

NURBS Method for Cycloidal Steel Ball Reducer Profile Design

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Abstract The profile design of cycloid disk in cycloidal steel ball reducer is the core of reducer design, which directly affects the performance of reducer. The cycloidal ball reducer is driven by a certain number of steel balls in a tooth profile composed of a pair of intermeshing internal and external cycloidal lines. The performance of the reducer can be optimized by adjusting the design equation of the cycloidal disc type line. However, in the process of design and optimization, the requirement of freedom and controllability of cycloid is getting higher and higher. The traditional design method of cycloidal profile based on cycloid parameter equation can no longer meet the requirements of design. The NURBS method can accurately represent the free curve and the conic quadratic curve. The cycloidal profile is actually a complex curve composed of a certain number of arcs. In this paper, the NURBS curve is applied to the design of cycloid disk of reducer, and the cycloidal profile based on NURBS is deeply studied. By sampling the known cycloidal profile, the sample point, the inverse control point, the weight factor, the node vector and the sample point are obtained, and the analysis verifies that the error between the fitted NURBS curve and the original curve is within the feasible range. Taking the BR85us-10G-6 reducer as an example, the fitting curve of the inner cycloid disk profile is obtained by using cubic NURBS curve, which can improve the flexibility and local adjustment of the cycloidal disc profile design.

Keywords Cycloidal Steel Ball Reducer, Profile Design, Non-uniform Rational B-spline, Curve Fitting

1. Introduction

Reducer is an important part of industrial production, especially in industrial robot industry. The innovation and development of reducer is the key to the progress of robot industry in the future. Cycloidal steel ball reducer is a kind

of cycloidal meshing high-end precision transmission device. It has the advantages of large transmission ratio, compact structure, high transmission efficiency and especially without backlash. It is used in aerospace, robot, machine tool and so on. Military industry has higher requirements for transmission accuracy of the device [1]. Cycloidal steel ball planetary transmission mechanism is essentially a kind of two-tooth difference K-H-V planetary transmission device. This deceleration drive mechanism transmits motion and power through steel ball, and the tooth profile of steel ball motion is formed by meshing the inner and outer cycloids [2]. The deceleration drive is accomplished by the continuous rotation of the steel ball in the tooth profile [3,4]. Design calculation method steps, transmission characteristics and principle, cycloid equation [5], transmission ratio calculation, parameter selection, force and strength analysis [6], dynamic balance [7] and vibration analysis [8], CAD/CAM and other aspects of research have achieved a lot of research results, and there are few studies on the design of the profile of the cycloidal disc.

The cycloidal disc is one of the important components of the cycloidal ball reducer, and the performance of the reducer is directly affected by its cycloidal disc profile. The design parameters of the cycloidal disc are closely related to whether the cycloidal groove is undercut, and the efficiency and load bearing of the reducer are significant. Cai Feng et al [9], analyzed in detail the influence of the value of short-amplitude coefficient on the transmission performance of the steel ball reducer, and studied the range of the short-amplitude coefficient. Wang Lixing [10] and Xi Qingkun [11] et al. optimized the efficiency, volume and bearing of the steel ball reducer based on genetic algorithm. The research optimized the design parameters of the steel ball reducer to some extent. At present, the design method of drawing the inner and outer cycloidal disc profile according to the cycloid equation and the given condition parameters is widely used in the design of the cycloidal disc profile, but the overall profile will be changed when the design parameters of the profile is

modified, avoiding the undercutting while reducing the efficiency and carrying capacity, the optimization of the overall performance of the reducer cannot be achieved, so the introduction of the NURBS curve design method to achieve a partial adjustment of the cycloidal disc profile. The NURBS curve design method has been widely used in various types of profile design and free curve design [12]. In the design of screw rotor profile, Shyh Hau [13] proposed to use the cubic spline curve to fit the point on the original rotor tool profile line to obtain the screw type. Line; Long Ji et al [14] studied the continuity in the design of NURBS curves and improved the curvature stability of the profile design; He Xueming et al [15] use the straight line, arc, and cubic B-spline curve for the reverse design of the yin-yang rotor for the meshing line segment. The NURBS curve design method provides a new idea for the design of the cycloidal disc profile. On the basis of the original profile design, the subsequent partial optimization is performed by the NURBS representation profile, which can improve the flexibility of the line design.

2. Forward Design Principle of Cycloidal Steel Ball Reducer Profile

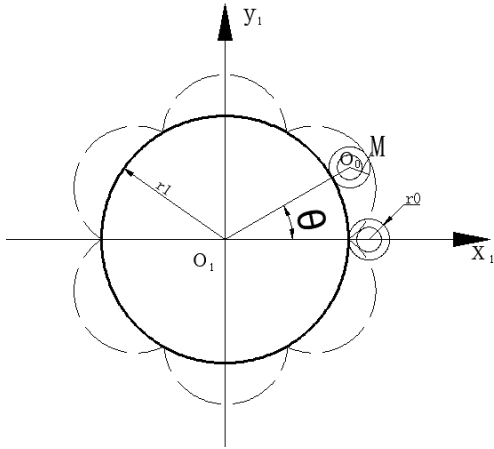


Figure 1. Outer cycloid forming principle diagram

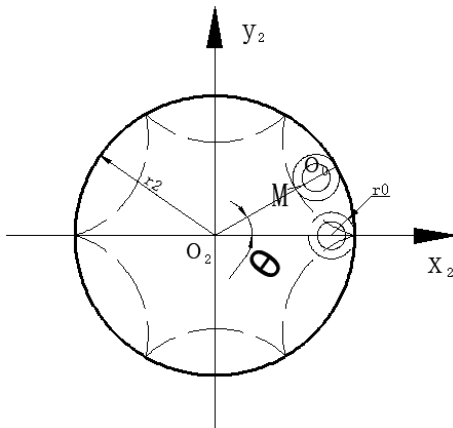


Figure 2. Inner cycloid forming principle diagram

The theoretical tooth profile curve of deceleration mechanism is a pair of meshing inner and outer cycloid of short amplitude. The forming method of inner and outer cycloid of short amplitude includes two kinds of methods: enclosing method and non-enveloping method. The equation of inner and outer cycloid is deduced according to the non-enveloping method as an example. Figure 1 shows a schematic diagram of cycloid generation. In that figure, The center of a rolling circle " O_0 " rolls purely around the center of a base circle " O_1 ", M is the point in the circle and the eccentricity, $e=O_0M$. The number of teeth generated from the outer cycloid is defined as Eq. (1), the motion track of the M point is the outer cycloid. Figure 2 shows the generating principle diagram of inner cycloid. In the figure 2, The center of a rolling circle " O_0 " rolls purely around the center of a base circle " O_2 ", M' is the inner point of the circle, the eccentricity, $A=O_0M'$. The number of teeth generated by the inner cycloid is defined as Eq. (2), and the motion track of the M' point is the inner cycloid.

$$Z_1=R_1/R_0 \quad (1)$$

$$Z_2=R_2/R_0 \quad (2)$$

Z_1 -- The number of teeth generated from the outer cycloid.

R_1 -- The radius of the outer cycloid base circle.

R_2 -- The radius of the inner cycloid base circle.

R_0 -- The radius of the base circle.

Z_2 -- The number of teeth generated by the inner cycloid.

The inner cycloid equation is as follows:

$$\begin{cases} X = d_s \times \cos \theta + c \times \cos(n_b \times \theta) \\ Y = d_s \times \sin \theta - c \times \sin(n_b \times \theta) \end{cases} \quad (3)$$

The outer cycloid equation is as follows:

$$\begin{cases} X = d_s \times \cos \theta - c \times \cos(n_b \times \theta) \\ Y = d_s \times \sin \theta - c \times \sin(n_b \times \theta) \end{cases} \quad (4)$$

d_s is the circle diameter of the steel ball distribution, and its calculation equation is as follows:

$$d_s = r_d / \sin(180/n_h) \quad (5)$$

r_d -- Radius of the steel ball.

θ -- The angle at which the base circle is rolled when the unwrapped circle is rolled purely on the base circle.

c -- Half of the eccentricity.

n_b -- Number of steel balls.

Referring to the traditional forward design method of cycloidal steel ball reducer, the parameter design process of cycloidal steel ball reducer mainly includes the following steps. The main calculation formulas for the forward design are shown in Table 1.

Table 1. Cycloid Design Formula

Name	Symbol	Calculation formula
Number of steel balls	n_b	Determined by transmission ratio
Eccentricity	A	Transfer power and model
Diameter of steel ball	d_b	Transfer power and model
Radius of base circle of epicycloid	r_e	$r_e = R_b n_e / n_b$
Radius of base circle of hypocycloid	r_h	$r_h = R_b n_h / n_b$
Radius of roll	r_0	$r_0 = R_b / n_b$
Distribution radius of steel ball	r_s	$r_s = n_b r_0$
Wave number of epicycloid	n_e	$n_e = n_b - 1$
Wave number of hypocycloid	n_h	$n_h = n_b + 1$
Short amplitude coefficient	K	$k = A n_b / 2 R_b$
Spherical diameter coefficient	K_2	$K_2 = d_z \sin(\pi/n_b) / d_b$

- (1) Select the reduction ratio.
- (2) Determine the number of steel balls based on the reduction ratio, input speed, and input power.
- (3) Determine the number of inner and outer cycloidal waves by determining the relationship between the number of inner and outer cycloidal waves and the number of steel balls determined according to the meshing principle.
- (4) Select the eccentricity and the standard steel ball diameter in the series table.
- (5) Calculate the basic parameters of the epicycloid and hypocycloid.

3. NURBS Curve Theory in Profile Design

NURBS curve is a kind of non-uniform rational B-spline curve with excellent smoothness and controllability, and its highly free curve characteristics greatly increase the flexibility of the curve construction. The one-time NURBS curve has the following definition.

$$C(u) = \frac{\sum_{i=0}^n N_{i,p(u)} \omega_i P_i}{\sum_{i=0}^n N_{i,p(u)} \omega_i}, \quad a \leq u \leq b \quad (6)$$

$\{P_i\}$ -- The control point.

$\{\omega_i\}$ -- The weight factor of the control point which determines the degree of control points from the curve.

$\{N_{i,p(u)}\}$ -- A p times B-spline basis function defined on the aperiodic node vector U .

A set of node vectors $u \in [u_0, u_1, \dots, u_{n+p+1}]$ satisfies the non-decreasing sequence of nodes. Let s be the repeatability of two endpoint nodes, then $s \leq p + 1$. Let s' be the repeatability of inner nodes, then $s' \leq p$. The

non-uniform B-spline basis functions are defined by such a set of node vectors. In order to control the characteristics at the end of NURBS curve, the definition domain of NURBS curve is generally taken as $u \in [u_p, u_{n+1}] = [0, 1]$, where $u_0 = u_1, \dots, u_p = 0$ and $u_{n+1} = u_{n+2}, \dots, u_{n+p+1} = 1$. The inner nodes $u_{p+1}, u_{p+2}, \dots, u_n$ are determined according to the calculation method of node vectors.

The B-spline basis function can be defined by the difference quotient definition of the truncated power function, the definition of flowering, and the recursion formula proposed by Debour, Cox and Mansfield, etc. This paper uses the recursive definition method, as follows:

$$N_{i,0(u)} = \begin{cases} 1, & \text{if } u_i \leq u < u_{i+1} \\ 0, & \text{others} \end{cases}$$

$$N_{i,p(u)} = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1(u)} + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1(u)} \quad (7)$$

u_i -- node.

$N_{i,0(u)}$ -- A step function which is equal to zero outside the half-open interval $u \in [u_i, u_{i+1})$.

$N_{i,p(u)}$ -- A linear combination of two $p - 1$ times basis functions when $p > 0$.

According to the above calculation method, a NURBS profile can be obtained by defining the control point and the corresponding weight factor of the profile and calculating the node vector and the basis function.

4. NURBS Profile Design Method

4.1. Getting Profile Data of BR85us-10G-6

It can be seen from the design of the reducer profile that directly expressing the profile of the reducer with the

NUBRS curve firstly requires obtaining the data of the original profile, and using the control points and weight factor of the profile, the endpoint of the profile is used as the node of the NURBS curve. The cumulative chord length method is used to calculate the node vector of the entire section line, making it a one-stage reducer type profile. In the process of NURBS curve profile design, it is necessary to control the vectors of the endpoints of each segment, and then the designed profile can achieve first-order continuity. Taking the model BR85us-10G-6 reducer as an example, the cubic spline curve interpolation is used to design the reducer. The cubic spline curve is used because it can reach the minimum number of consecutive G^2 . The profile data of the reducer prototype are obtained. The design process is shown in Figure 3.

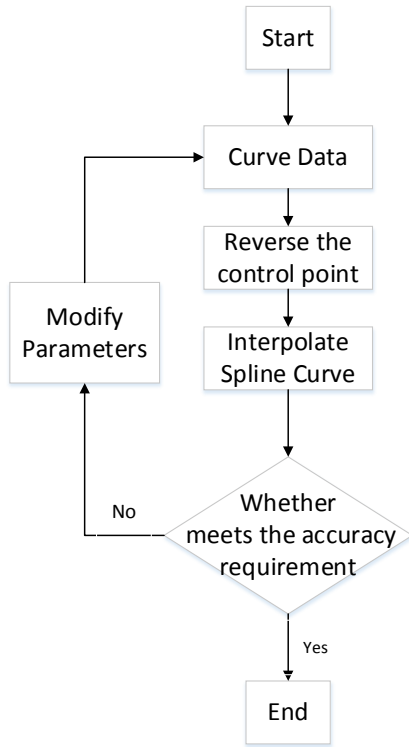


Figure 3. NURBS curve design process

The design method of cycloidal disc profile of reducer is the same as that of inner cycloidal disc. Taking the inner cycloidal disc profile as an example, the design of outer cycloidal disc profile can be completed by referring to the design process of inner cycloidal disc profile. The design parameters of the cycloidal disk profile of BR85us-10G-6 reducer are shown in the Table 2. According to the parameter equation, the cycloidal disk profile of the reducer can be obtained as shown in Figure 4.

Table 2. Cycloidal disk profile design parameters

Number of steel balls	Eccentricity [mm]	Diameter of steel ball [mm]	Short amplitude coefficient	Radius of roll [mm]
21	2	3.5	0.89425	1.11825

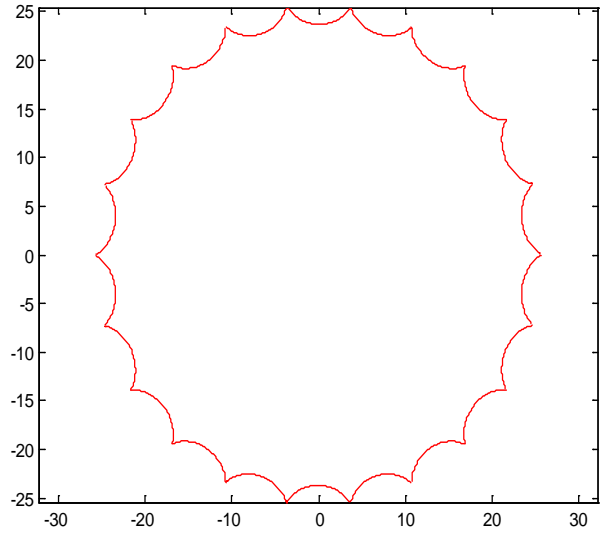


Figure 4. BR85us-10G-6 cycloidal disk profile

According to the geometric characteristics of the inner cycloid disk profile, a complete profile is connected by the same curve of n segments, in which n is determined by the deceleration ratio of the reducer. In order to simplify the fitting process, one segment is fitted, and then the other segments are fitted and linked into the whole profile with the same design method. Because the curvature of the inner cycloid disc varies greatly, the sampling method and sampling points have great influence on the accuracy of curve fitting design. Different sampling methods and NURBS of different control points are used to design the inner cycloid disk profile.

4.2. Node Vector Determination of Cycloidal Disk Profile Data Points

The parameterization of data points is an important step in curve construction and plays an important role in curve fairness. In order to obtain better smoothness, the cumulative chord length parameter method can be used to parameterize the data points of profiles.

$$L \text{ is the total length of the curve. } L = \sum_{i=1}^n |Q_i - Q_{i-1}|,$$

$$\text{when } u_0=0 \text{ } u_n=1, \quad u_i = u_{i-1} + \frac{|Q_i - Q_{i-1}|}{L}, \quad i=1, \dots, n-1.$$

This parameterization truly reflects the distribution of data points according to chord length. The interpolation curve obtained has good fairness and has been widely used.

4.3. Inversely Find the Control Vertex of NURBS Interpolation Curve

When the cubic NURBS curve is used to express a series of data points on the measured cycloidal disk profile, the control points of the NURBS curve must be inversely calculated according to the node vectors determined by the type points on the curve.

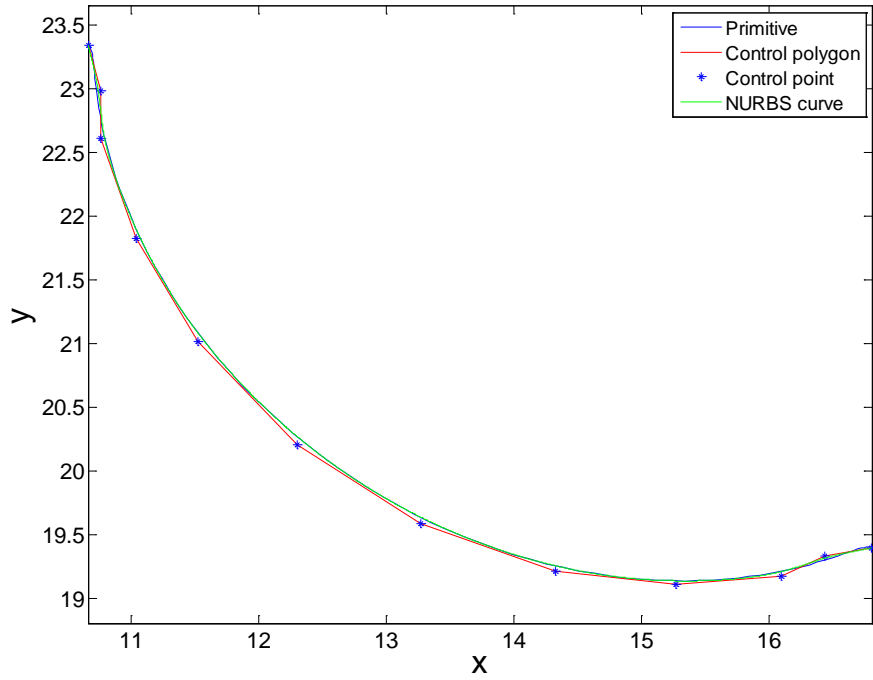
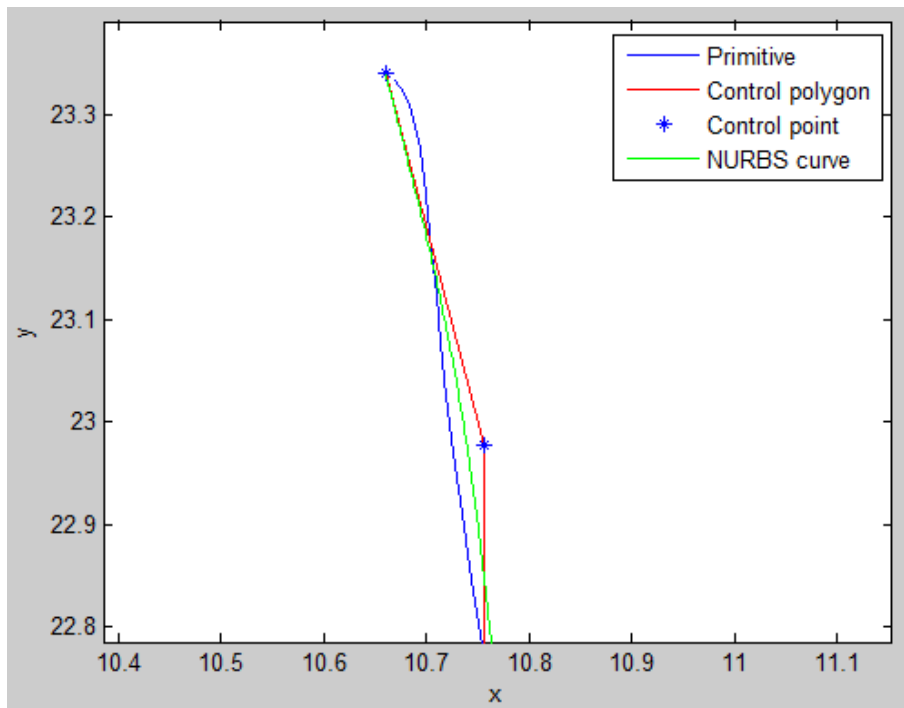
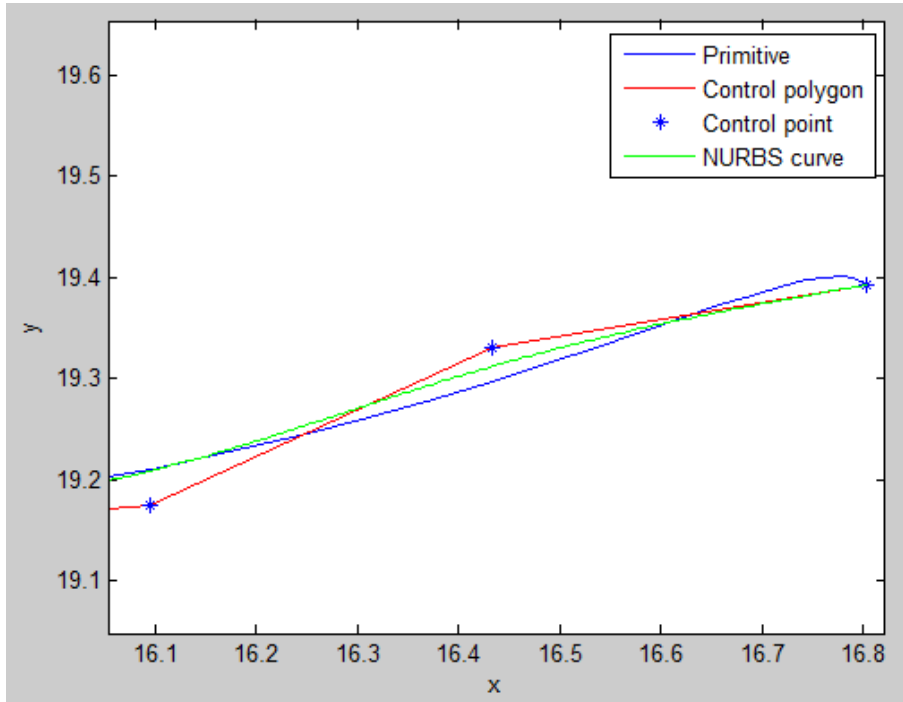


Figure 5. The first type of NURBS curve fitting profile



(a) Partial enlargement of the left end



(b) Partial enlargement of the right end

Figure 6. Local enlargement of NURBS curve fitting profile

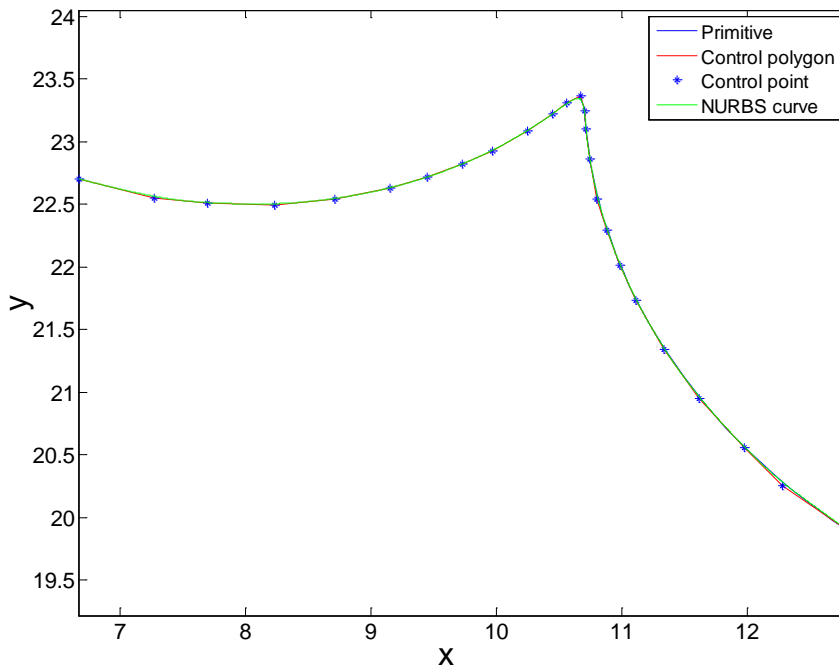


Figure 7. The second type of NURBS curve fitting profile

Figure 8 shows the complete cycloidal disk profile obtained by using NURBS profile design method.

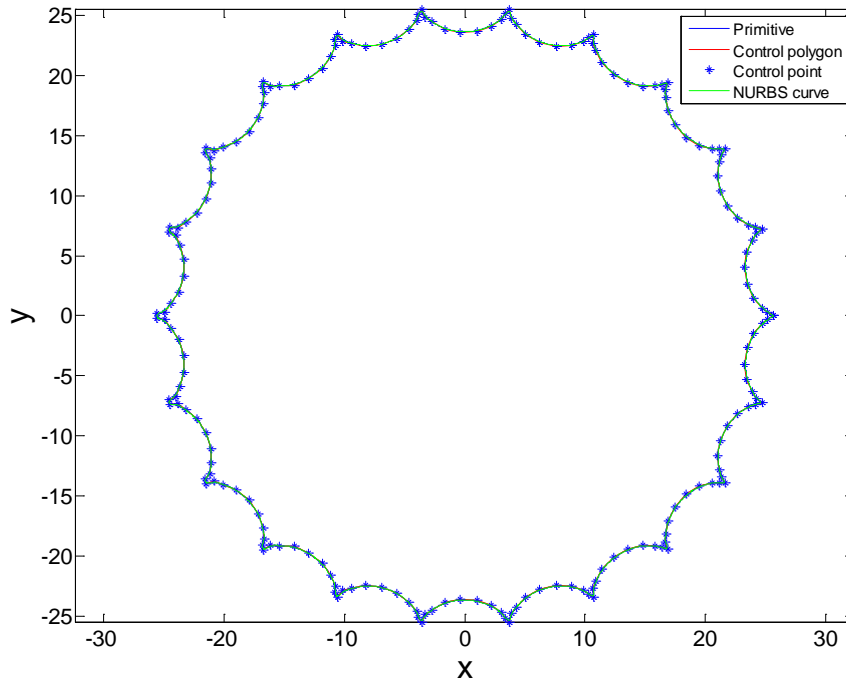


Figure 8. NURBS curve fitting of the whole shape of the cycloid

4.5. Error Analysis of Fitting Accuracy

The basic method for calculating fitting error between fitting curve and data points is to calculate the distance between data points and curves. Average distance and maximum distance are the main evaluation indexes. In order to better compare the errors of various fitting methods, two fitting curve error analysis methods are used to analyze the accuracy error of the fitting results of the last stage.

Sum of Squares due to Error (SSE):

$$SSE = \sum_{i=1}^n \omega_i (y_i - \hat{y}_i)^2 \tag{12}$$

The statistical parameter calculates the sum of the error squares of the corresponding points between the fitting data and the original data. The closer SSE is to 0, the better model selection and fitting are, and the more successful data prediction is.

Sum of Squares for total (SST):

$$SST = \sum_{i=1}^n \omega_i (y_i - \bar{y})^2 \tag{13}$$

$$R\text{-square} = 1 - \frac{SSE}{SST} \tag{14}$$

n -- Sample size.

ω_i -- The weight factor of data.

y_i -- The value of sample data.

\hat{y}_i -- The average of sample data.

R-square represents the fitness of a fitting by the change of data. The normal range of determined coefficients is [0, 1], and the closer to 1, it shows that the stronger the explanatory power of the variables of the equation to y , the better the model fits the data.

As shown in figure 9, error 1 is the fitting accuracy error of the fitting curve obtained by the first sampling method, error 2 is the fitting accuracy error of the fitting curve obtained by the second sampling method, and error 3 is the fitting accuracy error of the fitting curve obtained by increasing the number of control points on the basis of the second sampling method.

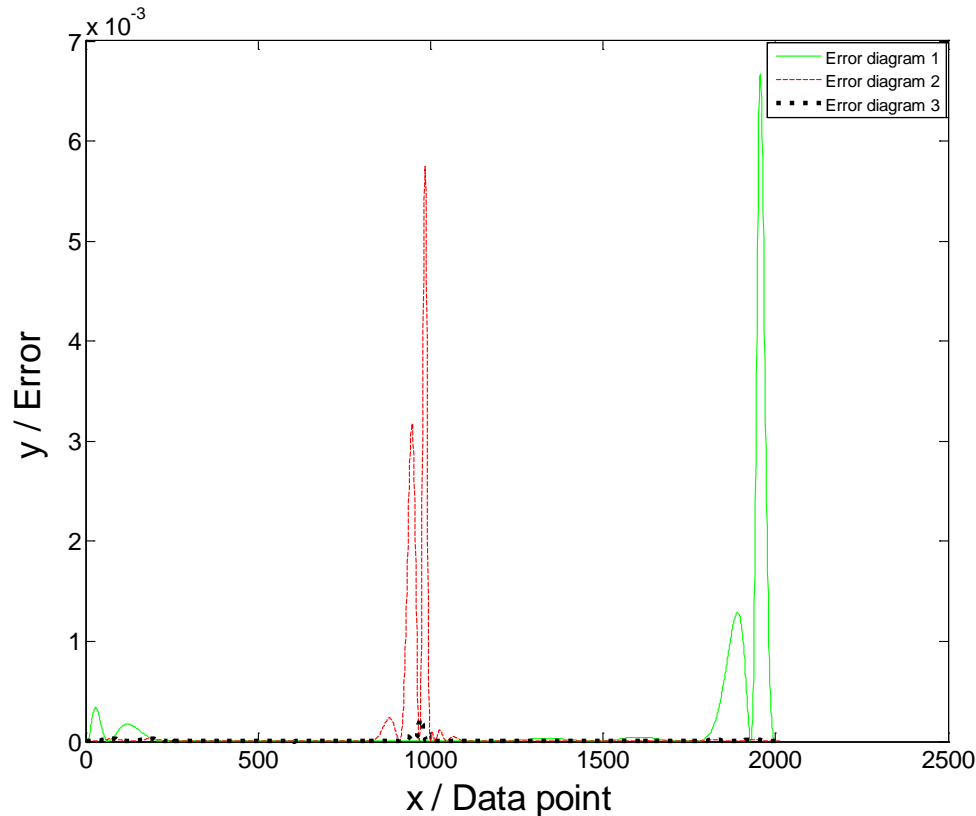


Figure 9. Error diagram of NURBS curve fitting profile

5. Conclusions

For the optimization design process of the cycloidal disc, the relative optimization of the objective function is achieved by adjusting the values of each parameter within a certain range, and the optimal value of each target is not obtained. The degree of freedom of the traditional rule curve cannot satisfy the problem of contour design flexibility. A method for optimizing and redesigning traditional rule curves using NURBS curves is proposed. By sampling the known cycloidal contours, the sampling points, inverse control points, weighting factors, node vectors and sampling points are obtained, and the analysis is verified. The error between the fitted NURBS curve and the original curve is within the feasible range to obtain the cycloidal contour with higher degree of freedom and curve continuity. The profile expressed by the method can not only accurately represent the cycloidal profile features, but also has excellent local modification characteristics.

The design method of NURBS profiles is proposed, and the design of the cycloidal profile of the reducer prototype is carried out with 3 NURBS curves. The cycloidal disk profiles with a precision error of 0.1 micron from the original profiles are designed by this method. According to the local controllability of NURBS curve, the cycloidal disk profile can be modified locally. The local characteristics of the cycloidal disc profile can be improved without affecting the performance of other parts, thereby

optimizing the cycloidal disc profile and further optimizing the performance of the reducer.

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