An Analytical Solution of Dynamics of Self-gravitating Spherical Gas-Dust Cloud

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Abstract In this paper, we present a simple model for the dynamics of one dimensional of a self-gravitating spherical symmetrical gas-dust cloud. We take two analytical approaches to study the dynamics of a gravitating system of a gas-dust cloud. The first approach solves a set of non-linear equation of dynamics of a gravitating system. The second approach is a Cole-Hopf transformation, which is used to simplify the equations of dynamics and after that, we applied the method of characteristics to reduce partial differential equations to a system of entirely solvable ordinary differential equations. The results found by the analytical method and the Cole-Hopf method are compared with each other, showing that both lead to the same result. The obtained results in this study are presented in plots. We used the Mathematica software package in performing calculation and plotting graphs.

Keywords Hydrodynamics, Non-linear PDE, Cole-Hopf Method, Gravitating System

1 Introduction

One of the general problems in astrophysics is a description of the fluid flow of gas-dust medium in the gravitating field. Mechanical theories of gravitating system of gas-dust cloud can be developed from two quite different starting points: We can introduce either the model of N gravitating mass points [1], or the model of a compressible fluid streaming in the phase space [2, 3]. In this work we will study a model of a compressible fluid of a gas-dust cloud.

In order to simplify the equations of motion, we must make several approximations. We shall assume that self-gravitating gas-dust cloud is spherically symmetric. The assumption of spherical is playing a fundamental role in explaining qualitative behaviour through the analytical solving of equations of dynamics. The magnetic field, radiation force, rotation probably play important roles but to simplify the problem in this paper we will be ignoring these factors.

In this work, The model is described by systems of non-linear partial differential equations, which no existing general theory for solving them, so the derivation of analytical solution is a important problem, and search for analytical solutions is now motivated by the desire to understand the mathematical structure of the solutions and, hence, a deeper understanding of the physical phenomena described by them.

The dynamics of a gravitating system of gas sphere have been studied analytically since the mid of the 1960s, by a number of authors (e.g. [3–6]). Several authors have applied similarity technique [4, 5, 7] to study the dynamics of gas cloud, the central element of these studies involves the form of the equation of state used.

Many numerical simulation have been performed in order to study the dynamics of the collapce of clouds [8–10]. The numerical simulation techniques have become popular with the development of the computing capabilities, and although they give approximate solutions, have sufficient accuracy for engineering purposes. However, numerical methods often do not provide an opportunity to understand the internal nature of the solutions obtained. Due to this search for the methods for constructing exact solutions , will remain one of the important research areas in hydrodynamics.

Many analytical methods for solving nonlinear partial differential equations were suggested during the last decades [11, 12]. For problems of the dynamics of a compressible medium, one of the main methods for constructing exact solutions is the Hodograph transformation method [12, 13], and some other similar methods. However, these methods allow obtaining solutions in a specific form of dependence of coordinates and time on fluid flow parameters, which complicates their interpretation and the construction of solutions to initial and boundary-value problems. As an alternative approach is the Cole-Hopf transformation method [14], in contrast to the Hodograph transformation method allows us to construct solutions either explicitly or in the form of integral of motion, setting the solution in an implicit form.

In this study, we intend to investigate the dynamics of the gravitating system of gas-dust cloud by the analytical method and Cole-Hopf transformation method. Both methods will reduce the set of partial differential equations to the solvable ordinary differential equations.

The Cole-Hopf transformation provides an interesting method for solving Burger's equation and can simplify some non-linear partial differential equation and thus makes them analytically

solvable.

2 Fundamental equations

In this paper, we consider one-dimensional compressible spherical cloud. Assume the spherical cloud has mass M, radius R and uniform density ρ_0. All the physical quantities will depend on two independent variables; radius r and time r. Let $p(r,t)$, $\rho(r,t)$, $v(r,t)$, and $\Phi(r,t)$ be the pressure, mass density, radial velocity, and gravitational potential respectively.

In a fluid description, the dynamics of a spherically symmetrical compressible gas-dust cloud is governed by the continuity equation

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v) = 0, \tag{1}$$

and the momentum equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = -\frac{\partial\Phi}{\partial r}. \tag{2}$$

The gravitational potential Φ is given by Poisson's equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) = 4\pi G\rho, \tag{3}$$

where G is Newton's gravitational constant.
The system of equations (1)–(3) are non-linear partial differential equations are quite complicated, and the general solution cannot be obtained.
We shall simplify our model even further and assume that the cloud collapses as pressureless dust, which corresponds to the equation of state $p = 0$; like many authors have been neglected the pressure to simplify the problem [16, 17]. Now we can rewrite the (1)–(3) as:

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v) = 0, \tag{4}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{\partial\Phi}{\partial r}, \tag{5}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) = 4\pi G\rho. \tag{6}$$

3 Initial and boundary conditions

One of the main problems with model calculations the formation of planets is the fact that initial conditions of the cloud are not known. The simplest case when the initial distribution density, and velocity are uniform. Let us look for the solution of the problem with initial conditions
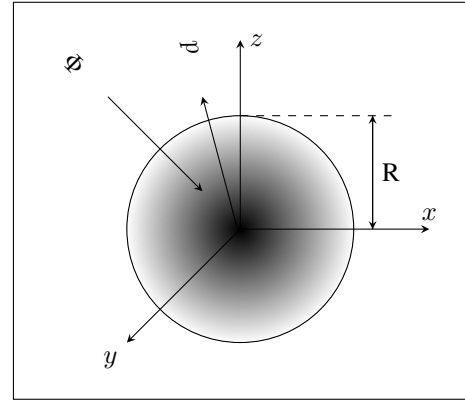
$$\rho(r,0) = \rho_0, \tag{7}$$

$$v(r,0) = 0, \tag{8}$$

$$\frac{\partial\Phi}{\partial r}(r,0) = \frac{4\pi G}{3}r\rho_0, \tag{9}$$

and boundary conditions

$$\frac{\partial\Phi}{\partial r}(0,t) = 0, \tag{10}$$

$$v(0,t) = 0. \tag{11}$$

Figure 1. The plot of a spherical symmetric gas-dust cloud, the gravity force Φ is balanced by pressure force p.

4 Cole-Hopf transformation method

Many methods for solving nonlinear differential equations were independently suggested during the last decades [18], among these methods Cole-Hopf transformation [14, 17]. Cole-Hopf transformation provides an interesting method for solving Burger's equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = \nu\frac{\partial^2 u}{\partial r^2}, \tag{12}$$

with transformation:

$$u(r,t) = 2\mu\frac{\varphi_r}{\varphi}; \qquad \varphi = \varphi(r,t), \tag{13}$$

also opened up other doors to solve other non-linear partial differential equations through similar methodologies.
Let us introduce an auxiliary function $\sigma(r,t) = r^2\rho(r,t)$. In this case, we can rewrite (4)–(6) in this way:

$$\frac{\partial\sigma}{\partial t} + \frac{\partial}{\partial r}(\sigma v) = 0, \tag{14}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{\partial\Phi}{\partial r}, \tag{15}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) = 4\pi G\sigma. \tag{16}$$

It is an amazing fact that the equations like (14)–(16) may be solved exactly using a trick discovered independently by Cole and Hopf [14]. After Hopf and Cole introduced the transformation, several attempts have been made to generalised Cole-Hopf transformation, we shall use here modified generalized Cole-Hopf method [17, 18]. Let us we change the fluid velocity v in the following form:

$$v(r,t) = -\frac{\theta_t}{\theta_r}, \tag{17}$$

where $\theta = \theta(r,t)$ is the auxiliary function (generalised Cole-Hopf transformation), $\theta_t = \frac{\partial\theta}{\partial t}$, $\theta_r = \frac{\partial\theta}{\partial r}$.
The equivalent representation (17) has the form of the equation

$$\theta_t + v(r,t)\theta_r = 0. \tag{18}$$

Let

$$v = S(r)T(\theta),\qquad(19)$$

where $S(r)$, and $T(\theta)$ are so far undefined functions. Substitution of (19) into the left side of (15), and using (18), we obtain

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = S'(r)S(r)T^2(\theta).\qquad(20)$$

We reduce the Poisson equation (16) to the form

$$\frac{\partial \Phi}{\partial r} = 4\pi G\theta.\qquad(21)$$

From (15), (21), and (20), we obtain

$$S'(r)S(r)T^2(\theta) = -4\pi G\theta.\qquad(22)$$

From (22), we obtain

$$T^2(\theta) = 4\pi G\theta,\qquad(23)$$

and

$$S'(r)S(r) = -1.\qquad(24)$$

Thus, we obtain

$$T(\theta) = \sqrt{4\pi G}\sqrt{\theta},\qquad(25)$$

and

$$S(r) = \pm\sqrt{2}\sqrt{r^{-1}+c},\qquad(26)$$

where c is constant of integration.
From (17), (19), (25), and (26)

$$v(r,t) = -\frac{\theta_t}{\theta_r} = \pm\sqrt{8\pi G}\sqrt{r^{-1}+c}\sqrt{\theta},\qquad(27)$$

or

$$\theta_t \pm \sqrt{8\pi G}\sqrt{r^{-1}+c}\sqrt{\theta}\theta_r = 0.\qquad(28)$$

By the substitution of new variable

$$\xi(r) = \int (r^{-1}+b)^{-1/2}dr,\qquad(29)$$

to (28), it takes the form

$$\theta_t \pm \sqrt{8\pi G}\sqrt{\theta}\theta_\xi = 0.\qquad(30)$$

This is quasi-linear partial differential equation. We can solve it by the method of characteristics [7].
Let us consider differential form

$$\frac{dt}{1} = \pm\frac{d\xi}{\sqrt{8\pi G}\sqrt{\theta}} = \frac{d\theta}{0},\qquad(31)$$

from (31), we have

$$\theta = c_1,\qquad(32)$$

where c_1 is a constant. Also

$$\xi \pm \sqrt{8G\lambda}\sqrt{\theta}t = c_2,\qquad(33)$$

Combining (32), and (33) we obtain

$$c_2 = F(c_1),\qquad(34)$$

where F is an arbitrary function, then the general solution to the partial differential equation (30) my be written in implicit form

$$\xi \pm \sqrt{8\pi G}\sqrt{\theta}t = F(\theta).\qquad(35)$$

Let us look for the solution of the problem with initial conditions:

$$\rho(r,0) = \frac{1}{r^2}\frac{\partial\theta_0(r)}{\partial r},\qquad(36)$$

where $\theta_0(r) = \theta(r,0)$.
Let

$$\theta_0(r) = \frac{\rho_0}{3}r^3.\qquad(37)$$

Differentiating (37), and using (36), we find $\rho(r,0)$

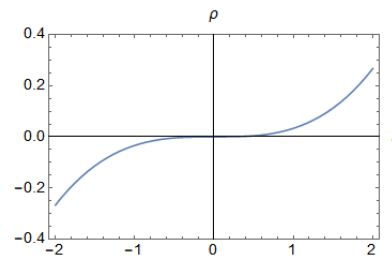$$\rho(r,0) = \rho_0 > 0.\qquad(38)$$

When $b = 0$ in (29), we obtain

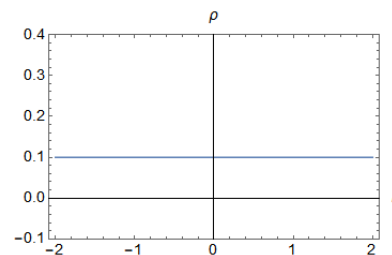**Figure 2.** Graph of function θ_0.

Figure 3. Graph of initial density $\rho(r) = \rho_0$.

$$\xi(r) = \frac{2}{3}r^{\frac{3}{2}}.\qquad(39)$$

Then we can rewrite (37) as follow

$$\theta_0(\xi) = \frac{3\rho_0}{4}\xi^2.\qquad(40)$$

We seek now the solution of (35) with initial condition (40). By plugging initial condition (40) into (35), we get

$$F(\theta_0(\xi)) = \xi.\qquad(41)$$

Then

$$F(\theta) = \pm\left(\frac{4\theta}{3\rho_0}\right)^{\frac{1}{2}}.\qquad(42)$$

Substituting the foregoing equation into (35), we obtain that

$$\pm\left(\frac{4\theta}{3\rho_0}\right)^{\frac{1}{2}} = \xi \pm \sqrt{8\pi G}\sqrt{\theta}t.\qquad(43)$$

Therefore

$$\theta - \frac{3\rho_0}{4}(\xi \pm \sqrt{8\pi G}\sqrt{\theta}t)^2 = 0, \tag{44}$$

replacing the variable ξ by its expression from (39), we finally obtain

$$\theta - \rho_0 \left(\frac{r^{\frac{3}{2}}}{\sqrt{3}} \pm \sqrt{6\pi G}\sqrt{\theta}t \right)^2 = 0, \tag{45}$$

which is non-linear algebraic equation, we can solve it numerically to find θ.

Differentiating (45) with respect to r, we find

$$\theta_r = \frac{\sqrt{3}\rho_0 \sqrt{r}\sqrt{\theta}w(r,t)}{(\sqrt{\theta} \pm \rho_0 \lambda t w(r,t))}, \tag{46}$$

where

$$w(r,t) = \frac{r^{\frac{3}{2}}}{\sqrt{3}} \pm \lambda\sqrt{\theta}t, \qquad \lambda = \sqrt{6\pi G}. \tag{47}$$

Now we can calculate density ρ (Figure 4, (Figure 5) through function $\theta(r,t)$

$$\rho(r,t) = \frac{\theta_r}{r^2} = \frac{\sqrt{3}\rho_0 \sqrt{\theta}(\frac{r^{\frac{3}{2}}}{\sqrt{3}} \pm \lambda\sqrt{\theta}t)r^{-\frac{3}{2}}}{(\sqrt{\theta} \pm \rho_0 \lambda t(\frac{r^{\frac{3}{2}}}{\sqrt{3}} \pm \lambda\sqrt{\theta}t))}. \tag{48}$$

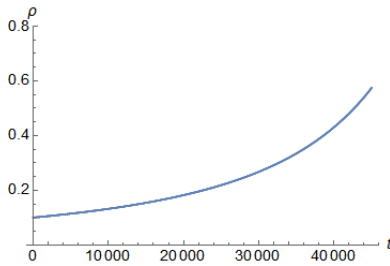Differentiating (45) with respect to t, we obtain

Figure 4. Graph of density ρ as function of t for a variety of radius r.
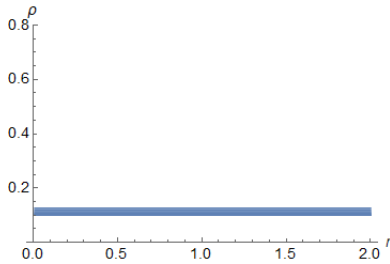

Figure 5. Graph of density ρ as function of r for a variety of times t.

$$\theta_t = \frac{\pm 2\rho_0 \lambda\theta w(r,t)}{\sqrt{\theta} \pm \rho_0 \lambda t w(r,t)}. \tag{49}$$

Now we can calculate velocity v (Fugire 6) through function θ

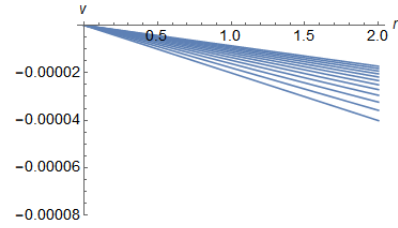$$v(r,t) = \pm \frac{2\lambda\sqrt{\theta}}{\sqrt{3}\sqrt{r}}. \tag{50}$$

Figure 6. Graph of velocity $v(r)$ for a variety of times t.

5　Derivation of the analytical solution of the fundamental equations

As mentioned before it is not possible to find a general solution of (4)–(6), so we obtain here only partial solution. The system of equations (4)–(6) with initial condition (7)–(9), and boundary conditions (10), and (11) dmits a solution characterised by the fact that:

$$\rho(r,t) = \rho(t). \tag{51}$$

Now we can rewrite the equation (4)–(6) as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial v}{\partial r}\rho + \frac{2}{r}\rho v = 0, \tag{52}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{\partial \Phi}{\partial r}, \tag{53}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r} \right) = 4\pi Gt\rho(t)r^2. \tag{54}$$

From (52), we obtain

$$-\frac{\dot{\rho}}{\rho} = \frac{\partial v}{\partial r} + \frac{2v}{r} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v). \tag{55}$$

Integrating both sides of (55) with respect to r, we obtain

$$v(r,t) = -\frac{r}{3}\frac{\dot{\rho}}{\rho} + \frac{c(t)}{r^2}, \tag{56}$$

where $c(t)$ is function, determined from the initial conditions (8), obviously, in this case $c(t) = 0$, and so

$$v(r,t) = -\frac{r}{3}\frac{\dot{\rho}}{\rho}. \tag{57}$$

Differentiating the foregoing equation with respect to t, we get

$$\frac{\partial v}{\partial t} = \frac{r}{3}\left(\frac{\dot{\rho}^2}{\rho^2} - \frac{\ddot{\rho}}{\rho} \right). \tag{58}$$

Differentiating (57) with respect to r, we get

$$\frac{\partial v}{\partial r} = -\frac{1}{3}\frac{\dot{\rho}}{\rho}. \tag{59}$$

By integration (54) with respect to r, we obtain

$$\frac{\partial \Phi}{\partial r} = \frac{4}{3}\pi G\rho r + \frac{H(t)}{r^2}. \tag{60}$$

Using boundary condition (10), we get $H(t) = 0$, and then (60) becomes

$$\frac{\partial \Phi}{\partial r} = \frac{4}{3}\pi G\rho r. \tag{61}$$

From (5) and (61), we get

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{4}{3}\pi G r\rho(t) = 0. \tag{62}$$

Substituting (57), (58), and (59) into (62), we obtain that ρ satisfies the equation

$$3\rho\ddot{\rho} - 4\dot{\rho}^2 - 12\pi G\rho^3 = 0. \tag{63}$$

This is second order nonlinear ordinary differential autonomous equation, we will try to solve it, to find the density $\rho(t)$ as function of time t.
Treating ρ as independent variable, let

$$\Psi(\rho) = \frac{d\rho(t)}{dt}, \tag{64}$$

which gives

$$\frac{d^2\rho(t)}{dt^2} = \frac{d\Psi(\rho)}{dt} = \frac{d\Psi(\rho)}{d\rho}\frac{d\rho}{dt} = \\ = \Psi(\rho)\frac{d\Psi(\rho)}{d\rho}. \tag{65}$$

Substituting (64), and (65) into (63), we get:

$$3\rho\Psi(\rho)\frac{d\Psi(\rho)}{d\rho} - 4\Psi^2(\rho) - 12\pi G\rho^3 = 0. \tag{66}$$

This is Bernoulli's equation, rewrite it as

$$2\frac{d\Psi(\rho)}{d\rho}\Psi(\rho) - \frac{8\Psi^2(\rho)}{3\rho} = 8\pi\rho^2. \tag{67}$$

Let

$$\varphi(\rho) = \Psi^2(\rho), \tag{68}$$

then (67) became

$$\frac{d\varphi(\rho)}{d\rho} - \frac{8\varphi(\rho)}{3\rho} = 8\pi G\rho^2. \tag{69}$$

Let

$$\mu(\rho) = e^{\int -\frac{8}{3\rho}d\rho} = \frac{1}{\rho^{\frac{8}{3}}}, \tag{70}$$

multiply both sides of (69) by $\mu(\rho)$ and integrate both sides with respect to ρ, we get

$$\varphi(\rho) = 24\pi G\rho^3 + a\rho^{\frac{8}{3}}, \tag{71}$$

where a is an arbitrary constant. But $\varphi(\rho) = \Psi(\rho)^2$, then we get

$$\Psi(\rho) = \pm\sqrt{24\pi G\rho^3 + a\rho^{\frac{8}{3}}}. \tag{72}$$

Substitute back for $\Psi(\rho) = \frac{d\rho(t)}{dt}$:

$$\frac{d\rho(t)}{dt} = \pm\sqrt{24\pi G\rho^3 + a\rho^{\frac{8}{3}}}. \tag{73}$$

Applying the initial condition (7), (8), and (57), we obtain

$$a = -24\pi G\rho_0^{\frac{1}{3}}. \tag{74}$$

Then we can rewrite (73) as

$$\frac{d\rho(t)}{dt} = \pm\sqrt{24\pi G}\rho^{\frac{4}{3}}\sqrt{\rho^{\frac{1}{3}} - \rho_0^{\frac{1}{3}}}. \tag{75}$$

Integrate both sides with respect to t:

$$\int \frac{d\rho}{\rho^{\frac{4}{3}}\sqrt{\rho^{\frac{1}{3}} - \rho_0^{\frac{1}{3}}}} = \pm\sqrt{24\pi G}t + b, \tag{76}$$

where b is an arbitrary constant. Evaluating the integral on the left-hand side of (76), we obtain

$$\frac{3\left(\frac{\sqrt[6]{\rho_0}\sqrt{\sqrt[3]{\rho} - \sqrt[3]{\rho_0}}}{\sqrt[3]{\rho}} + \arctan\left(\frac{\sqrt{\sqrt[3]{\rho} - \sqrt[3]{\rho_0}}}{\sqrt[6]{\rho_0}}\right)\right)}{\sqrt{\rho_0}} = \\ = \pm\sqrt{24\pi G}t + b. \tag{77}$$

Applying the initial condition (7), we find that $b = 0$. Hence,

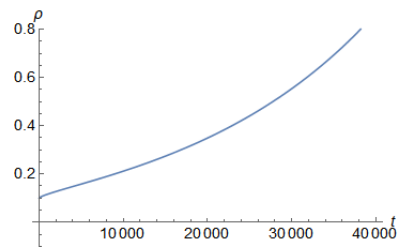**Figure 7.** Graph of the density $\rho(t)$ as function of time t.

we have finally

$$3\frac{\sqrt[6]{\rho_0}\sqrt{\sqrt[3]{\rho} - \sqrt[3]{\rho_0}}}{\sqrt[3]{\rho}} + 3\arctan\left(\frac{\sqrt{\sqrt[3]{\rho} - \sqrt[3]{\rho_0}}}{\sqrt[6]{\rho_0}}\right) = \\ = \pm\sqrt{24\pi G}\sqrt{\rho_0}t. \tag{78}$$

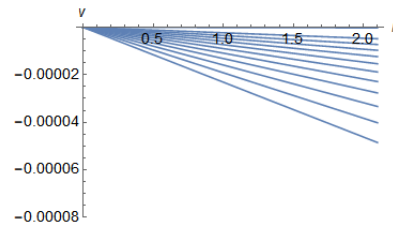From (57), and (75) it follows that

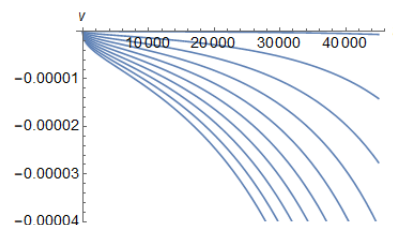**Figure 8.** Graph of velocity $v(r)$ for a variety of times t.

Figure 9. Graphs of velocity $v(t)$ for a variety of radius r.

$$v(r,t) = -\frac{\sqrt{24\pi G}}{3}r\rho^{\frac{1}{3}}\sqrt{\rho^{\frac{1}{3}} - \rho_0^{\frac{1}{3}}}. \qquad (79)$$

The equation (78) is an implicit function, which is not defined explicitly. The MATHEMATICA software package, can be used to draw the implicit function curve.

Using the equations (78) and (79), we can plot velocity v as a function of time t and as function of radius r.

6 Results and discussion

The Mathematica package has been used to solve the non-linear algebraic equation (45) numerically, and to generate a plot of the density function (48) as a function of time t.

It can be observed from Figure 4, and Figure 7, that the density ρ inside the cloud increases with the increasing value of the time t.

Figure (6) showed graphs of velocity $v(r)$ for a variety of time t which have been found by the Cole Hopf transformation method. Figure (5) showed graphs of the density as a function of radius r for a variety of time t which have been found by the Cole-Hopf transformation method. It can be noted from that the density $\rho(r)$ for a variety of time t is very near to initial density ρ_0. Figure (8) showed graphs of velocity $v(r)$ for a variety of time t, and Figure (9) showed graphs of velocity $v(t)$ for a variety of radius r which both have been found by the analytical method.

Figure (10) showed graphs of the density, comparing results obtained using the Cole-Hopf transformation method with the analytical method. Figure (11) showed graphs of the velocity v as function of radius r, comparing results obtained using the Cole-Hopf transformation method with the analytical method. Figure (12) showed graphs of the density v as function of time t for different values of the initial density ρ_0. It can be observed from the figure, that the density inside the cloud increase with the increasing value of initial density ρ_0.
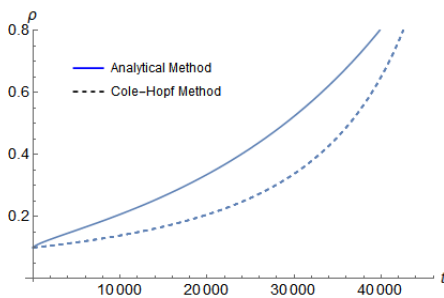

Figure 10. Obtained solutions for density ρ as a function of time t, by the Cole-Hopf transformation method and by the analytical method.

7 Conclusion and future work

We investigated the motion of the spherically symmetrical compressible fluid flow of self-gravitating dust-gas cloud. We have tried to find a solution for the system of equations presented in (4)–(6). In the case when $p = 0$, i.e., the pressure
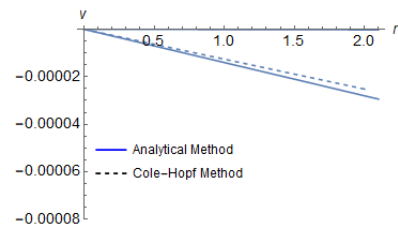

Figure 11. Obtained solutions for velocity v as a function of radius r, by the Cole-Hopf transformation method and by the analytical method.
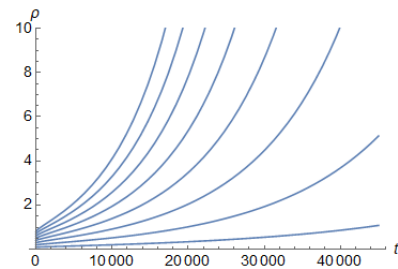

Figure 12. Graph of the density $\rho(t)$ as function of time t for different values of the initial density ρ_0

in the medium is zero, we found the particular analytical solutions with the help of analytical method and special initial condition.

In the present paper, we have applied two methods to obtain solutions to equations of dynamics gravitating system of a gas-dust cloud. Firstly, we investigated the problem using the Cole-Hopf, which reduces the problem from non-linear partial differential equation into an ordinary differential equation which, in turn, can be solved effectively. Secondly we discussed the analytical method which allows us to get an exact analytical solution to the problem.

The validity of the result which obtained by the Cole-Hopf transformation method is verified by the result obtained by the analytical method. Also, our results are similar to those solutions obtained by several authors [3, 4, 8] but with similarity technique.

8 Acknowledgment

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