n)_{xx}(x,t) dt + \int_0^t b(u_n)(x,t) dt - \int_0^t c(u_n)^m(x,t) dt \tag{19}$$

4. Illustrative Examples

Three numerical examples are considered to be solved numerically by SAIM in this section to reveal the reliability and accuracy of the method.

4.1. Example 1

Given the Newell-Whitehead-Segel equation as follows

$$u_t(x,t) = 5u_{xx}(x,t) + 2u(x,t) + u^2(x,t), u(x,0) = \lambda \tag{20}$$

and the exact solution of this equation is given by

$$u(x,t) = \frac{2e^{2t}\lambda}{2 + (1 - e^{2t})\lambda} \tag{21}$$

By using SAIM, (19) becomes

$$u_t(x,t) - 5u_{xx}(x,t) - 2u(x,t) - u^2(x,t) = 0 \tag{22}$$

where

$$L(u(x,t)) = u_t(x,t) \tag{23}$$

$$N(u(x,t)) = -5u_{xx}(x,t) - 2u(x,t) - u^2(x,t) \tag{24}$$

$$f(x,t) = 0 \tag{25}$$

The initial problem with initial condition that needs to be solved is

$$L(u_0(x,t)) = 0, u_0(x,0) = \lambda \tag{26}$$

Where

$$\int_0^t u_{0t}(x,t) dt = 0 \tag{27}$$

Thus, we obtained

$$u_0(x,t) = \lambda \tag{28}$$

For the second iterative, we solve the following equation

$$L(u_1(x,t)) = 5(u_0)_{xx}(x,t) + 2(u_0)(x,t) + (u_0)^2(x,t), u_1(x,t) = \lambda \tag{29}$$

The next iterative to the solution can be obtained by solving

$$L(u_{n+1}(x,t)) = 5(u_n)_{xx}(x,t) + 2(u_n)(x,t) + (u_n)^2(x,t), u_{n+1}(x,t) = \lambda \tag{30}$$

Thus, with SAIM the first few iterative solutions are

$$u_0(x,t) = \lambda \tag{31}$$

$$u_1(x,t) = (1 + (\lambda + 2)t)\lambda \tag{32}$$

$$u_2(x,t) = \frac{(3 + \lambda(\lambda + 2)^2 t^3)\lambda}{3} + \frac{(3\lambda^2 + 9\lambda + 6)\lambda t^2}{3} + \frac{(3\lambda + 6)\lambda t}{3} \tag{33}$$

$$u_3(x,t) = \left(\frac{1}{63}(63 + \lambda^3(\lambda + 2)^4 t^7 + 7\lambda^2(\lambda + 1)(\lambda + 2)^3 t^6 + 21(\lambda + 2)^2\left(\lambda^2 + 2\lambda + \frac{3}{5}\right)\lambda t^5 + 42\lambda(\lambda + 1)(\lambda + 2)^2 t^4 + (63\lambda^2 + 252\lambda^2 + 294\lambda + 84)t^3 + (63\lambda^2 + 189\lambda + 126)t^2 + (63\lambda + 126)t)\right)\lambda \tag{34}$$

By letting $\lambda = 12$, the fifth iterative solution is

$$\begin{aligned}
 u_4(x,t) = & \left(\frac{53293212499968}{5}\right)t^{15} + 12371638616064t^{14} + \left(\frac{435108944019456}{65}\right)t^{13} \\
 & + \left(\frac{11630629011456}{5}\right)t^{12} + \left(\frac{164580693774336}{275}\right)t^{11} + \left(\frac{3048418394112}{25}\right)t^{10} \\
 & + \left(\frac{102161446912}{5}\right)t^9 + \left(\frac{14638076544}{5}\right)t^8 + \left(\frac{1844579072}{5}\right)t^7 + \left(\frac{206000704}{5}\right)t^6 \\
 & + 4130112t^5 + 367640t^4 + 28336t^3 + 2184t^2 + 168t + 12
 \end{aligned} \tag{35}$$

In this study, we let $t \in [0, 0.05]$ to execute the solution The results will be plotted in Figure 1.

4.2. Example 2

Consider that the Newell-Whitehead-Segel equation is as follows

$$\begin{aligned}
 u_t(x,t) = u_{xx}(x,t) + 2u(x,t) - 3u^2(x,t), \\
 u(x,0) = \lambda
 \end{aligned} \tag{36}$$

and the exact solution to this equation is

$$u(x,t) = -\frac{2e^{2t}\lambda}{-2 + 3(1 - e^{2t})\lambda} \tag{37}$$

By using SAIM, (36) becomes

$$u_t(x,t) - u_{xx}(x,t) - 2u(x,t) + 3u^2(x,t) = 0 \tag{38}$$

where

$$\begin{aligned}
 L(u(x,t)) = u_t(x,t) \\
 N(u(x,t)) = -u_{xx}(x,t) - 2u(x,t) + 3u^2(x,t) \\
 f(x,t) = 0
 \end{aligned} \tag{39}$$

The initial problem with initial condition that needs to be solved is

$$L(u_0(x,t)) = 0, \quad u_0(x,0) = \lambda \tag{40}$$

$$\begin{aligned}
 u_3(x,t) = & \lambda - \lambda(3\lambda - 2)t + \lambda(3\lambda - 1)(3\lambda - 2)t^2 - \frac{1}{3}\lambda(3\lambda - 2)(27\lambda^2 - 18\lambda + 2)t^3 + 2\lambda^2(3\lambda - 1)(3\lambda - 2)t^4 \\
 & - \frac{3}{5}\lambda^2(15\lambda^2 - 10\lambda + 1)(3\lambda - 2)^2t^5 + \lambda^3(3\lambda - 1)(3\lambda - 2)^3t^6 - \frac{3}{7}\lambda^4(3\lambda - 2)^4t^7
 \end{aligned} \tag{48}$$

By letting $\lambda = 0.1$, the fifth iterative solution is

where

$$\int_0^t u_{0t}(x,t) dt = 0 \tag{41}$$

Thus, we obtained

$$u_0(x,t) = \lambda \tag{42}$$

For the second iterative, we solve the following equation

$$\begin{aligned}
 L(u_1(x,t)) = (u_0)_{xx}(x,t) + 2(u_0)(x,t) - 3(u_0)^2(x,t), \\
 u_1(x,t) = \lambda
 \end{aligned} \tag{43}$$

The next iterative to the solution can be obtained by solving

$$\begin{aligned}
 L(u_{n+1}(x,t)) = (u_n)_{xx}(x,t) + 2(u_n)(x,t) \\
 - 3(u_n)^2(x,t),
 \end{aligned} \tag{44}$$

$$u_{n+1}(x,t) = \lambda$$

By SAIM, the first few iterative solutions are

$$u_0(x,t) = \lambda \tag{45}$$

$$u_1(x,t) = -3t\lambda^2 + 2t\lambda + \lambda \tag{46}$$

$$\begin{aligned}
 u_2(x,t) = & -\lambda - \lambda(3\lambda - 2)t + \lambda(3\lambda - 1)(3\lambda - 2)t^2 \\
 & - \lambda^2(3\lambda - 2)t^3
 \end{aligned} \tag{47}$$

$$\begin{aligned}
 u_4(x,t) = & 0.1000000000000000 - 2.56252314210^{-8}t^{15} + 5.27578293910^{-7}t^{14} \\
 & - 0.0000315910381200000t^{13} - 0.00000276872115000000t^{12} \\
 & + 0.0000792527515100000t^{11} - 0.0000925412679599999t^{10} \\
 & - 0.0007317562569999999t^9 + 0.000539201749599995t^8 \\
 & + 0.00488974238000001t^7 + 0.00310193333299999t^6 \\
 & - 0.0252585999999999t^5 - 0.0210233333400001t^4 \\
 & + 0.0266333333999998t^3 + 0.119000000000000t^2 + 0.170000000000000t
 \end{aligned}
 \tag{49}$$

In this example, we let $t \in [0, 2.2]$ to execute the solution. The results will be plotted in Figure 3.

4.2. Example 3

Given the Newell-Whitehead-Segel equation is as follows

$$u_t(x,t) = u_{xx}(x,t) - 3u(x,t), \quad u(x,0) = e^{2x} \tag{50}$$

By using SAIM, the first few iterative solutions are

$$u_0(x,t) = e^{2x} \tag{51}$$

$$u_1(x,t) = e^{2x}(t+1) \tag{52}$$

$$u_2(x,t) = \frac{1}{2}e^{2x}(t^2 + 2t + 2) \tag{53}$$

$$u_3(x,t) = \frac{1}{6}e^{2x}(t^3 + 3t^2 + 6t + 6) \tag{54}$$

$$u_4(x,t) = \frac{1}{24}e^{2x}(t^4 + 4t^3 + 12t^2 + 24t + 24) \tag{55}$$

By using the identity $u(x,t) = \lim_{n \rightarrow \infty} u_n(x,t)$ leads

to $u(x,t) = e^{2x+1}$, which is the exact solution of (50).

5. Results and Discussion

In this section, we compare the fifth iterative solution of our result as the first example, with the exact solution, four term iterative solution with those obtained from NIM by Patade and Bhalekar [8] and LADM by Pue-On [10].

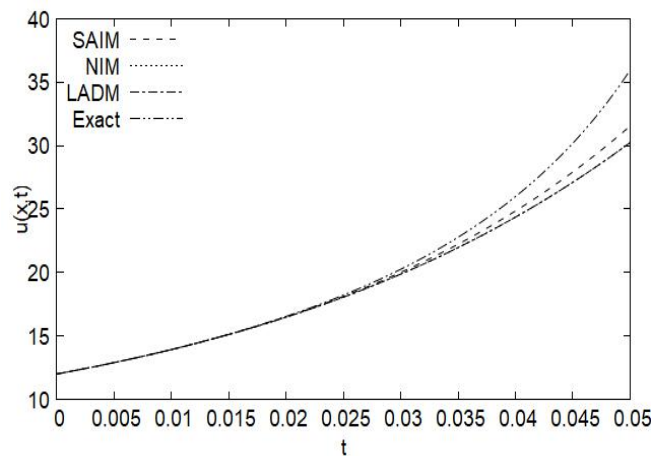


Figure 1. Comparison of solutions between SAIM, NIM, LADM and Exact Solution for $\lambda = 12$.

For the second example, we compare the fifth iterative solution of our results with the exact solution, four term solution NIM by Patade and Bhalekar [8], ADM and RDTM by Saravanan and Magesh [5].

Figure 1 shows the value of $u(x,t)$ of SAIM which is close to the exact solution compared to NIM and LADM. Therefore, SAIM is proved to be efficient and more accurate. In addition, the value of $u(x,t)$ for SAIM is converging due to the increase from zero until the top of the record.

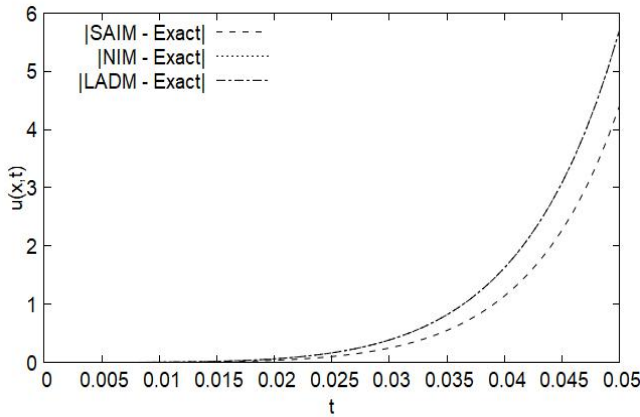


Figure 2. Comparison of magnitude error between SAIM, Exact Solution, NIM and LADM for $\lambda = 12$.

From Figure 2, at $t = 0$, the accuracy of these three methods is similar since the magnitude of error is 0. However, as t increases, the result obviously shows that the magnitude errors of SAIM are lower than NIM and LADM. The error increases due to the range of utility of the power series which is limited to the neighborhood of the origin by its convergence radius that is determined by the singularity closest to that point.

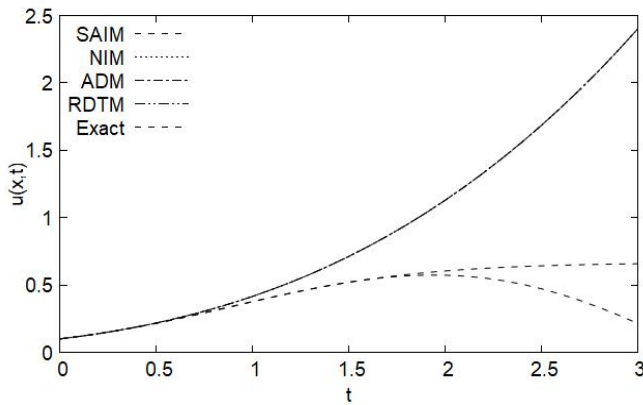


Figure 3. Comparison of solutions between SAIM, NIM, ADM, RDTM and Exact Solution for $\lambda = 0.1$.

Based on Figure 3, at $t = 0$ there is no difference among these four methods but when t increases, obviously we can see that SAIM is in good agreement with the exact solution compared to NIM, ADM and RDTM.

From Figure 4, in the beginning all these three methods stayed constant at $u(x,t) = 0$. As t increases, SAIM shows better accuracy compared to NIM, ADM and RDTM. Thus, the result has shown that the accuracy of the magnitude value of SAIM is much better since the value of $u(x,t) = 0$ is close enough to zero compared to the other methods.

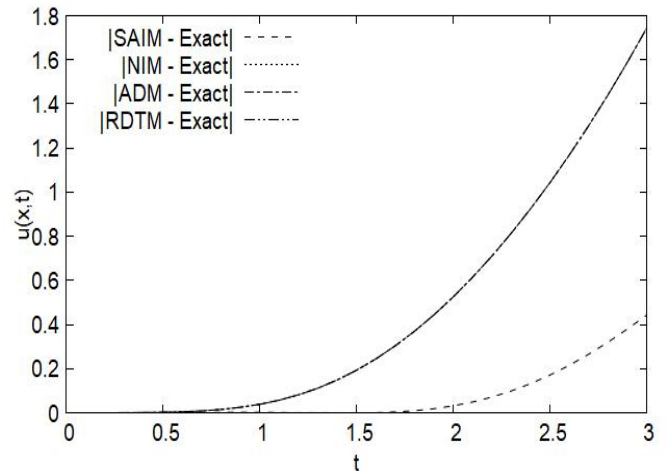


Figure 4. Comparison of magnitude error between SAIM, Exact Solution, NIM, ADM and RDTM for $\lambda = 0.1$.

6. Conclusion

In this study, the Semi Analytical Iterative method (SAIM) is used to solve the Newell-Whitehead-Segel equation (NWS). As mentioned above, for the first example, the fifth iterative solution of the SAIM is compared with the exact solution, the four term iterative solution of the NIM and LADM in Figure 1 at $t \in [0, 0.05]$ and $\lambda = 12$. The result obtained shows SAIM is in excellent agreement with the exact solution. A comparison of the magnitude error values between SAIM, exact solution, NIM and LADM has been plotted in Figure 2. It can be seen that as the time increases, the magnitude error value of the solution obtained by SAIM is much lower than the others. This indicates the efficiency and accuracy of SAIM in solving NWS equation.

On the other hand, for the second example, the fifth iterative solution of the SAIM has been compared with the exact solution, the four term iteration solution of the NIM, ADM and RDTM in Figure 3 at $t \in [0, 2.2]$ and $\lambda = 0.1$. It can be seen that, as time increases, SAIM shows better approximations than the others. A comparison of the magnitude error values between SAIM, Exact Solution, NIM, ADM and RDTM has been plotted in Figure 4. It shows that the relative error is consistently very small compared to the others. Again, this indicates the reliability, efficiency and accuracy of SAIM.

The iteration of SAIM is direct and straightforward. SAIM is simple, accurate, reliable and useful in solving nonlinear problems. It is derivative free, does not require the calculation of multiple integrals and does not require the usage of Adomian polynomial.

REFERENCES

- [1] Newell, A. C., & Whitehead, J. A. (1969). Finite bandwidth, finite amplitude convection. *Journal of Fluid Mechanics*, 38(2), 279-303.
- [2] Segel, L. A. (1969). Distant side-walls cause slow amplitude modulation of cellular convection. *Journal of Fluid Mechanics*, 38(1), 203-224.2. Patade, J., & Bhalekar, S. (2015). Approximate analytical solutions of Newell-Whitehead-Segel equation using a new iterative method. *World Journal of Modelling and Simulation*, 11(2), 94-103
- [3] Ezzati, R., & Shakibi, K. (2011). Using Adomian's decomposition and multiquadric quasi-interpolation methods for solving Newell-Whitehead equation. *Procedia Computer Science*, 3, 1043-1048.
- [4] Manaa, S. A. (2011). An Approximate Solution to The Newell-Whitehead Equation by Adomian Decomposition Method. *simulation*, 6(12), 14-17.
- [5] Saravanan.A & Magesh.N. (2013). A comparison between the reduced differential transform method and the Adomian decomposition method for the Newell-Whitehead-Segel equation. *Journal of The Egyptian Mathematical Society*. 21(3), 259-265.
- [6] Nourazar, S. S., Soori, M., & Nazari-Golshan, A. (2011). On the exact solution of Newell-Whitehead-Segel equation using the homotopy perturbation method. *Journal of Applied Sciences Research*,7(8).
- [7] Jassim, H. K. (2015). Homotopy perturbation algorithm using Laplace transform for Newell-Whitehead-Segel equation. *Int J Adv Appl Math Mech*, 2, 8-12.
- [8] Patade, J., & Bhalekar, S. (2015). Approximate analytical solutions of Newell-Whitehead-Segel equation using a new iterative method. *World Journal of Modelling and Simulation*, 11(2), 94-103.
- [9] Prakash, A., & Kumar, M. (2016). He's variational iteration method for the solution of nonlinear Newell-Whitehead-Segel equation. *Journal of Applied Analysis and Computation*, 6(3), 738-748.
- [10] Pue-on, P. (2013). Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation. *Applied Mathematical Sciences*, 7(132), 6593-6600.
- [11] Temimi, H., & Ansari, A. R. (2011). A semi-analytical iterative technique for solving nonlinear problems. *Computers & Mathematics with Applications*, 61(2), 203-210.
- [12] Temimi, H., & Ansari, A. (2011). A new iterative technique for solving nonlinear second order multi-point boundary value problems. *Applied Mathematics and Computation*, 218(4), 1457-1466.
- [13] AL-Jawary, M. A., Radhi, G. H., & Ravnik, J. (2017). Semi-analytical method for solving Fokker-Planck's equations. *Journal of the Association of Arab Universities for Basic and Applied Sciences*, 24, 254-262.
- [14] AL-Jawary, M. A., & Raham, R. K. (2017). A semi-analytical iterative technique for solving chemistry problems. *Journal of King Saud University-Science*, 29(3), 320-332.
- [15] Fokas, A., Flyer, N., Smitheman, S., & Spence, E. (2009). A semi-analytical numerical method for solving evolution and elliptic partial differential equations. *Journal of Computational and Applied Mathematics*, 227(1), 59-74.
- [16] Al-Jawary, M. A., Azeez, M. M., & Radhi, G. H. (2018). Analytical and numerical solutions for the nonlinear Burgers and advection-diffusion equations by using a semi-analytical iterative method. *Computers & Mathematics with Applications*, 76(1), 155-171.
- [17] Khuri.S. (2001). A laplace decomposition algorithm applied to a class of nonlinear differential equations. *Journal of Applied Mathematics*, 1(4):141-155.
- [18] Kiyamaz, O. (2009). An algorithm for solving initial value problems using Laplace Adomian decomposition method. *Applied Mathematical Sciences*, 3(30), 1453-1459.
- [19] Fadaei, J. (2011). Application of Laplace-Adomian decomposition method on linear and nonlinear system of PDEs. *Applied Mathematical Sciences*, 5(27), 1307-1315.
- [20] Khan.M, Hussain.M , Jafari.H and Khan.Y. (2010). Application of Laplace Decomposition Method to Solve Nonlinear Coupled Partial Differential Equations, *World Applied Sciences Journal*, 9, 13-19.
- [21] Naghipour, A., & Manafian, J. (2015). Application of the Laplace Adomian decomposition and implicit methods for solving Burgers' equation. *TWMS Journal of Pure and Applied Mathematics*, 6(1), 68-77.
- [22] Adomian.G (1988). A review of the decomposition method in applied mathematics, *J. Math. Anal. Appl.*, 135, 501-544.
- [23] Wazwaz, A. M. (2005). Adomian decomposition method for a reliable treatment of the Emden-Fowler equation. *Applied Mathematics and Computation*, 161(2), 543-560.
- [24] Wazwaz, A. M. (2005). Adomian decomposition method for a reliable treatment of the Bratu-type equations. *Applied Mathematics and Computation*, 166(3), 652-663
- [25] Daftardar-Gejji, V., & Jafari, H. (2007). Solving a multi-order fractional differential equation using Adomian decomposition. *Applied Mathematics and Computation*, 189(1), 541-548.
- [26] Jafari, H., & Daftardar-Gejji, V. (2006). Positive solutions of nonlinear fractional boundary value problems using Adomian decomposition method. *Applied Mathematics and Computation*, 180(2), 700-706.
- [27] Jafari, H., & Daftardar-Gejji, V. (2006). Solving a system of nonlinear fractional differential equations using Adomian decomposition. *Journal of Computational and Applied Mathematics*, 196(2), 644-651.
- [28] Jafari, H., & Daftardar-Gejji, V. (2006). Solving linear and nonlinear fractional diffusion and wave equations by Adomian decomposition. *Applied Mathematics and Computation*, 180(2), 488-497.
- [29] Daftardar-Gejji, V., & Jafari, H. (2006). An iterative method for solving nonlinear functional equations. *Journal of mathematical analysis and applications*, 316(2), 753-763.
- [30] Daftardar-Gejji, V., & Bhalekar, S. (2010). Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method. *Computers & Mathematics with Applications*, 59(5), 1801-1809.

- [31] Bhalekar, S., & Daftardar-Gejji, V. (2008). New iterative method: application to partial differential equations. *Applied Mathematics and Computation*, 203(2), 778-783.
- [32] Bhalekar, S., & Daftardar-Gejji, V. (2010). Solving evolution equations using a new iterative method. *Numerical Methods for Partial Differential Equations: An International Journal*, 26(4), 906-916.
- [33] Hemeda, A. (2013). New iterative method: an application for solving fractional physical differential equations. Paper presented at the Abstract and applied analysis.
- [34] Keskin, Y., & Oturanc, G. (2009). Reduced differential transform method for partial differential equations. *International Journal of Nonlinear Sciences and Numerical Simulation*, 10(6), 741-750.
- [35] Al-Amr, M. O. (2014). New applications of reduced differential transform method. *Alexandria Engineering Journal*, 53(1), 243-247.
- [36] Jafari, H., Jassim, H. K., Moshokoa, S. P., Ariyan, V. M., & Tchier, F. (2016). Reduced differential transform method for partial differential equations within local fractional derivative operators. *Advances in Mechanical Engineering*, 8(4), 1687814016633013.
- [37] Rawashdeh, M., & Obeidat, N. A. (2014). On finding exact and approximate solutions to some PDEs using the reduced differential transform method. *Applied Mathematics & Information Sciences*, 8(5), 21
- [38] Mahgoub, M. M. A., & Sedeeg, A. K. H. (2016). On the solution of Newell-Whitehead-Segel equation. *American Journal of Mathematical and Computer Modelling*, 1(1), 21-24.