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Long-ranged Interaction Forces and Real Spaces Related to Them Including Anisotropic Cases

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Abstract This paper is aimed to find a connection between i-dimensional spaces (**i**=0,…, '**n**') and the long-range **j**-dimensional attractive forces (**j**=0,…, **'m'**) creating these spaces. The connection is fundamental and unrelated to any processes going in the spaces being studied. A theorem is formulated and strictly proved showing in which cases the long-ranged attractive forces can form real spaces of different dimensions (**i**=0,…,**n**). The existence of the attraction between masses is defined by divergence of the vector of interaction between masses. Weak anisotropic real spaces are studied by rotating an ellipsoid for **(3 ± ζ)D**-cases when its eccentricity ε<<1. Such spaces cannot be in equilibrium, the time of their existence is substantially limited. The greater is anisotropy, the shorter is the lifetime of such substance. The latter cannot be in equilibrium, the time of their existence is substantially limited.

Keywords Attractive Forces, Spaces of Different Dimensions, Real Spaces, Attraction between Masses, Divergence

1. Introduction

As is well known from affine geometry (see, e.g., [1]), there are spaces with the admissible systems of orthogonal coordinates having a common origin, an identical unit volume and orientation. Such is our real isotropic 3d-space. Why? Because our real space could be created only owing to long-ranged attractive forces, e.g., by the forces of gravitation. An empty space, i.e., the space without any matter, can have any dimension – from zero up to '**n**'. The mathematical space is empty.

The main goal of this article is to find connection between the real spaces of different dimensions **i** = 0,…,n and the long-range attractive forces that create these spaces and have its dimension **j** = 0,…,m. By the dimension of long-range attraction forces F is meant a value of the exponent j in the denominator of the formula $F = km_1m_2/r^j$, where m_1 and m_2 are the interacting masses (kg); k is a coefficient; r the distance between these masses (m); **j**(j)=1,2,3,…,**m**(m).

Using the result obtained for the real spaces with an integer dimension, the author is studying two specific cases connected with weakly anisotropic real spaces whose dimension differs from the integer number by a very small quantity.

Our problem should not be confused with problem that P. Ehrenfest was solving 100 years ago (see, e.g., [2 – 5]). He made attempt to link the dimension of space with fundamental laws of physics but he did not concern the problems connected with the creature of spaces under influence of long-ranged interaction forces of a definite kind.

We should note the following as well. This article is only the first step in studying the spaces of any dimensions. The time will come, and the results obtained in this article will be united with results which were obtained by other researchers, e.g., by A. G. Horvath [6].

2. Real Spaces and Forces Creating them

Call any space containing matter a real space. Any real space has to contain sources of long-range interactions, viz. an attraction between masses in our case. The existence of these sources is defined by a divergence of the vector **a** of interaction between masses. For example, for our **3-D** real space the divergence of **a** will have the following form if **a** is only a function of the coordinate ρ (in a spherical coordinate system):

$$div_3\mathbf{a} = {}_{Lim}\frac{\oint_3 a_\rho n_\rho dS}{V}\,,\ V=\frac{4}{3}\pi r^3,\ \ S = 4\pi r^2,\ V \to \mathrm{o}\,, \quad (1)$$

$$\mathbf{F} = -km_1m_2\mathbf{r}/r^3;\ \mathbf{a} = \mathbf{E} = (\mathbf{F}/m_2) = -\mathbf{r}\frac{km_1}{r^3}; \quad (2)$$

where index "3" in (1) indicates that the above formulae refer to **3D**-space; *dS* is the surface element of a spherical surface; n_ρ the unit vector perpendicular to *dS*; *V* the volume; **F** the interaction force between masses *m* 1 and 2; *k* the constant of gravitation (kg$^{-1}\cdot$ m$^{3}\cdot$ s^{-2}); **E** the vector of gravitation field intensity (m $\cdot$ s^{-2}), as $m_2=1$

If $V\to 0$, then we can write down (1) as

$$div_3\mathbf{a}=-\oint_3 n_\rho \frac{k}{r^2}\frac{\delta m_1}{\delta V}dS\,. \tag{3}$$

Since $dS=\rho^2\sin\theta d\theta d\varphi$ in spherical coordinates (ρ,θ,φ), we have after integration (3):

$$div_3\mathbf{a}=-4\pi k n_\rho \frac{\delta m_1}{\delta V}\,. \tag{4}$$

If we use another formula instead of $\mathbf{F}=-km_1m_2\mathbf{r}/r^3$, e.g.,

$$\mathbf{F}=-km_1m_2\mathbf{r}/r^2, \tag{5}$$

then we shall have

$$div_3\mathbf{a}=-4\pi k r n_\rho \frac{\delta m_1}{\delta V}\,, \tag{6}$$

which means that $div_3\mathbf{a}$ depends on "*r*", and, in turn, the law of energy conservation is broken. Indeed, if $V\to 0$ (see (1)), then $r\to 0$ and $div_3\mathbf{a}=0$. This means that the gravitation source at this point of space is unobserved. An analogous picture takes place at other points of our space. In fact, it means as well that there is no real space, since the interaction (5) cannot maintain its existence.

Now take such a law instead of (5):

$$\mathbf{F}=-km_1m_2\mathbf{r}/r^4, \tag{7}$$

then instead (6) we have

$$div_3\mathbf{a}=-4\pi k r^{-1} n_\rho \frac{\delta m_1}{\delta V}\,, \tag{8}$$

and $div_3\mathbf{a}\to\infty$ if *V* and, consequently, *r* tends to zero. Hence, our space collapses into a point, i.e., we obtain a black hole.

Here we should make an important remark. As seen, we use the sphere of dimension 3, which means that we study an isotropic space. If the space investigated had a fractional dimension, e.g., 2.9, then we had to take for our investigations not a sphere but an ellipsoid. Consequently, we cannot use an expression for the element of the ellipsoid surface as $ds=\rho^2\sin\theta d\theta d\varphi$, since in this case ρ should be a function of the angles φ and θ.

Now we can put a question: how will things be going for spaces of other dimensions – from zero to **n**? To answer this question, first of all write down expressions for volumes and surfaces of different ranks. We begin to study spaces whose dimensions $\mathbf{i}\le 3$, i.e., 0,1,2.

If **i**=2, then we consider a circumference and a space inside it (we call this space a flat sphere). Therefore we have:

$$div_2\mathbf{a}=\lim_{S\to 0}\frac{\oint_2 a_\rho n_\rho dL}{S},\ S=\pi r^2,\quad L=2\pi r, \tag{9}$$

$$\mathbf{F}=-km_1m_2\mathbf{r}/r^2;\ \mathbf{a}=\mathbf{E}=(\mathbf{F}/m_2)=-\mathbf{r}\frac{km_1}{r^2}; \tag{10}$$

here the coefficient *k* in kg$^{-1}\cdot$ m$^{2}\cdot$ s^{-2} units.

If $S\to 0$, then we can write down (9) as

$$div_2\mathbf{a}=-\oint_2 n_\rho \frac{k}{r}\frac{\delta m_1}{\delta S}dL\,. \tag{11}$$

Since $dL=\rho d\varphi$ in polar coordinates (ρ,φ), we have after integration in (11):

$$div_2\mathbf{a}=-2\pi k n_\rho \frac{\delta m_1}{\delta S}\,. \tag{12}$$

If we have another formula instead of $\mathbf{F}=-km_1m_2\mathbf{r}/r^2$, e.g.,

$$\mathbf{F}=-km_1m_2\mathbf{n}_\rho, \tag{13}$$

then we shall have

$$div_2\mathbf{a}=-2\pi k r n_\rho \frac{\delta m_1}{\delta S}\,. \tag{14}$$

Hence, $div_2\mathbf{a}$ depends on "*r*", it means, in turn, that the law of energy conservation is broken. Indeed, if $S\to 0$ (see (9)), then $r\to 0$ and $div_2\mathbf{a}=0$. Thus, the gravitation source at this point of space is unobserved. An analogous picture takes place at other points of this space.

In fact, it means as well that there is no real **2D**-space, since the interaction (13) cannot maintain its existence.

Now take such a law instead of (10):

$$\mathbf{F}=km_1m_2\mathbf{r}/r^3, \tag{15}$$

then instead of (14) we have

$$\mathrm{div}_2\mathrm{a}=-2\pi k r^{-1} n_\rho \frac{\delta m_1}{\delta S}\,, \tag{16}$$

and $div_3\mathbf{a}\to\infty$, if $r\to 0$, i.e., the flat sphere collapses into point and we have black hole but in **2D**-space.

Below the Greek letters φ and θ will be replaced by the Greek letter ζ_m with index *m*=1,2,… , since we shall study spaces with $\mathbf{i}\ge 4$.

If **i** =1, a straight-line segment will be as if an analogue of the above flat sphere and a pair of points will be as if analogue of the above circumference bounding the above flat one. In this case we have:

$$div_1\boldsymbol{a}=-2a\,, \tag{17}$$

$$F = km_1 m_2, \quad a = E \tag{18}$$

here the coefficient k is in $kg^{-1} \cdot m \cdot s^{-2}$ units.

At last, if **i**=0, then the space is a point here and its $div_0\, a = \frac{0}{0}$, i.e. we have uncertainty.

Now we shall study the spaces having the dimensions from **i**=4 up to **i** =**n.**

If **i**=4, then we have [7]:

$$div_4\mathbf{A}_{(4)} = Lim \frac{\oint_4 a_\rho n_\rho d\Lambda_{(4)}}{\Omega_{(4)}}, \quad \Omega_{(4)} = \frac{1}{2}\pi^2 R_{(4)}^4;$$

$$\Lambda_{(4)} = 2\pi^2 R_{(4)}^3 \tag{19}$$

$$\Omega_{(4)} \to o$$

$$\mathbf{F}_{(4)} = -km_1m_2\mathbf{R}_{(4)} / R_{(4)}^4;$$

$$\mathbf{A}_{(4)} = \mathbf{E}_{(4)} = (\mathbf{F}_{(4)}/m_2) = \mathbf{R}_{(4)} \frac{km_1}{R_{(4)}^4}. \tag{20}$$

where index "4" in formulae (19 – 20) indicates referring them to **4D**-space, $d\Lambda_{(4)}$ is the element of 3D-surface, n_ρ the component of the unit vector perpendicular to each point of this **3D**-surface, $\Omega_{(4)}$ the **4-D** volume, the $\mathbf{F}_{(4)}$ interaction force between masses m 1 and 2 in **4D**-space, k the constant of gravitation in **4D**-space ($kg^{-1} \cdot m^4 \cdot s^{-2}$), **E** the vector of gravitation field intensity ($m \cdot s^{-2}$), as $m_2 = 1$.

If $\Omega_{(4)} \to o$, then we can write down (19) as

$$div_4\mathbf{A}_{(4)} = -\oint_4 n_\rho \frac{k}{R^3} \frac{\delta m_1}{\delta\Omega_{(4)}} d\Lambda_{(4)}. \tag{21}$$

Since $d\Lambda_{(4)} = \rho^3 F(\zeta_1, \zeta_2, \zeta_3) d\zeta_1 d\zeta_2 d\zeta_3$ in spherical coordinates $(\rho, \zeta_1, \zeta_2, \zeta_3)$, then we have after integration (21):

$$div_4\mathbf{A}_{(4)} = -2\pi^2 k n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}. \tag{22}$$

If we have another formula instead of $\mathbf{F}_{(4)} = -km_1m_2\mathbf{R}_{(4)} / R_{(4)}^4$, e.g.,

$$\mathbf{F}_{(4)} = -km_1m_2\mathbf{R}_{(4)} / R_{(4)}^3, \tag{23}$$

then we shall have

$$div_4\mathbf{A}_{(4)} = -2\pi^2 k R_{(4)} n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}. \tag{24}$$

Hence, $div_4\mathbf{A}_{(4)}$ depends on "R", it means, in turn, that the law of energy conservation is broken. Indeed, if $\Omega_{(4)} \to 0$ (see (19), then $R \to o$ and $div_4\mathbf{A}_{(4)} = 0$. The analogous picture takes place at other points of our space.

In fact, it means as well that there is no real space, since the interaction (23) cannot maintain its existence.

Now take such a law instead of (23):

$$\mathbf{F}_{(4)} = -km_1m_2\mathbf{R}_{(4)} / R_{(4)}^5, \tag{25}$$

then instead of (24) we have

$$div_4\mathbf{A}_{(4)} = -2\pi^2 k R_{(4)}^{-1} n_\rho \frac{\delta m_1}{\delta\Omega_{(4)}}, \tag{26}$$

and $div_4\mathbf{A}_{(4)} \to \infty$ if $\Omega_{(4)}$ and, consequently, $\mathbf{R}_{(4)}$ tends to zero. It means that our space collapses into a point, i.e., we obtain a black hole.

In principle, we get a similar picture for the cases **i**=5 – **n.** Show it for the case **i=n.**

$$\text{divnA(n)=Lim} \frac{\oint_n a_\rho n_\rho d\Lambda_{(n-1)}}{\Omega_{(n)}}; \Omega_{(n)} = C_{(n)}\rho^n.$$

$$\Lambda_{(n-1)} = nC_{(n)}\rho^{n-1}, \quad C_{(n)} = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}, \tag{27}$$

$$\Omega_{(n)} \to 0$$

$$\mathbf{F}_{(n)} = -km_1m_2\mathbf{R}_{(n)} / R_{(n)}^n;$$

$$\mathbf{A}_{(4)} = \mathbf{E}_{(4)} = (\mathbf{F}_{(4)}/\boldsymbol{m}_2) = \mathbf{R}_{(4)} \frac{km_1}{R_{(4)}^4}. \tag{28}$$

where index "n" in formulae (27 – 28) indicates referring them to **n**D-space,

$d\Lambda_{(n-1)}$ is the element of **nD**-surface, n_ρ the component of the unit vector perpendicular to each point of this **(n – 1)D**-surface, $\Omega_{(n)}$ the **nD**-volume, $\mathbf{F}_{(n)}$ the interaction force between of masses m_1 and m_2 in **nD**-space, k the constant of gravitation in **nD**-space ($kg^{-1} \cdot m^n \cdot s^{-2}$), **E** the vector of gravitation field intensity ($m \cdot s^{-2}$) at m_2=1 the $\Gamma\left(\frac{n}{2}+1\right)$ gamma function.

If $\Omega_{(n)} \to o$, then we can write down (27) as

$$div_n\mathbf{A}_{(n)} = -\oint_n n_\rho \frac{k}{R^{n-1}} \frac{\delta m_1}{\delta\Omega_{(n)}} d\Lambda_{(n)}\,. \qquad (29)$$

Since

$$d\Lambda_{(n)} = \rho^{n-1}F(\zeta_1,\zeta_2,\zeta_3,...,\zeta_{n-1})d\zeta_1 d\zeta_2 d\zeta_3,...,d\zeta_{n-1}$$

in spherical coordinates $(\rho,\zeta_1,\zeta_2,\zeta_3,...,\zeta_{n-1})$, we have after integration (29):

$$div_n\mathbf{A}_{(n)} = -kn_\rho \frac{n\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)} \frac{\delta m_1}{\delta\Omega_{(n)}}\,. \qquad (30)$$

If we have another formula instead of $F_{(n)}=km_1m_2/R^{n-1}$, e.g.,

$$\mathbf{F}_{(n)} = -km_1m_2\mathbf{R}_{(n)}/R_{(n)}^{n-1}\,, \qquad (31)$$

then we shall have

$$div_n\mathbf{A}_{(n)} = -kn_\rho \frac{nR_{(n)}\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)} \frac{\delta m_1}{\delta\Omega_{(n)}}\,. \qquad (32)$$

Hence $div_n\mathbf{A}_{(n)}$ depends on "$R_{(n)}$", it means, in turn, that the law of energy conservation is broken. Indeed, if $\Omega_{(n)} \to 0$ (see (27), then $R_{(n)} \to 0$ and $div_n\mathbf{A}_{(n)} = 0$, which means that the gravitation source at this point of space is not observed. An analogous picture takes place at other points of this space.

In fact, it means as well that there is no real space, since the interaction (31) cannot maintain its existence.

Now take such a law instead of (31):

$$\mathbf{F}_{(n)} = -km_1m_2\mathbf{R}_{(n)}/R_{(n)}^{n+1}\,, \qquad (33)$$

then instead of (24) we have

$$div_n\mathbf{A}_{(n)} = -kn_\rho \frac{nR_{(4)}^{-1}\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)} \frac{\delta m_1}{\delta\Omega_{(n)}}\,, \qquad (34)$$

and $div_n\mathbf{A}_{(n)} \to \infty$ if $\Omega_{(n)}$ and, consequently, $R_{(n)}$ tends to zero. It means that our space collapses into a point, i.e., we obtain a black hole.

Now we can assume that vacuum is a **nD**-space where the interaction law between masses has the rank **n**+1. There are fluctuations of the number **n**+1 in the interaction law and the rank of the interaction may become less than the space dimension. As a result, there occurs the Big Bang.

There occurs an ejection of substance to an empty space after the Big Bang, and the matter begins to convert a mathematical space into a space of dimension **n**. The number **n** depends on the law of mass interactions for this matter. If it is the law (2), then we obtain our three dimensional space.

3. The Formulation of the Theorem on Spaces and Forces

We can generalize the above-mentioned to a theorem, viz., the long-raged interaction forces of dimensions i =0,1, 2,…,n can form real isotropic Euclidean spaces if and only if, when the dimensions j of these spaces equal j=i+1. Then we can affirm, using the method of mathematical induction, that the long-raged interaction forces of the dimensions i =n+1 can form a real isotropic Euclidean space of the rank j=i+1=n+2.

This is a theorem, which we call "the theorem on spaces and long-ranged interaction forces forming the former" or, more shortly, "the theorem on spaces and forces forming them".

Based on this theorem we can suppose that real spaces, where the theorem is valid, can be in equilibrium. In case this theorem does not hold the spaces fail to exist, at any rate collapsing into a black hole or an empty space. In this connection it will be interesting to study weak anisotropy $(3\pm\zeta)$**D**-spaces, $\varepsilon \ll 1$being oblate and prolate ellipsoids where ε is an eccentricity of ellipse of revolution.

4. The Case of the Weak Isotropic (3–Z)D-Space, Z=Z(Φ)<<1; Oblate Ellipsoid with the Eccentricity E<<1

To study the weak anisotropic space, we use the expression (3) having transformed it as follows:

$$div_{3-\zeta}\mathbf{a} = Lim \frac{\oint_{3-\xi} a_\rho n_\rho dS}{V}\,,\ V \to 0\,, \qquad (35)$$

where the quantity **ζ** depends on an angle **φ** between the radius of the spheroid ***ρ*** and its major axis, i.e., instead of the sphere limiting **3-D** isotropic Euclidean space we use an oblate ellipsoid limiting a weakly anisotropic space of fractional dimension. Then the expression ***dS*** for the element of the ellipsoid surface will significantly differ from the analogous expression for the sphere surface element. In the latter case, this element is representable as a differential of the ellipse arc dl_e multiplied by a differential of the circle arc dl_c formed by rotation of one or another point on the ellipse arc around its major axis. Then we have:

$$dS = dl_e dl_c = \sqrt{(dr)^2 + r^2(d\varphi)^2}\, r\sin\varphi d\theta = r^2\sin\varphi\sqrt{\left(\frac{dr}{rd\varphi}\right)^2 + 1}\, d\varphi d\theta\,, \qquad (36)$$

$$r = \rho = \frac{b}{(1-\varepsilon^2\cos^2\varphi)^{\frac{1}{2}}}\,, \qquad (37)$$

$$r^2 = \rho^2 = \frac{b^2}{1-\varepsilon^2\cos^2\varphi}; \tag{38}$$

where (37) is the equation of ellipse in polar coordinates, when its origin is at the center of the ellipse, $r=\rho$ is the radius of the ellipse, b its minor axis, ε the eccentricity of the ellipse, φ the angle between the radius r and the axis **X** in Cartesian coordinates aligned with major axis of coordinates around which it rotates, θ the angle of the ellipse rotation around of axis **X**.

Taking into account of (37), we obtain from (36):

$$dS = \frac{b^2\sin\varphi}{1-\varepsilon^2\cos^2\varphi}\sqrt{\left(-\frac{\varepsilon^2\cos\varphi\sin\varphi}{1-\varepsilon^2\cos^2\varphi}\right)^2+1}d\varphi d\theta$$
$$= \frac{b^2}{1-\varepsilon^2\cos^2\varphi}\sqrt{\frac{\varepsilon^4\cos^2\varphi\sin^2\varphi+1-2\varepsilon^2\cos^2\varphi+\varepsilon^4\cos^4\varphi}{\left(1-\varepsilon^2\cos^2\varphi\right)^2}}d\varphi d\theta \approx$$
$$\frac{b^2\sin\varphi}{(1-\varepsilon^2\cos^2\varphi)^2}\sqrt{1-2\varepsilon^2\cos^2\varphi}d\varphi d\theta \approx \frac{b^2\sin\varphi}{\sqrt{1-2\varepsilon^2\cos^2\varphi}}d\varphi d\theta. \tag{39}$$

The final part of the expression (39) was obtained providing $\varepsilon^4\cos^2\varphi\sin^2\varphi;\varepsilon^4\cos^4\varphi << 1,$ if the eccentricity ε is small enough, i.e., much less than unity. If the quantity ε=0, then we obtain an expression for the element of a spherical surface. The integration over the angle φ here is clockwise, since this angle is also read in the same way.

Now we should obtain an expression for the interaction of masses in the anisotropic space studied (in the oblate ellipsoid), when the ellipse has the eccentricity ε. With this aim we should return to the expression (2). We write down it for the studied case as

$$\mathbf{F} = -km_1m_2\mathbf{r}/r^{3-\zeta};\ \mathbf{a} = \mathbf{E} = (\mathbf{F}/m_2) = -\mathbf{r}\frac{km_1}{r^{3-\zeta}}, \tag{40}$$

Where

$$\zeta = \varepsilon(1-\cos 2\varphi), \tag{41}$$

i.e., we have here: $\boldsymbol{\zeta}$ **=0, if** $\boldsymbol{\varphi}$**=**0 and $\boldsymbol{\pi}$**,** и $\boldsymbol{\zeta}$ **=**2$\boldsymbol{\varepsilon}$**,** if $\varphi=\pi/2$**.**

To obtain the divergence of the quantity **a**, we use the expression (3), allowing for (38 – 41) and taking into account that the vector product **a** · **n** = $a_\rho n_\rho \cos$ **a^n** $\approx a_\rho n_\rho$ at any point of the surface of the ellipsoid because of a very small eccentricity of the figure of rotation and, hence, the cosines of the angle between these vectors being close to unity.

$$\begin{aligned}
&div_{3-\zeta}\mathbf{a}= \\
&-\oint_{3\text{-}\zeta} n_p \frac{k}{r^{2-\zeta}}\frac{\delta m_1}{\delta V}dS = -\oint_{3\text{-}\zeta} n_p \frac{kr^{\zeta}}{r^2}\frac{\delta m_1}{\delta V}\frac{b^2\sin\varphi}{\sqrt{1-2\varepsilon^2\cos^2\varphi}}d\varphi d\theta = \\
&-\oint_{3\text{-}\zeta} n_p kr^{\zeta}\frac{\delta m_1}{\delta V}\frac{1-\varepsilon^2\cos^2\varphi}{\sqrt{1-2\varepsilon^2\cos^2\varphi}}\sin\varphi d\varphi d\theta \approx \\
&-2\pi\int_0^{\pi} n_p kr^{\zeta}\frac{\delta m_1}{\delta V}\sin\varphi d\varphi = \\
&-2\pi\int_0^{\pi} n_p k\varepsilon^{-1}\frac{\delta m_1}{\delta V}r^{\varepsilon(1-\cos 2\varphi)}\varepsilon\sin\varphi d\varphi \approx 2\pi n_p k\varepsilon^{-1}\frac{\delta m_1}{\delta V}b2^{\varepsilon}\int_0^{\pi} b^{-2\varepsilon\cos^2\varphi}d\varepsilon\cos\varphi = \\
&A\int_1^2 b^{-\frac{x^2}{\varepsilon}}dx, \\
&V \to 0
\end{aligned} \tag{42}$$

where $A = \sqrt{2}\pi n_\rho k\varepsilon^{-1}\dfrac{\delta m_1}{\delta V}b^{2\varepsilon}$, $\sqrt{2}\ \varepsilon\cos\varphi = x.$

Obtaining the expression (42), we proceeded especially, from approximately equality $(1-\varepsilon^2\cos^2\varphi)\approx\sqrt{1-2\varepsilon^2\cos^2\varphi}$ since we shall obtain $(1-\varepsilon^2\cos^2\varphi)$, having added under the root the expression $\varepsilon^4\cos^4\varphi << 2\varepsilon^2\cos^2\varphi$.

The integrated expression and its Maclaurin series expansion are well known; in this specific case we have:

$$\int_1^2 b^{-\frac{x^2}{\varepsilon}}dx = \int_1^2 (1-\frac{2\ln b}{1!\varepsilon}x^2+\frac{\ln^2 b}{2!\varepsilon^2}x^4-\frac{\ln^3 b}{3!\varepsilon^3}x^6+...)dx =$$

$$x-\frac{2}{1!}\frac{\ln b}{\varepsilon}\frac{x^3}{3}+\frac{1}{2!}\frac{\ln^2 b}{\varepsilon^2}\frac{x^5}{5}-\frac{1}{3!}\frac{\ln^3 b}{\varepsilon^3}\frac{x^7}{7}+\frac{1}{4!}\frac{\ln^4 b}{\varepsilon^4}\frac{x^9}{9}-...\Big|_1^2 =$$

$$= \sqrt{2}\varepsilon\cos\varphi-\frac{4\sqrt{2}}{1!}\frac{(2.3\log b)}{\varepsilon}\frac{\varepsilon^3\cos^3\varphi}{3}+\frac{4\sqrt{2}}{2!}\frac{(2.3\log b)^2}{\varepsilon^2}\frac{\varepsilon^5\cos^5\varphi}{5}-\frac{8\sqrt{2}}{3!}\frac{(2.3\log b)^3}{\varepsilon^3}\frac{\varepsilon^7\cos^7\varphi}{7}+ \quad (43)$$

$$\frac{16\sqrt{2}}{4!}\frac{(2.3\log b)^4}{\varepsilon^4}\frac{\varepsilon^9\cos^9\varphi}{9}...\Big|_1^\pi = -2\sqrt{2}\varepsilon+\frac{4\sqrt{2}\varepsilon^2}{1!3}2.3\log b-\frac{4\sqrt{2}\varepsilon^3}{2!5}(2.3\log b)^2+\frac{8\sqrt{2}\varepsilon^4}{3!7}(2.3\log b)^3-$$

$$\frac{16\sqrt{2}\varepsilon^5}{4!9}(2.3\log b)^4-....$$

Here and below, in ln$b$ and log$b$ the quantity b is dimensionless.

As a result, we have for the divergence **a**;

$$\underset{V\to 0}{\mathrm{div}_{3-\zeta}\mathbf{a}} = 2\pi n_\rho k\frac{\delta m_1}{\delta V}b^{2\varepsilon}\left[-2+\frac{4\varepsilon^1}{1!3}2.3\log b-\frac{4\varepsilon^2}{2!5}(2.3\log b)^2+\frac{8\varepsilon^3}{3!7}(2.3\log b)^3-\frac{16\varepsilon^4}{4!9}(2.3\log b)^4...\right],. \quad (44)$$

then the expression (44) transforms in the expression (4), as $V \to 0$, all terms in the right side of (44), except the first one, are tending to infinity, since $\varepsilon = 0$, but the quantities b are only tending to zero. Absolutely another picture takes place, when the quantity ε does not tend to zero but is taken equal to 0.01. In this case, we obtain in the right side of (44) an indeterminate form, which is very difficult to evaluate, if possible at all. However, this difficulty we can overcome, if instead of $V \to 0$ another expression $V \to V_{\min}$ is used, where $V_{\min}$ is the volume of the oblate ellipsoid having the small axis $b = 10^{-6}$ m. Then the right side of (44) converges quickly, and in this volume there can be all free atoms of Mendeleev table each taken separately and the most of molecules containing these atoms, i.e., an almost complete set of matter components, whose mass defines of the existence of gravitation.

The quantity $b^{2\varepsilon} = \left(10^{-6}\right)^{0.02} = 5 \cdot 10^{-5}$ м. It is very small. In turn, the major and minor axes of the ellipse are almost equal to each other. Then the interaction (40) can form and maintain a space whose dimension is very close to **3D,** but this space cannot exist arbitrarily long. Early or late, the space transforms to an empty space with all circumstances following from that.

5. Case of Weakly Anisotropy (3+Z)D-Space, E<<1; Prolate Ellipsoid of Rotation

Replace the oblate ellipsoid in (40) by a prolate ellipsoid of revolution. Then we again obtain a new weak anisotropic space, but now it will have other configuration and properties. This space is also anisotropic, but it should be tending, in the course of time, to the black hole and not to an empty space. In this case, the dependence (35) takes the form:

$$div_{3+\zeta}\mathbf{a} = {}_{Lim} \frac{-\oint_{3+\zeta} a_\rho n_\rho dS}{V} \qquad (45)$$

$$V \to 0.$$

To obtain formulae similar to the dependences (36) – (38), but for the prolate ellipsoid of revolution, we should, in particular, turn the ellipsoid, studied in the previous section, by ninety degrees anticlockwise, after that we should rotate the upper arc of the ellipse around the horizontal axis **X** of Cartesian coordinates. Then the formula for the element of surface of this geometrical object takes form:

$$dS = dl_e dl_c = \sqrt{(dr)^2 + r^2 (d\varphi)^2}\, r\cos\varphi d\vartheta = r^2 \sin\varphi \sqrt{\left(\frac{dr}{rd\varphi}\right)^2 + 1}\, d\varphi d\vartheta, \qquad (46)$$

where the differential of the ellipse arc dl_e is again multiplied by the differential of circle arc dl_c formed by the rotation of one or another point on the arc of the ellipse around the axis **X** of the Cartesian coordinates, $d\vartheta$ the differential of the angle of the rotation arc around this axis. Then the dependences (37) and (38) are written down as:

$$r = \rho = \frac{b}{(1-\varepsilon^2 \sin^2\varphi)^{\frac{1}{2}}}, \qquad (47)$$

$$r^2 = \rho^2 = \frac{b^2}{1-\varepsilon^2 \sin^2\varphi}; \qquad (48)$$

but instead of (39) we finally have:

$$dS \approx \frac{b^2 \sin\varphi}{\sqrt{1-2\varepsilon^2 \sin^2\varphi}} d\varphi d\vartheta, \qquad (49)$$

and instead of (40)

$$\mathbf{F} = -k m_1 m_2 \mathbf{r}/\mathbf{r}^{3+\zeta};\ \mathbf{a} = \mathbf{E} = (\mathbf{F}/m_2) = -\mathbf{r}\frac{km_1}{r^{3+\zeta}}. \qquad (50)$$

Then instead of (42) we have

$$\begin{aligned} &div_{3+\zeta}\mathbf{a} = \\ &-\oint_{3+\zeta} n_p \frac{k}{r^{2+\zeta}} \frac{\delta m_1}{\delta V} dS = -\oint_{3+\zeta} n_p \frac{k}{r^{2+\zeta}} \frac{\delta m_1}{\delta V} \frac{b^2 \sin\varphi}{\sqrt{1-2\varepsilon^2\sin^2\varphi}} d\varphi d\vartheta = \\ &-\oint_{3+\zeta} n_p k r^{-\zeta} \frac{\delta m_1}{\delta V} \frac{(1-\varepsilon^2\sin^2\varphi)}{\sqrt{1-2\varepsilon^2\sin^2\varphi}} \sin\varphi d\varphi d\vartheta \approx \\ &-2\pi\int_0^\pi n_p k r^{-\zeta} \frac{\delta m_1}{\delta V} \sin\varphi d\varphi = -2\pi \int_1^2 n_p \varepsilon^{-1} k \frac{\delta m_1}{\delta V} r^{-\varepsilon(1-\cos 2\varphi)} \varepsilon \sin\varphi d\varphi = \\ &\sqrt{2}\pi n_p \varepsilon^{-1} b^{-2\varepsilon} k \frac{\delta m_1}{\delta V} \int_1^2 r^{\frac{(\sqrt{2})^2 \varepsilon^2 \cos^2\varphi}{\varepsilon}} d\sqrt{2}\varepsilon\cos\varphi = A\int_1^2 b^{\frac{x^2}{\varepsilon}} dx \\ &V \to 0 \end{aligned} \qquad (51)$$

где $A = \sqrt{2}\pi n_\rho k\varepsilon^{-1} \frac{\delta m_1}{\delta V} b^{-2\varepsilon}$, $\sqrt{2}$ $\varepsilon \cos\varphi = x$.

The main difference (51) from (42) is in the exponent of the magnitude *b* in the expression (51) having a diametrically opposite sign as compared with the exponent in (42). Consequently, $div_{3+\zeta}\mathbf{a}$ will tend to infinity as $V \to 0$. As to the integrand as a whole, so it expands without difficulties into a series which converges very quickly at ε<<1. It makes no sense to represent here a final expression using (51), since its principal difference from the expression (42) is in the sign of the exponent of the magnitude *b*, which defines the tending of *div***a** to zero or to infinity.

As in the previous case, the interaction (50) can keep a space created by it, however, not infinitely long. Finally, this space should be converted into a black hole, with the ensuing consequences.

6. Discussion

The results obtained and, first of all, the above theorem permits one to make a proposal concerning an evolution of the real spaces created by a certain cataclysm, e.g., by the Big Bang. These spaces are Euclidean or almost Euclidean ones; they can have different dimensions and be isotropic as well as anisotropic. They can exist infinitely long if the above theorem is kept or a limited time if the theorem is broken. As to curved spaces, the author of this article does not deny their possible existence but a theory of their own is required. Here it is clear that in the case of curved real spaces there should exist a connection between their dimensions and the dimensions of the long-ranged interactions created these spaces.

How will evolve the real Euclidean and almost Euclidean spaces (isotropic and anisotropic) created by the certain long-ranged interactions? Different cases, which are possible here, will be studied below.

The first case. The space is isotropic, and the theorem is valid.

In this case, the substance, having got an initial impulse because of a cataclysm, generates an expanding space, e.g., three-dimensional one**.** During the expansion of space, there occurs a transformation of an empty space in the three-dimensional one. Later on, the non-equilibrium space can come to an equilibrium state remaining there arbitrarily long.

The second case. The space is weak anisotropic, and the theorem is broken.

In this case, the space cannot be in equilibrium, it will be weakly non-equilibrium. At some instant, the expansion will be replaced by the compression owing to the long ranged forces of interaction. Further evolution depends on the law of interaction between the masses in collapsing spaces. For example, if the space is close to **3D-**space and the exponent in the law of interaction somewhat less than two, then the space will be finally converted into an empty space. If the exponent is, in contrary, somewhat greater than two, then the collapsing space will be converted into a black hole.

The space where we live, can be considered a three-dimensional Euclidean one, showing a few patches of curved space volumes containing masses of substance of higher density. Evidently, “the theorem on spaces and forces” is invalid for them.

7. Conclusions

1. Formulated and proved is a theorem according to which real Euclidean spaces can be formed by long-ranged interaction forces. These spaces are in equilibrium if and only if the integer dimensions of the space **i** and those of interacting forces **j** are connected by the relation **i = j + 1**.
2. It is shown that weakly curved anisotropic spaces cannot be in equilibrium and cannot exist arbitrarily long. Early and late, they have either to transform to an empty space or to collapse into a black hole.

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