

Effect of Thermal Radiation on a Three-dimensional Stagnation Point Region in Nanofluid under Microgravity Environment

Mohamad Hidayad Ahmad Kamal¹, Noraihan Afiqah Rawi², Mohd Rijal Ilias³,
Anati Ali¹, Sharidan Shafie^{1,*}

¹Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, Malaysia

²Universiti Kuala Lumpur Malaysian Institute of Industrial Technology, Malaysia

³Department of Mathematical Sciences, Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Malaysia

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Abstract The unsteady three dimensional boundary layer flow near a stagnation point region is studied numerically under the influence of microgravity environment. The boundary layer plate was embedded by the nanofluid with nanosized copper particles and water as a based fluid together with thermal radiation effect. The problem was mathematically formulated in term of coupled governing equations consisting of continuity, momentum and energy equations derived from the fundamental physical principles with Tiwari and Das nanofluid model. Boundary layer and Boussinesq approximation were then applied to the coupled equations and then reduced into non-dimensional equations to lessen the complexity of the problem using semi-similar transformation technique. Implicit finite different method known as Keller box method was used in this problem. The problem was then analyzed in terms of physical quantities of principal interest known as skin frictions and Nusselt number which explained the flow behavior and heat transfer analysis. From the outcome of the analysis, it was found that the parameter values for curvature ratio lead to the different cases of the stagnation point flow which is either plane stagnation flow or asymmetry stagnation flow. On the other hand, by increasing the nanoparticles volume fraction which is one of the nanofluid parameter may increase the skin frictions on both x - and y - directions. The presence of thermal radiation parameter was found to have increased the rate of change of heat transfer at the boundary layer flow.

Keywords g-Jitter, Stagnation-point Region, Nanofluid, Thermal Radiation

1. Introduction

A part of fluid dynamic discipline which is concerned with the mechanic of the fluid and the extended force on them; boundary layer flow is a slice of bigger picture which studies transportation phenomena happen to the flow such as flow behavior, heat transfer and concentration distribution. A bunch of studies has been conducted to discover and get more understanding on this research field either experimentally [1-6] or theoretically [7-11]. Hence, boundary layer concept is the most successful in achieving simplification of the equation of motion and energy and has been applied to a large variety of practical situation importantly in engineering applications such as hot rolling, skin friction drag reduction, grain storage, glass fiber and paper production [12]. In general, the boundary layer concept provides a good description of the velocity and temperature field.

Stagnation point is a part of boundary layer flow which holds important properties as proven by Bernoulli principal is having the higher pressure on the flow [13]. For a past few decades, stagnation point flow has engrossed the attention of many researchers due to its growing industry and scientific applications such as cooling of electronic devices by fan, cooling of nuclear reactor during emergency shutdown and hydrodynamic process. Hiemenz [14] was the first person who has studied the two-dimensional forward stagnation point and reduced the Navier-Stokes equation to nonlinear ordinary differential equations. Then, Mahapatra and Gupta [15] focus on the heat transfer on steady two-dimensional stagnation-point flow of an incompressible viscous fluid over a flat deformable sheet. The problem is then extended using non-Newtonian fluid by Nazar *et al.* [16] by studying

steady two-dimensional stagnation point flow of an incompressible micropolar fluid over a stretching sheet.

Generally, there are three different ways of heat transfer, namely conduction, convection and radiation. Radiation is a form of electromagnetic energy transmission and it is independent from any medium between the emitter and the receiver. In addition, the studies of radiative heat transfer in a fluid flow is currently undergoing great enlargement of many researcher. Raptis, Perdakis and Takhar [17] have done an analysis on the steady MHD asymmetric flow of an electrically conducting fluid past a semi-infinite stationary plate with presence of thermal radiation. For the case of heat and mass transfer, Pal [18] analyses two-dimensional stagnation point flow of an incompressible viscous fluid over a stretching vertical sheet in the presence of buoyancy force and thermal radiation. Next, Bhattacharyya and Layek [19] add effect of suction/blowing on steady boundary layer stagnation-point flow over a porous shrinking sheet with the presence of thermal radiation.

Nanofluid is engineered by dispersing a small quantity of nanosized particles usually less than 100nm, which are uniformly and stably suspended into a conventional fluid called base fluid such as ethylene glycol, oil, water, bio-fluids and polymer solution [20]. Choi [21] was the first person who has developed this special class of fluid by elaborated it using Maxwell equation. For the boundary layer problem, there are two famous mathematical models which are proposed by Tiwari and Das [22] and Buongiorno [23] in studying the effect of nanofluid at boundary layer. Mustafa *et al.* [24] focus on the effect of Brownian motion and thermophoresis for two-dimensional stagnation-point flow of a nanofluid towards a stretching sheet. Haq *et al.* [25] on the other hand present a model for the stagnation point flow of nanofluid with magnetohydrodynamics (MHD) and thermal radiation effects passed over a stretching sheet.

On the other hand, g-jitter effect can be defined as inertia effect due to quasi-steady, oscillatory or transient accelerations arising from crew motions and machinery vibrations in parabolic aircrafts, space shuttles or other microgravity environments. Theoretical studies on g-jitter indicated convection and temperature gradient on microgravity environment at boundary layer flow has also been conducted previously [26] and [27]. Theoretical study on the boundary layer flow induced by g-jitter has been conducted by Sharidan *et al.* [28] by considering g-jitter in a three-dimensional stagnation point flow problem with free convection flow. As for the nanofluid with g-jitter effect, Rawi *et al.* [29] studies the unsteady two-dimensional convective boundary layer flow of nanofluids past a vertical permeable stretching sheet associated with the effect of g-jitter.

Based on this motivation, a part of the present work focusses on the conjugate study of a flow and heat transfer with several physical effects. The nanofluid model used in

this study is proposed by Tiwari and Das [22] and effects such as g-jitter and thermal radiation that is considered with boundary layer flow is studied near a three-dimensional stagnation point region. The mathematical model is solved using Keller box method and the effects considered in this problem will be analyzed in terms of skin frictions and Nusselt number graphically.

2. Mathematical Formulations

Consider an incompressible viscous nanofluid which unsteadily flows at the boundary layer near a three-dimensional stagnation point region with uniform temperatures at the wall. Thermal radiation effect is considered in this boundary layer problem and the flow is conducted under microgravity environment. Copper nanoparticles and water that carries their own thermophysical properties are considered in representing the presence of nanofluid in this flow problem. The temperature for both; boundary body, T_w and ambient nanofluid, T_∞ are assume be in a uniform temperature initially. Three dimensional stationary orthogonal Cartesian coordinate (x, y, z) is chosen such that x – and y – coordinates are measured along with the surface while the z – coordinate is measured normal to the surface of origin nodal point N. For the flow cases near a stagnation point region, nodal and saddle point are the points that represent the flow behavior in which they are vice versa in term of the pattern of the curvatures. Under the microgravity environment, the gravitational field depend on time t^* defined in [30] as ,

$$g^*(t^*) = g_0 \left[1 + \varepsilon \cos(\pi \omega t^*) \right] \quad (1)$$

where the gravitational field represents in term of mean gravitational acceleration g_0 , ε is the amplitude of the gravitational modulation and ω is the frequency of oscillation driven from the g-jitter effect. From the fundamental of physical principle, the nature of the fluid flow near the boundary layer can be presented mathematically in terms of coupled equations. Under the boundary and Boussinesq approximations, the coupled partial differential equations that represent the flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

$$\rho_{nf} \left(\frac{\partial u}{\partial t^*} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \mu_{nf} \frac{\partial^2 u}{\partial z^2} + g^*(t^*) (\rho\beta)_{nf} \alpha x (T - T_\infty), \quad (3)$$

$$\rho_{nf} \left(\frac{\partial v}{\partial t^*} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \tag{4}$$

$$\mu_{nf} \frac{\partial^2 v}{\partial z^2} + g^*(t^*)(\rho\beta)_{nf} by(T - T_\infty),$$

$$\frac{\partial T}{\partial t^*} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} =$$

$$\alpha_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial q_r}{\partial z}. \tag{5}$$

The appropriate initial and boundary conditions are given by

$$\begin{aligned} t^* < 0 : u = v = w = 0, T = T_\infty \\ \text{for any } x, y \text{ and } z, \\ t^* \geq 0 : u = v = w = 0, T = T_w \\ \text{on } z = 0, x \geq 0, y \geq 0, \\ u = v = w = 0, T = T_\infty \\ \text{as } z \rightarrow \infty, x \geq 0, y \geq 0. \end{aligned} \tag{6}$$

where u, v and w are the velocity components along the directions x, y, z axes and T is the temperature of the nanofluid. The nanofluid model used in this problem is initially proposed by Tiwari and Das [22] on 2016 which focuses on the nanoparticles volume fraction. Here ρ_{nf} and μ_{nf} are the density and dynamic viscosity while β_{nf}, α_{nf} and $(C_p)_{nf}$ are the thermal expansion, thermal diffusivity and specific heat at constant pressure carried out by the chosen nanofluid. Parameter a and b present the principle curvature happening at the nodal point corresponding to the flow near the stagnation point region measured at the plane $x = 0$ and $y = 0$. Both a and b are to be assumed a positive values in which lead to the nodal point by holding the properties $|a| \geq |b|$ with $a > 0$ without loss the generality. It corresponds to the characteristic carried by the principle curvature at the nodal point. Introduced parameter c such that $c = b/a$ and c is the curvature ratio, with range 0 to 1. Conversely, the negative value of parameters a or b is able to occur but it is at the saddle point in which most of the practical does not lies on it. Due to that, the analysis on the stagnation point region will focus on the nodal point in which the geometry of the cylinder ($c = 0$) is included and sphere ($c = 1$) lies on it. As for the thermal radiation parameter, q_r is a nonlinear radiative

heat flux and according to the Rosseland approximation from Sheikholeslami *et al.* [31] for radiation, it can be simplified as,

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial z}, \tag{7}$$

where σ^* is Stefan-Boltzman constant and k^* is the mean absorption coefficient. By using Taylor's series expansion and neglecting the higher order, the temperature difference within the flow T^4 about T_∞ can be expressed as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{8}$$

Substituting equation (8) into equation (7), then equation (5) becomes

$$\begin{aligned} \frac{\partial T}{\partial t^*} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \\ \alpha_{nf} \frac{\partial^2 T}{\partial z^2} + \frac{1}{(\rho C_p)_{nf}} \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial z^2}. \end{aligned} \tag{9}$$

Since water is used as the based fluid, which is a type of viscous Newtonian fluid, the expression of thermophysical characteristic of the nanofluid derived from Maxwell equation defined in Rawi *et al.* [29] in term of nanoparticles and the based fluid are

$$\begin{aligned} \mu_{nf} &= \frac{\mu_f}{(1-\phi)^{2.5}}, \\ \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}}, \\ \rho_{nf} &= (1-\phi)\rho_f + \phi\rho_s, \\ (\rho\beta)_{nf} &= (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s, \\ (\rho C_p)_{nf} &= (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \end{aligned} \tag{10}$$

where ϕ is an important parameter which represents nanofluid properties stressed out by Tiwari and Das nanofluid model known as nanoparticles volume fraction and k hold the thermal conductivity parameter. The subscript f and s represent the fluid and solid characteristics carried by the nanofluid chosen in this research and the thermophysical properties are shown in Table 1.

Table 1. Thermophysical of nanoparticles and based fluid

Thermophysical Properties	Water	Copper
ρ (kgm ⁻³)	997.1	8933
C_p (Jkg ⁻¹ K ⁻¹)	4179	385
k (Wm ⁻¹ K ⁻¹)	0.613	401
$\beta \times 10^{-5}$ (K ⁻¹)	5.21	1.67

Equations (1)-(3) and (9) are then undertake semi similar transformation technique together with the initial and boundary conditions equation (6) to reduce the complexity of the problem with the following semi-similar solution

$$\begin{aligned}
 \tau &= \Omega t, \\
 \eta &= Gr^{1/4} \alpha z, \\
 t &= \nu \alpha^2 Gr^{1/2} t^*, \\
 u &= \nu \alpha^2 x Gr^{1/2} f'(t, \eta), \\
 v &= \nu \alpha^2 y Gr^{1/2} h'(t, \eta), \\
 w &= -\nu \alpha Gr^{1/4} (f + h), \\
 \theta(t, \eta) &= \frac{(T - T_\infty)}{(T_w - T_\infty)}, \\
 \Omega &= \frac{\omega}{\nu \alpha^2 Gr^{1/2}}, \\
 g(t^*) &= \frac{g^*(t^*)}{g_0}
 \end{aligned}
 \tag{11}$$

where ν is the kinematic viscosity of the base fluid and Grashof number is denoted by Gr and defined as $Gr = g_0 \beta (T_w - T_\infty) / (\alpha^3 \nu^2)$. The prime at the function f and h indicated the partial differential equation with respect to η . Here θ and Ω are the dimensionless variable for temperature and frequency of oscillation. The partial differential equations on the equation (1) – (3) and (9) are now become

$$\begin{aligned}
 C_1 f'' + C_2 [(f + h)f'' - f'^2] + \\
 C_3 [1 + \varepsilon \cos(\pi\tau)] \theta = C_2 \Omega \frac{\partial f'}{\partial \tau},
 \end{aligned}
 \tag{12}$$

$$\begin{aligned}
 C_1 h'' + C_2 [(f + h)h'' - h'^2] + \\
 C_3 c [1 + \varepsilon \cos(\pi\tau)] \theta = C_2 \Omega \frac{\partial h'}{\partial \tau},
 \end{aligned}
 \tag{13}$$

$$\frac{1}{Pr} \left[\frac{C_4 + Nr}{C_5} \right] \theta' + (f + h)\theta' = \Omega \frac{\partial \theta}{\partial \tau}, \tag{14}$$

where $Pr = \frac{C}{3k^*}$ is the Prandtl number, $Nr = \frac{16\sigma^* T_\infty^3}{3k^*}$ is the thermal radiation parameter and

$$\begin{aligned}
 C_1 &= \frac{1}{(1 - \phi)^{2.5}}, \\
 C_2 &= \left(1 - \phi + \frac{\phi \rho_s}{\rho_f} \right), \\
 C_3 &= \left(1 - \phi + \frac{\phi \rho \beta_s}{\rho \beta_f} \right), \\
 C_4 &= \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \\
 C_5 &= 1 - \phi + \frac{\phi(\rho C_p)_s}{(\rho C_p)_f}.
 \end{aligned}$$

The dimensional boundary conditions (6) become,

$$\begin{aligned}
 f(\tau, 0) = f'(\tau, 0) = 0, \\
 h(\tau, 0) = h'(\tau, 0) = 0, \\
 \theta(\tau, 0) = 1, \\
 f' \rightarrow 0? \quad h' \rightarrow 0? \quad \theta \rightarrow 0? \text{ as } \eta \rightarrow \infty
 \end{aligned}
 \tag{15}$$

The dimensional physical quantities of principal interest considered in this research are skin friction on both directions and Nusselt number defined as,

$$\begin{aligned}
 C_{fx} &= \mu_{nf} \left(\frac{\partial u}{\partial z} \right)_{z=0} / (\rho_f \nu^2 \alpha^3 x), \\
 C_{fy} &= \mu_{nf} \left(\frac{\partial v}{\partial z} \right)_{z=0} / (\rho_f \nu^2 \alpha^3 y), \\
 Nu &= \left(\alpha^{-1} k_{nf} \left(\frac{\partial T}{\partial z} \right)_{z=0} + q_r \right) / k_f (T_w - T_\infty).
 \end{aligned}$$

By using semi solution transformation technique (11), physical quantities of principal interest in terms of non-dimensional obtained as,

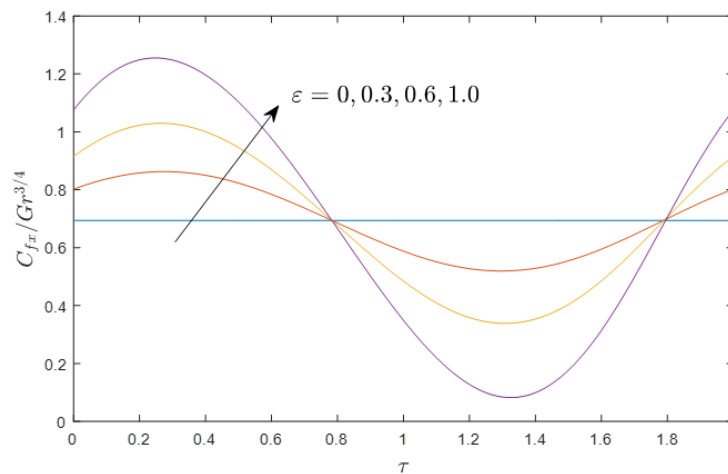
$$\begin{aligned}
 C_{fx} / Gr^{3/4} &= f''(\tau, 0) / (1 - \phi)^{2.5}, \\
 C_{fy} / Gr^{3/4} &= h''(\tau, 0) / (1 - \phi)^{2.5}, \\
 Nu / Gr^{1/4} &= (-k_{nf} / k_f + Nr) \theta'(\tau, 0).
 \end{aligned}$$

3. Results and Findings

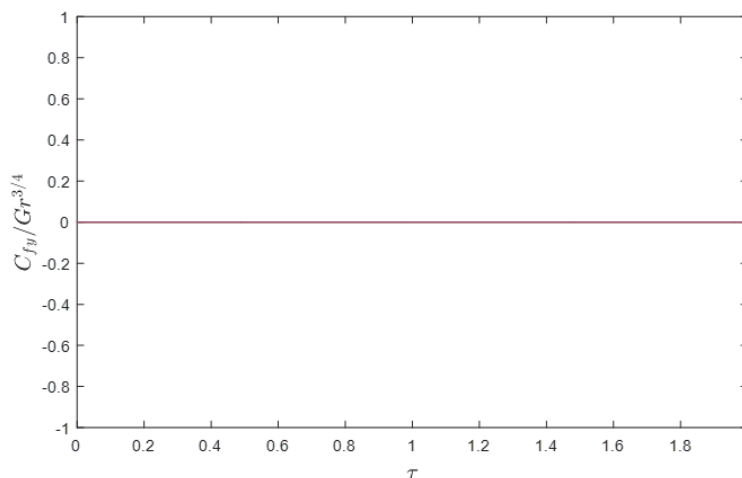
The system of non-dimensional partial differential equations (12)-(14) together with boundary equations (15) are solved numerically using implicit finite different methods known as Keller box method which is firstly developed by Keller [32]. This method is widely used in solving non-linear parabolic problem either for two-dimensional [33-34] or three- dimensional problem [35-36]. This method is chosen since it seems to be most flexible of the common methods, being easily adaptable and unconditionally stable while achieves the exceptional accuracy. All the results are obtained by using a uniform grids in both τ and η directions with $\Delta\tau = 0.1$ and $\Delta\eta = 0.04$. A solution will be considered converged when the maximum absolute point change between iteration is 10^{-10} . The flow behavior and heat transfer distributions are analyzed based on physical quantities of principal interest such as skin frictions on x – and y –directions together with Nusselt number. Parameters

involved are amplitude of modulation ε , and frequency of oscillation ω , which represent g-jitter effect, curvature ratio c , which symbolize the flow near a stagnation point region, nanoparticle volume fraction ϕ , and thermal radiation Nr . The detailed results are presented for the amplitude of the gravity modulation in the range $0 < \varepsilon < 1$ with values of frequency of the single-harmonic components 0.2 and 5 as well as the values for the curvature ratio at the stagnation point c , 0, 0.5 and 1. The results presented on each Figures are marked as skin friction on x – and y –direction on Figure A and B while Figure C represent the analysis for Nusselt number.

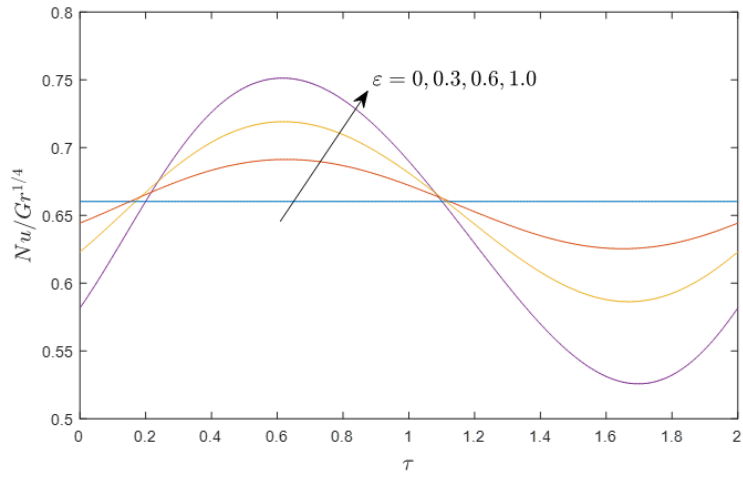
The analysis on Figures 1-3 show a same behavior in terms of skin frictions and Nusselt number when parameter ε , is analyzed. The increasing of parameter ε gives a fluctuating behavior on skin friction for both directions and Nusselt number with almost proportional increases and decreases. It shows that there exists critical point on the flow due to the fluctuating behavior and singularity in nanofluid flow.



A

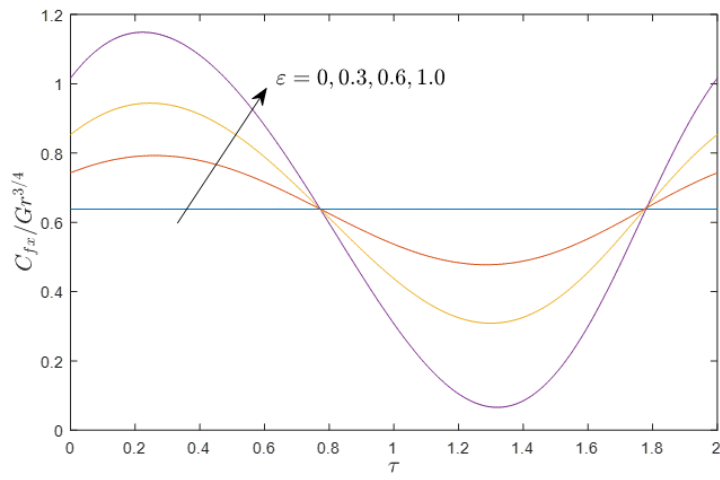


B

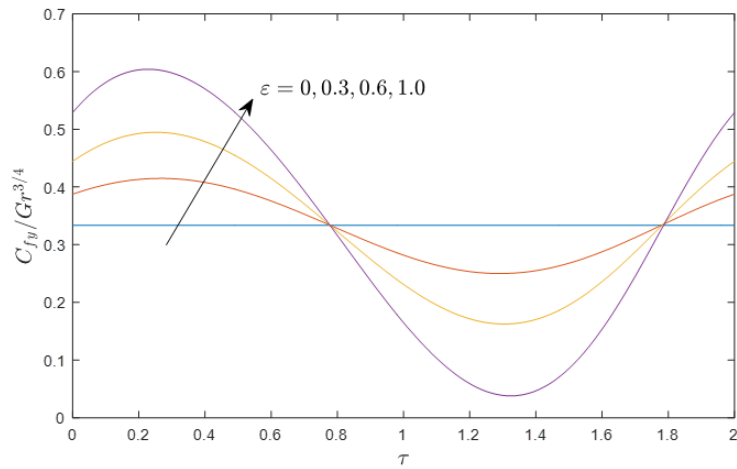


C

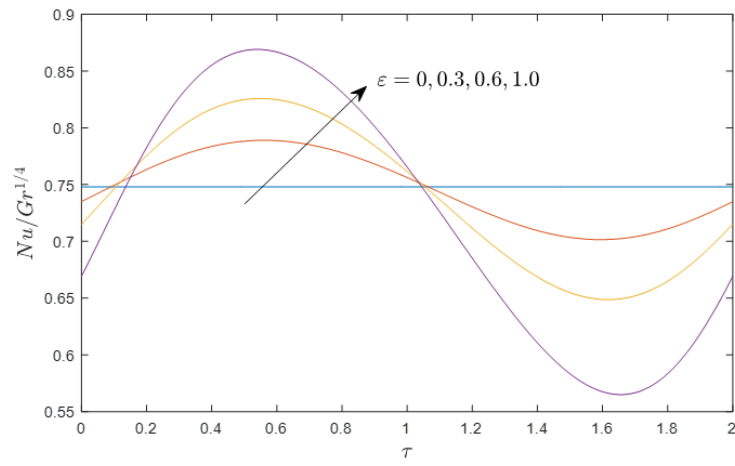
Figure 1. Skin Frictions on A and B while Nusselt number on C for $c = 0$, $\phi = 0.05$, $\Omega=0.2$ and $Nr = 1$ with different values of ϵ



A

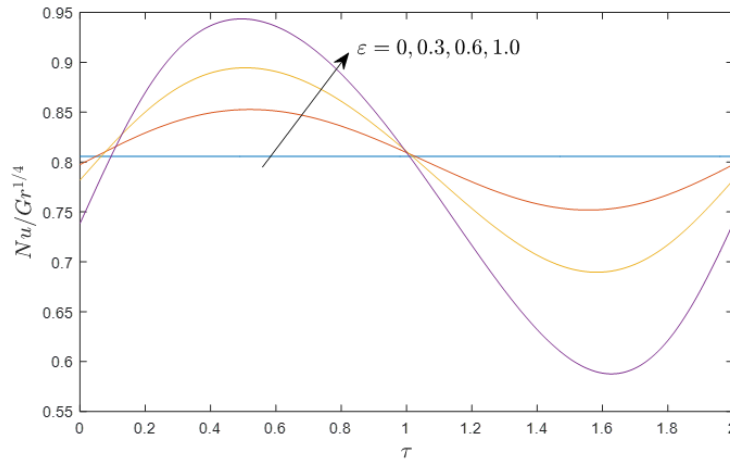


B

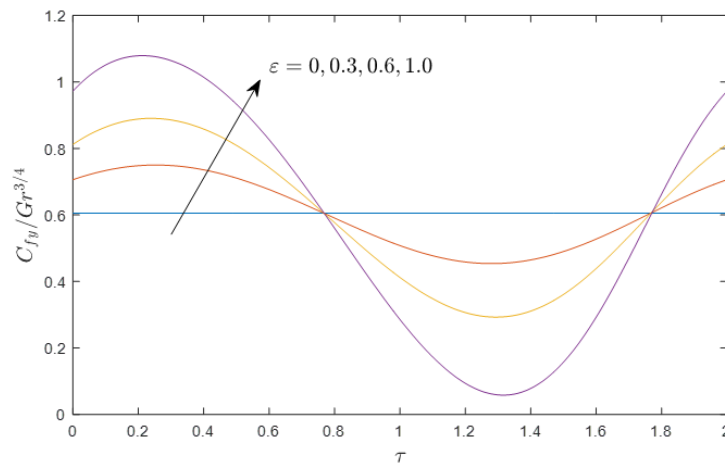


C

Figure 2. Skin Frictions on A and B while Nusselt number on C for $c = 0.5$, $\phi = 0.05$, $\Omega=0.2$ and $Nr = 1$ with different values of ϵ



A



B

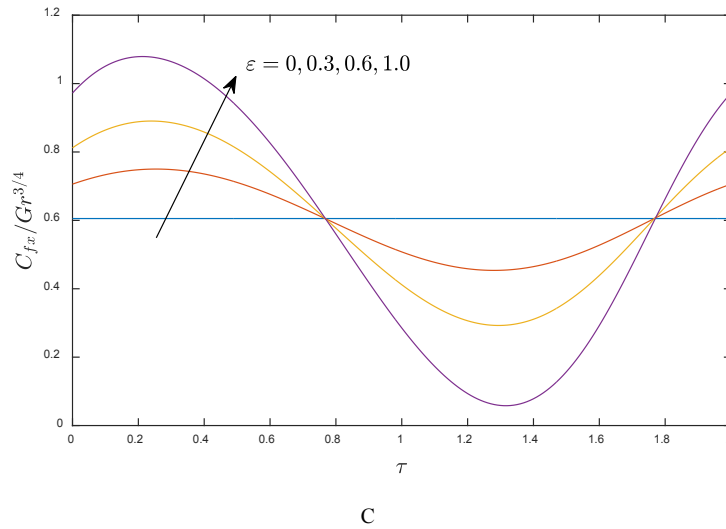
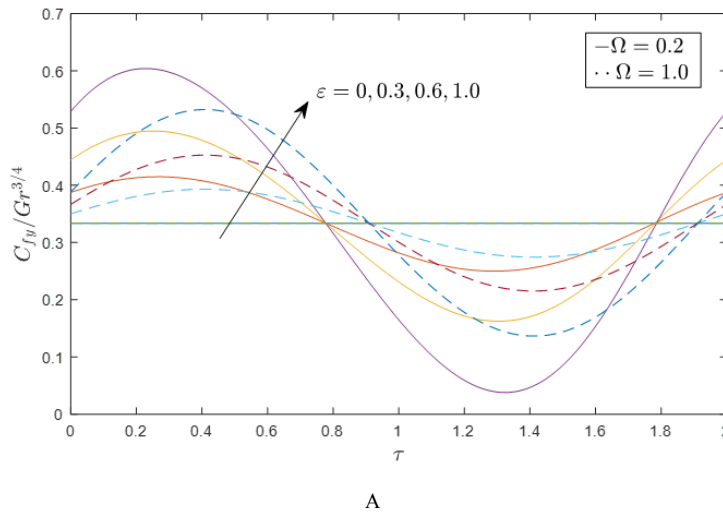


Figure 3. Skin Frictions on A and B while Nusselt number on C for $c = 1.0$, $\phi = 0.05$, $\Omega = 0.2$ and $Nr = 1$ with different values of ε

On the other hand, since stagnation point parameter is presented by parameter c , the different values chosen for parameter c will lead to a different case flow near a stagnation point region. It is interesting to discuss that in Figure 1B; there are no changes in terms of magnitude for the skin friction on y -direction as parameter ε increases. The value $c = 0$ represents the specific geometry for the plane which is the sphere which leads to the plane stagnation case flow. In addition, there is another type of flow happening on Figure 3A and 3B when the magnitude of the skin friction for both directions area found the same as the parameter ε increases. Cylindrical geometry on Figure 3 caused by the chosen parameter value c leads to the axisymmetric stagnation case flow.



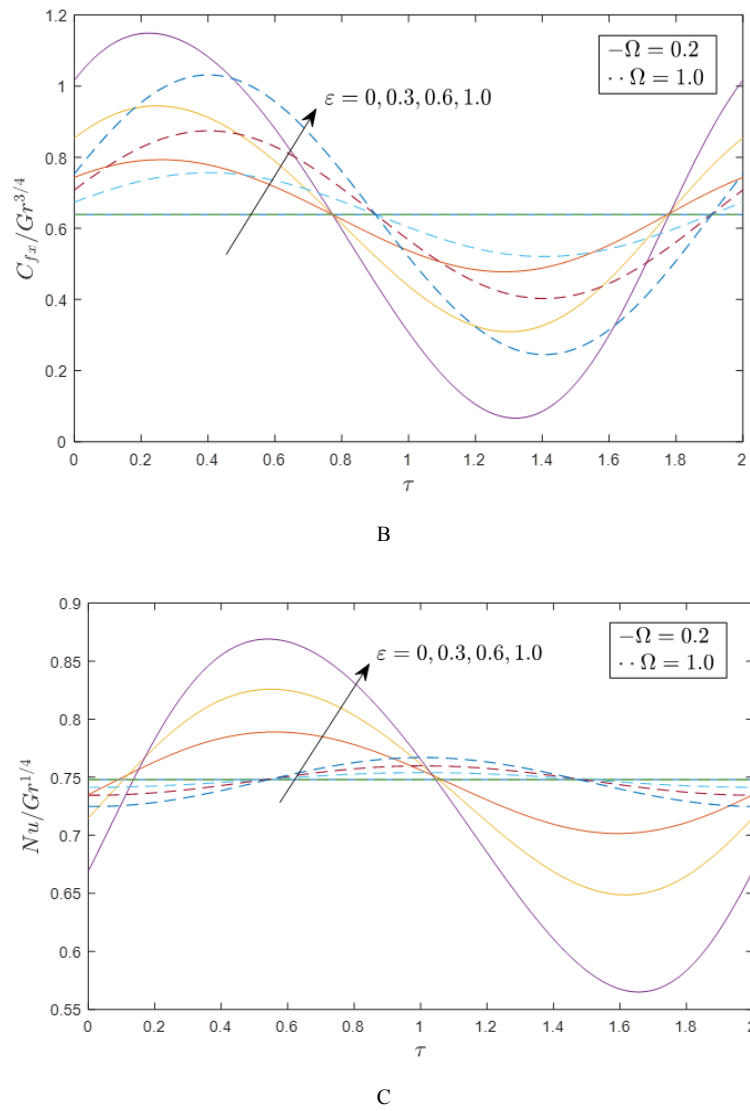


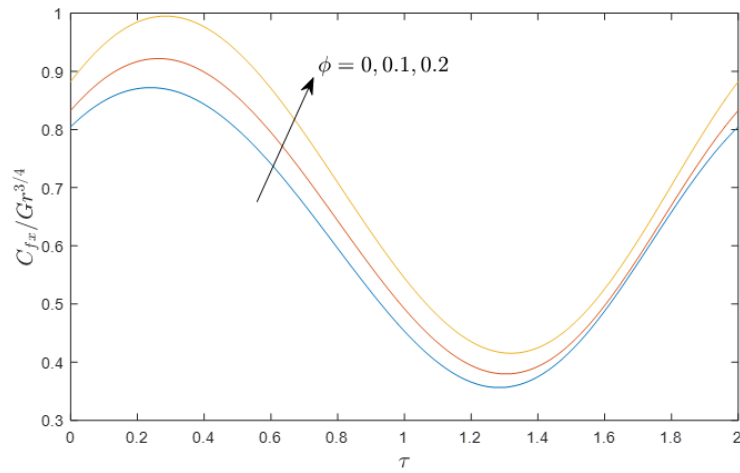
Figure 4. Skin Frictions on A and B while Nusselt number on C for $c = 0.5$, $\phi = 0.05$ and $Nr = 1$ with different values of ε and Ω

Figure 4 on the other hand presents an analysis of skin frictions and Nusselt number when two different values of Ω ; $\Omega = 0.2$ and $\Omega = 1.0$. It is found that, from the physical quantities of principal interest, there are some significant differences for different size of frequency of oscillation as parameter ε increases. The bigger size of Ω provided a decreasing value of the peak variation response for skin frictions and Nusselt number. Generally, for the larger size frequency of oscillation, the convergence rate occurs faster than the smaller size of frequency of oscillation in the microgravity environment.

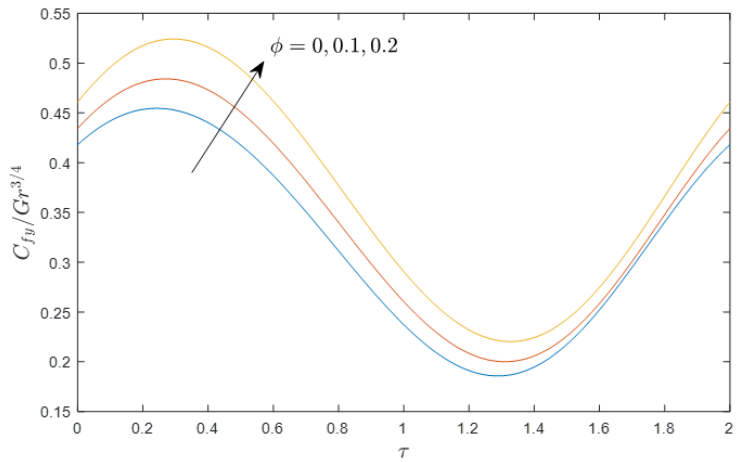
The effect of nanofluid is analyzed on Figure 5. Increasing values of parameter ϕ , both skin frictions and Nusselt number are also found to have increased. The purpose of nanofluid which is to enhance the thermal conductivity of conventional fluid is proven in this analysis when the Nusselt number results which represent the heat

transfer at the boundary layer are found to increase as parameter ϕ increases. Corresponding with that, by adding nano size particles on the fluid, the frictions on the boundary layer are found to be increased due to the increases of resistance which disturb the flow. It will lead to the increases of skin friction on both directions and will decrease the velocity of the flow.

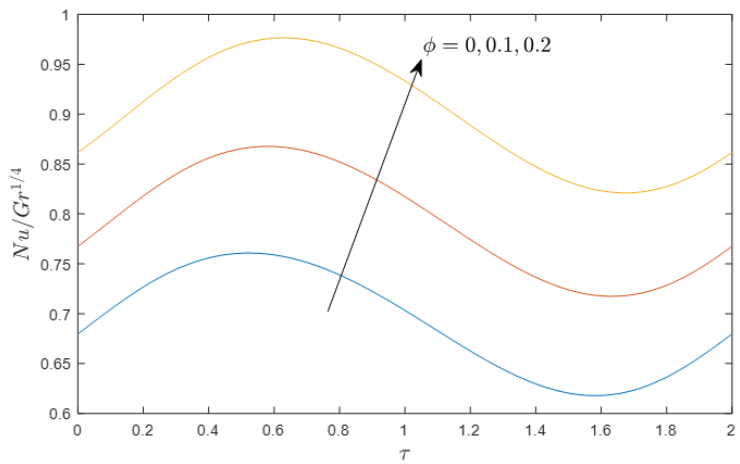
Thermal radiation Nr , effect is analyzed in this boundary layer flow and from Figure 6, skin friction for both directions together with Nusselt number are increased as Nr parameter increases. Physically, increases of Nr parameter will increase the thermal conductivity of the fluid and decreases viscosity of the fluid. As a result, better heat transfer in the fluid can be achieved by considering thermal radiation effect because of the increases of rate of energy transported in the fluid.



A



B



C

Figure 5. Skin Frictions on A and B while Nusselt number on C for $c = 0.5$, $\varepsilon = 0.5$, $\Omega=0.2$ and $Nr = 1$ with different values of ϕ

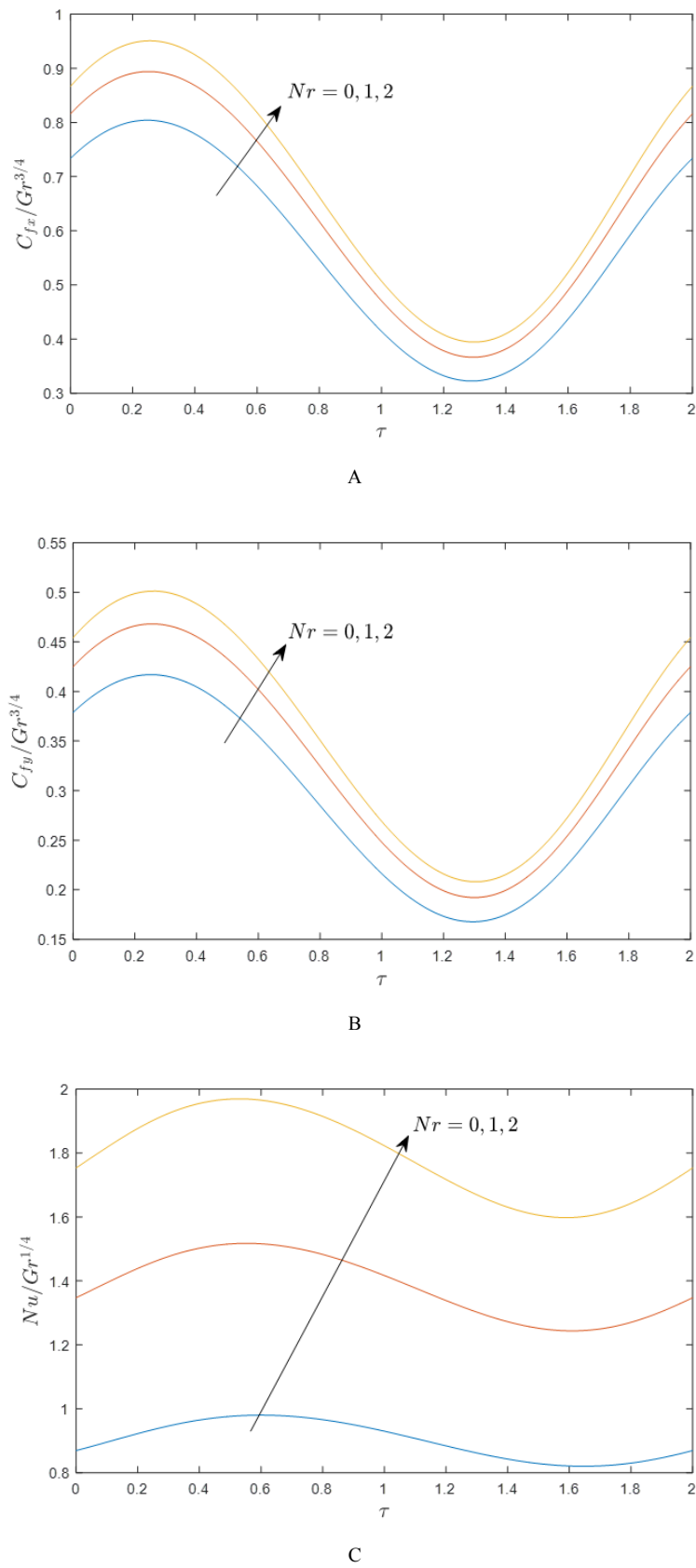


Figure 6. Skin Skin Frictions on A and B while Nusselt number on C for $c = 0.5$, $\varepsilon = 0.5$, $\Omega=0.2$ and $\phi=0.05$ with different values of Nr

4. Conclusions

In this article, three-dimensional boundary layer nanofluid flow near a stagnation point region with thermal radiation induced by g -jitter was investigated numerically. The flow problem was governed into mathematical formulation based on physical principal together with considered effects and non-dimensional partial differential equations solved using implicit finite difference method known as Keller box method. The effect considered were then analyzed in terms of physical quantities of principal interest such as skin frictions and Nusselt number which were then presented graphically. It can be concluded that

- Parameter \mathcal{E} produced a singularity solution on the boundary layer flow as shown on fluctuating graph for skin friction and Nusselt number.
- Different values of curvature ratio c will lead to a special stagnation case flow and show a significant difference in terms of magnitude for skin friction and Nusselt number.
- Larger size of frequency of oscillation Ω , converge faster compared to smaller size frequency of oscillation.
- Both parameter ϕ and Nr enhanced the thermal conductivity of the conventional fluid as their parameter increase.
- Additional of nanoparticle on the fluid flow increased skin frictions due to the increases of frictional force at the boundary layer.

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REFERENCES

- [1] A. J. Faller. An experimental study of the instability of the laminar Ekman boundary layer. *Journal of Fluid Mechanics*. Vol.15, No.4, 560-76. 1963.
- [2] R. J. Lingwood. An experimental study of absolute instability of the rotating-disk boundary-layer flow. *Journal of Fluid Mechanics*. Vol.314. 373-405. 1996.
- [3] J. M. Carlton, C. A. Yowell, K. A. Sturrock, J. B. Dame, Biomagnetic separation of contaminating host leukocytes from plasmodium-infected erythrocytes. *Experimental parasitology*. Vol.97, No.2:111. 2001.
- [4] J. R. Garratt. The atmospheric boundary layer. *Earth-Science Reviews*. Vol.37, No1-2, 89-134. 1994.
- [5] R. B. Cal, J. Lebrón, L. Castillo, H. S. Kang, C. Meneveau. Experimental study of the horizontally averaged flow structure in a model wind-turbine array boundary layer. *Journal of Renewable and Sustainable Energy*. Vol.2, No.1. 013106. 2010.
- [6] E. Aljallis, M. A. Sarshar, R. Datla, V. Sikka, A. Jones, C. H. Choi. Experimental study of skin friction drag reduction on superhydrophobic flat plates in high Reynolds number boundary layer flow. *Physics of fluids*. Vol. 25, No. 2:025103. 2013.
- [7] Sakiadis, C. Byron. Boundary - layer behavior on continuous solid surfaces: I. Boundary - layer equations for two - dimensional and axisymmetric flow. *AIChE Journal* Vol.7, No. 1. 26-28. 1961.
- [8] T. Cebeci, P. Bradshaw. *Momentum transfer in boundary layers*. Washington, DC, Hemisphere Publishing Corp.; New York. 1977.
- [9] P. D. D Weidman, D. G. Kubitschek, A. M. J. Davis. The effect of transpiration on self-similar boundary layer flow over moving surfaces. *International journal of engineering science* Vol.44, No. 11-12. 730-737. 2006.
- [10] M. Sajid, T. Hayat. Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *International Communications in Heat and Mass Transfer* Vol.35, No. 3. 347-356. 2008.
- [11] D. A. Nield, A. V. Kuznetsov. The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid. *International Journal of Heat and Mass Transfer* Vol.52, No. 25-26: 5792-5795. 2009
- [12] Z. H. Khan, M. Qasim, Rizwan Ul Haq, and Qasem M. Al-Mdallal. Closed form dual nature solutions of fluid flow and heat transfer over a stretching/shrinking sheet in a porous medium. *Chinese journal of physics* Vol.55, No. 4: 1284-1293. 2017.
- [13] W. G. Park, J. H. Jang, H. H. Chun, M. C. Kim. Numerical flow and performance analysis of waterjet propulsion system. *Ocean Engineering*. Vol.32, No.14-15. 1740-61. 2005.
- [14] K. Hiemenz. Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder. *Dinglers Polytech. Journal*. Vol.326. 321-324. 1911.
- [15] Mahapatra, T. Ray, A. S. Gupta. Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass transfer* Vol.38, No 6: 517-521. 2002.
- [16] R. Nazar, N. Amin, D. Filip, I. Pop. Stagnation point flow of a micropolar fluid towards a stretching sheet. *International Journal of Non-Linear Mechanics* Vol.39, No.7. 1227-1235. 2004.
- [17] A. Raptis C. Perdakis, H. S. Takhar. Effect of thermal radiation on MHD flow. *Applied Mathematics and computation*. Vol.153, No.3: 645-649. 2004.
- [18] D. Pal. Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation. *Meccanica*, Vol.44, No.2. 145-158. 2009.
- [19] K. Bhattacharyya, G. C. Layek. Effects of suction/blowing

- on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer*. Vol.54, No.1-3. 302-307. 2011.
- [20] J. A. Eastman, S. U. S. Choi, S. Li, W. Yu, L. J. Thompson. Anomalous increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Applied physics letters*, Vol.78, No.6. 718-720. 2011.
- [21] S. U. Choi, J. A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. Argonne National Lab., IL (United States); 1995.
- [22] R. K. Tiwari, M. K. Das. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer*. Vol.50, No.9-10: 2002-2018. 2007.
- [23] J. Buongiorno. Convective transport in nanofluids. *Journal of heat transfer*. Vol.128, No.3. 240-250. 2006.
- [24] M. Mustafa, T. Hayat, I. Pop, S. Asghar, S. Obaidat. Stagnation-point flow of a nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*. Vol.54, No.25-26. 5588-5594. 2011.
- [25] R. U. Haq, S. Nadeem, Z. H. Khan, N. S. Akbar. Thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet. *Physica E: Low-dimensional Systems and Nanostructures*. Vol.6: 17-23. 2015.
- [26] B. Pan, D. Y. Shang, B. Q. Li, H. C. De Groh. Magnetic field effects on g-jitter induced flow and solute transport. *International journal of heat and mass transfer*. Vol.45, No. 1. 125-144. 2002.
- [27] Y. Shu, B. Q. Li, Henry C. D. Groh. Numerical study of g-jitter induced double-diffusive convection. *Numerical Heat Transfer: Part A: Applications*. Vol.39, No.3. 245-265. 2001.
- [28] S. Shafie S, Amin N, Pop I. g-Jitter free convection flow in the stagnation-point region of a three-dimensional body. *Mechanics Research Communications*. Vol.34, No.2. 115-22. 2007.
- [29] N. A. Rawi, N. A. Zin, A. R. Kasim, S. Shafie. g-Jitter induced MHD mixed convection flow of nanofluids past a vertical stretching sheet. *InAIP Conference Proceedings*. Vol. 1750, No. 1. 030017. 2016.
- [30] M. H. A. Kamal, A. Ali, S. Shafie. g-Jitter free convection flow of nanofluid in the three-dimensional stagnation point region. *MATEMATIKA* Vol.35, No.2. 260-270. 2019.
- [31] M. D. D. Sheikholeslami, M. M. Ganji, Rashidi. Ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation. *Journal of the Taiwan Institute of Chemical Engineers* Vol.47. 6-17. 2015.
- [32] H. B. Keller, *A new difference scheme for parabolic problems*. *Numerical Solution of Partial Differential Equations-II*, Academic Press, Maryland, 1971.
- [33] N. M. Sarif, M. Z. Salleh, R. Nazar. Numerical solution of flow and heat transfer over a stretching sheet with Newtonian heating using the Keller box method. *Procedia Engineering* Vol.53. 542-554. 2013.
- [34] A. R. M. Kasim, N. F. Mohammad, S. Shafie, I. Pop. Constant heat flux solution for mixed convection boundary layer viscoelastic fluid. *Heat and Mass Transfer*. Vol.49. No.2. 163-171. 2013.
- [35] N. Bachok, A. Ishak, I. Pop. Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. *International Journal of Thermal Sciences*. Vol.49. No.9. 1663-1668. 2010.
- [36] M. A. Admon, A. R. M. Kasim, S. Shafie. Unsteady free convection flow over a three-dimensional stagnation point with internal heat generation or absorption. *World Acad. Sci., Eng. Technol* Vol.51. 530-535. 2011.