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Effects of Double Interfacial Layers on the Local Field Enhancement Factor and Optical Induced Bistability of a Small Spherical Metal/Dielectric Composites

Berhanu Aragie

Department of Physics, CNCS, Wolkite University, P.O.Box 007, Wolkite, Ethiopia

Abstract We propose the way of enhancing the enhancement factor of local field, and increasing the input domain threshold of the optical induced bistability of a spherical metal/dielectric composites within a linear host matrixes. The local field enhancement factor of a metal particles with dielectric core in the presence of double interfacial layers shows two maxima at two different frequencies. By introducing double interfacial layers, we calculate the enhancement factor of the local field. Also using the cubic equation the optical induced bistability of the composite material is calculated. Because of the double interfacial layers, we observe an increasing of the input domain threshold of the optical induced bistability. In comparison with the same composite without interfacial layer and with having single interfacial layer, our finding shows that, introducing additional interfacial layer makes the composite having a better enhancement factor and much better input domain threshold of the optical induced bistability.

Keywords Nonlinear Optics; Enhancement Factor; Interfacial Layer; Optical Induced Bistability

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1 Introduction

The rapid growth of studding the effects of nonlinear optical property becomes an interesting area in modern photonic functionalities, including ultrafast optical switching, optical transistors and optical modulation [1, 2, 3]. Enhancing the local electric field inside the metal/dieletric composite is one of the important ingredient in the development of nonlinear optical effect. The dielectric properties differences of the composite and the host matrix and the nanometer sizes and shape of metal/dielectric composite are factors for enhancing the local electric field inside the metal/dielectric composite. Not only that, the surfaceplasmon resonance that arises from localized surface plasmonic also the other factors for enhancing the local electric field inside the metal/dielectric composite [4, 5, 6]. Moreover, the presence of interfacial layer enhances the local electric field inside the metal/dieletric composite and increase the threshold of observing the optical induced bistability(IOB) [7].

Optical induced bistability is an optical effect of a system that a single value of the input field gives two different values of the local field intensity [8]. It is one of the nonlinear optical effect observed phenomenon in nonlinear plasmonicnano metal/dielectric composites [9, 10]. Because of its wide range of potential application in optoelectronics and logic elements, induced optical bistability in nonlinear plasmonicnano composites becomes an interesting area [11].

This work proposes the way of enhancing the local field and increasing the threshold of OIB inside the metal/dielectric composite by introducing double interfacial layers. We consider a metal/dielectric composite of dielectric core within a metal shell in a linear dielectric host matrix. The first interfacial layer lies between the dielectric core and the metallic shell of the composite and the other one between the metallic shell and the linear dielectric host matrix. Regardless of the nature of the first interfacial layer, we observe that the presence of the second interfacial layer enhances the enhancement factor of the local field. Not only that, it also increases the input domain threshold of optical induced bistability.

The remaining part of paper is organized as follows: Section 2, be about the effect of double interfacial layers on the enhancement factor of local field in small spherical metal/dielectric composite. In Section 3, we study the effect of double interfacial layers on the optical induced bistability of the same composite. Section 4 deals with summary and conclusion.

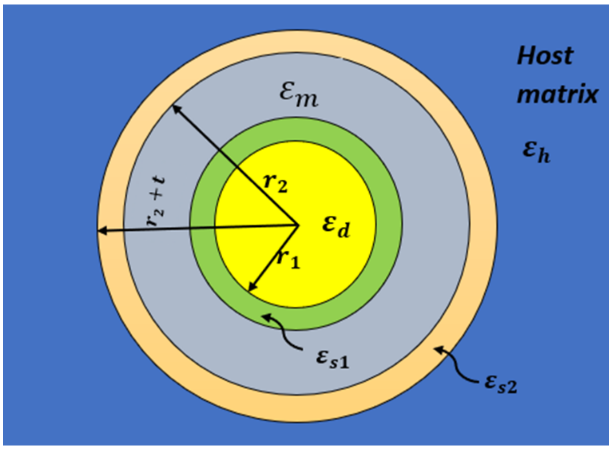

Figure 1. Metal/dielectric composite with double interfacial layers of dielectric ε_d in core, the first interfacial layer with dielectric ε_{s1}, metallic dielectric ε_m, the second interfacial layer of dielectric ε_{s2} and the external coverage host matrix ε_h.

2 Effects of double interfacial layers on the enhancement factor of local field in small spherical metal/dielectric composites

Consider a composite system that consists of spherical shaped dielectric core of dielectric function ε_d, with an interfacial layer of dielectric function ε_{s1} inside a metallic shell of dielectric function ε_m that enclosed by another interfacial layer having dielectric function ε_{s2} embedded in host matrix having a dielectric function ε_h as shown fig.1.

The distribution of the potential of small composite having double interfacial layers embedded in a linear dielectric host can be written as follows:

$$\Phi_d = -\mathbf{E}_h A r\cos\theta, \qquad r \le r_1 \tag{1}$$

$$\Phi_{s1} = -\mathbf{E}_h\left(Br - \frac{C}{r^2}\right)\cos\theta, \qquad r_1 \le r \le r_1 + t \tag{2}$$

$$\Phi_m = -\mathbf{E}_h\left(Dr - \frac{E}{r^2}\right)\cos\theta, \qquad r_1 + t \le r \le r_2 \tag{3}$$

$$\Phi_{s2} = -\mathbf{E}_h\left(Fr - \frac{G}{r^2}\right)\cos\theta, \qquad r_2 \le r \le r_2 + t \tag{4}$$

$$\Phi_h = -\mathbf{E}_h\left(r - \frac{H}{r^2}\right)\cos\theta, \qquad r > r_2 + t \tag{5}$$

where,Φ_d, Φ_{s1}, Φ_m, Φ_{s2}, and Φ_h are potential of the dielectric core, the first interfacial layer, the metal, the second interfacial layer, and the linear dielectric host matrix respectively. A, B, C, D, E, F, G, and H are the unknown coefficients. And r_1, $r_1 + t$, r_2 and $r_2 + t$ are the dielectric core, the first interfacial layer, the metal, and the second interfacial layer radiuses respectively. $\mathbf{E}_h$ is the applied filed and r and θ are the spherical coordinates respectively. Applying the conditions of continuity for the potential and displacement vector at the boundaries of each interfaces including the interfacial layers, we secure an expression for the unknown coefficients A, B, C, D, E, F, G and H. The solution of the unknown coefficients can be written as follows:

$$A = \frac{9\varepsilon_m\varepsilon_h}{2p\Delta} \tag{6}$$

$$B = \frac{3\varepsilon_m\varepsilon_h}{p\Delta} \tag{7}$$

$$C = \frac{-3\varepsilon_m\varepsilon_h r_1^3}{2p\Delta} \tag{8}$$

$$D = \frac{3\varepsilon_h(\varepsilon_d + 2\varepsilon_m + \frac{2I_1}{r_1})}{2p\Delta} \tag{9}$$

$$E = \frac{3\varepsilon_h(\varepsilon_d - \varepsilon_m + \frac{2I_1}{r_1})r_1^3}{2p\Delta} \tag{10}$$

$$F = \frac{\varepsilon_h((\varepsilon_d + \frac{2I_1}{r_1}) + \varepsilon_m(\frac{3}{p} - 1))}{\Delta} \tag{11}$$

$$G = \frac{-\varepsilon_h((\varepsilon_d + \frac{2I_1}{r_1}) + \varepsilon_m(\frac{3}{p} - 1))r_2^3}{2\Delta} \tag{12}$$

$$H = r_2^3 - \frac{3\varepsilon_h((\varepsilon_d + \frac{2I_1}{r_1}) + \varepsilon_m(\frac{3}{p} - 1))}{2\Delta} r_2^3 \tag{13}$$

where $\Delta = \varepsilon_m^2 + Q\varepsilon_m + (\varepsilon_d + \frac{2I_1}{r_1})(\varepsilon_h + \frac{I_2}{r_2})$ with $Q = (\frac{3}{p} - 1)(\varepsilon_h + \frac{I_2}{r_2}) + (\frac{3}{2p} - 1)(\varepsilon_d + \frac{2I_1}{r_1})$ and $p = (1 - (\frac{r_1}{r_2})^2)$ be the metal fraction in the inclusion. I_1 and I_2 are the interfacial layer factors between the dielectric core and the metal and between the metal and the dielectric host matrix respectively.

The interfacial layers have the properties that the limit, , $t \to 0$ (zero thicknesses), in the limit $\varepsilon_{s1}, \varepsilon_{s2} \to \infty$ with significant value of $\varepsilon_{s1}t, \varepsilon_{s2}t$ and defined as [12]

$$I_1 = \lim_{\varepsilon_{s1} \to \infty, t \to 0} \varepsilon_{s1} t, \qquad I_2 = \lim_{\varepsilon_{s2} \to \infty, t \to 0} \varepsilon_{s2} t \tag{14}$$

The values of I_1 and I_2 could be zero or positive(negative) depending on the perfectness or imperfectness of the interfacial layers. The values are zero when there is no jump in the normal component of the electric displacement across the interfaces; whereas, for imperfect interface jumps of the electric displacement across the interface occurs. A negative value of the interfacial layer shows metal-like behavior of the interfacial layer and dielectric-like behavior of the interfacial layer for positive value [7].

Let us choose the metal dielectric function in the inclusion of Drude form as

$$\varepsilon_m = \varepsilon_\infty - \frac{1}{Z(Z + i\gamma)} \tag{15}$$

with dimensionless frequencies

$$Z = \omega/\omega_p, \qquad \gamma = \nu/\omega_p \tag{16}$$

where ω, ω_p and ν are the incident radiation frequency, the metal shell frequency, and the electron collision frequency respectively. We can rewrite the above equation in the form of real and imaginary parts as

$$\varepsilon_m = \varepsilon_m' + i\varepsilon_m'' \tag{17}$$

where $\varepsilon_m' = \varepsilon_\infty - \frac{1}{(Z^2+\gamma^2)}$, $\varepsilon_m'' = \frac{\gamma}{Z(Z^2+\gamma^2)}$ and ε_∞ is the high frequency dielectric constant.

Furthermore, consider a nonlinear form of the inclusion dielectric function as

$$\varepsilon_d = \varepsilon_{d0} + \chi|\mathbf{E}|^2 \tag{18}$$

where ε_{d0}, χ, and $\mathbf{E}$ are the linear part of inclusion dielectric, the nonlinear kerr coefficient and the local field in the dielectric core respectively. For a weak incident field $\chi|\mathbf{E}|^2 \ll \varepsilon_{d0}$ the contribution of the nonlinear kerr coefficient part of the inclusion dielectric becomes negligible. The local field inside the dielectric core using eq. 1 becomes $\mathbf{E} = A\mathbf{E}_h$ with $\varepsilon_d = \varepsilon_{d0}$. However, because of the complex quantity of ε_m in the metal, the quantity A becomes a complex quantity. So, it is convenient to deal with $|A|^2$ called the enhancement factor and is given by

$$|A|^2 = \frac{81\varepsilon_h^2(\varepsilon_m'^2 + \varepsilon_m''^2)}{4p^2\{(\varepsilon_m'^2 - \varepsilon_m''^2 + Q\varepsilon_m' + (\varepsilon_d + \frac{2I_1}{r_2})(\varepsilon_h + \frac{I_2}{r_2}))^2 + \varepsilon_m''^2(2\varepsilon_m' + Q)^2\}} \tag{19}$$

Throughout this work I used the following parameters as fixed parameters. Those are $\varepsilon_\infty = 4.5$, $\varepsilon_d = 6$, $\varepsilon_h = 2.25$, $\omega_p = 1.46 \times 10^{16}$ and $\nu = 1.68 \times 10^{14}$.

The enhancement factor of the metal/dielectric composite with double interfacial layer versus Z for $p = 0.8$ (black line) and $p = 0.9$ (blue line) are as shown in fig. 2a. From the figure we observed that the strength of the two enhancement factor peak increase with increasing p. The first maxima peak of the enhancement factor shifted to the right where as the second maxima peak shifted to the left with increasing p. We listed out the data of the first and second maxima peaks of Z and their difference ΔZ of the two maxima peaks in table 1 1. From the data we observed that with increasing p, the value of Zt at the first maxima peak of $|A|^2$ shifted to the left and the second maxima peak shifted to the right. As a result the difference ΔZ of the two maxima peaks decreases with increasing p. This decreasing of ΔZ continued until the two maxima peaks of the enhancement factor merged. Moreover, changing the second interfacial layer I_2 from dielectric property (blue line) to metallic property (red line) as shown in fig. 2a, the strength of the two peak of the enhancement factor of the local field increase regardless of the nature of the

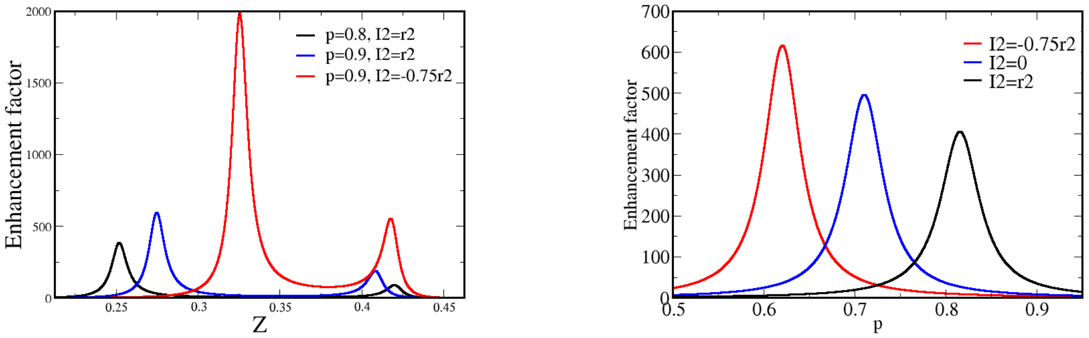

Figure 2. (a) (Color online) Enhancement factor versus Z of the metal/dielectric composite with $p = 0.8$ and $I_2 = 1r_2$(black line), $p = 0.9$ and $I_2 = 1r_2$(blue line) and $p = 0.9$ and $I_2 = -0.75r_2 =$ (red line). In both plots of a $I_1 = -1r_1$. (b) Enhancement factor versus p of the metal/dielectric composite with $I_2 = 1r_2$(black line), $I_2 = 0$ (blue line), and $I_2 = -0.75r_2$ (red line). In both plots $I_1 = -1r_1$ and of b $Z = 0.255$.

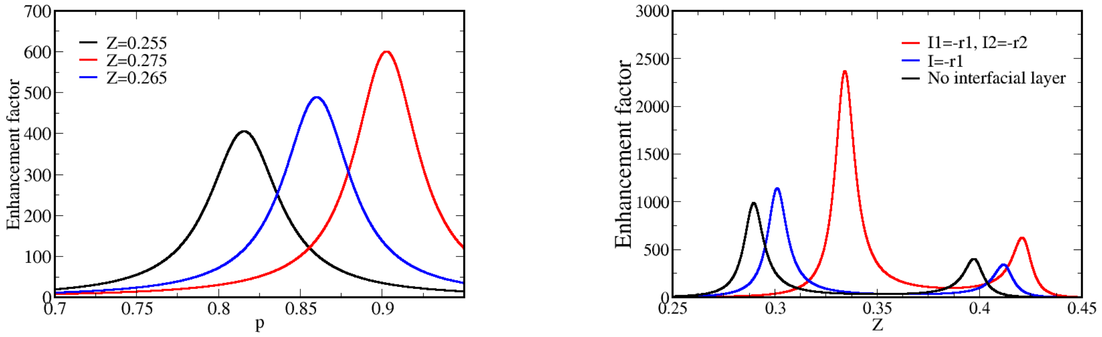

Figure 3. (a) (Color online) Enhancement factor versus p of the metal/dielectric composite with $Z = 0.255$ (black line), $Z = 0.265$ (blue line), and $Z = 0.275$ (red line). In both plots of a $I_1 = -1r_1$, and $I_2 = 1r_2$. (b) Enhancement factor versus Z of the metal/dielectric composite with no interfacial layer (black line), with single interfacial layer of $I = -1r_1$ (blue line), and double interfacial layer with $I_1 = -1r_1$ and $I_2 = -1r_2$ (red line). In both plots of b $p = 0.9$.

first interfacial layer. Not only the first maxima peak of the enhancement factor, but also the second maxima peak increases in changing the second interfacial layer factor, I_2, from dielectric property to metallic property as we listed out in table 2 2.

In fig. 2b we also plot the enhancement factor versus metallic fraction (p) for different values of second interfacial factor (I_2) with fixed Z . With the same value of Z, we observed that maximum peak of the enhancement factor occurred for metallic property of the second interfacial factor as shown fig. 2b. Our data shows that with fixed $Z = 0.255$ the values of p in which maximum enhancement factor peak appears for $I_2 = -0.75r_2$, $I_2 = 0$, and $I_2 = 1r_2$ at $p = 0.62044$, $p = 0.71047$, and $p = 0.81563$ respectively. This shows that maxima peak appears in early value of p for metallic property of the second interfacial layer. Furthermore, from fig 3a we observed that with fixed values of $I_1 = -r_1$ and $I_2 = r_2$, the values p of maximum enhancement factor peaks for $Z = 0.255$, $Z = 0.265$, and $z = 0.275$ occurred at $p = 62044$, $p = 0.8603$, and $p = 0.90259$ respectively. This indicates that the enhancement factor shifts to the right with increasing Z as shown in fig. 3a. Not only shifting of the enhancement factor peak , but the strength of $|A|^2$also increases and this result agreed with the comparison of the blue and red colors of fig. 2a and the data of table 1 1.

Table 1. Z of first and second maxima peak of $|A|^2$ at different values of p. $I_2 = 1r_2$ and the rest of the parameters of the composite are specified in Fig. 2a.

p	Z_1	Z_2	$\Delta Z = Z_2 - Z_1$
0.8	0.25188	0.41986	0.16798
0.85	0.26302	0.41439	0.15137
0.9	0.27475	0.40814	0.13339

In comparison with the same composite without interfacial layer (black line) and having only one interfacial layer (blue line) in fig. 3b, the strength of the enhancement factor becomes stronger in the presence of double interfacial layer with metallic property. We observe that for a composite with no interfacial layer (black line) has stronger second maxima peak as compared

Table 2. First and second maxima peak of $|A|^2$ at different values of I_2. Rest of the parameters of the composite are specified in Fig. 2a.

I_2	1^{st} peak of $\|A\|^2$	2^{nd} peak of $\|A\|^2$
$1r_2$	594.9082	188.9562
0	1142.3	342.9483
$-0.75r_2$	1983.6	553.7222

with the same composite with single interfacial layer (blue line), the strength of the in the presence of the interfacial layer with metallic property. However, in the presence of double interfacial layer both the first maxima and the second maxima of the enhancement increases with metallic property of the second interfacial layer. This is one of our finding that observed in the presence of double interfacial of the same composite. In the next section we will see the effect of double interfacial layer on the optical induced bistability.

3 Effects of double interfacial layers on the optical induced bistability of small spherical metal/dielectric composites

It is clear that the field intensity inside the cavity increases with increasing the input intensity. More over, increasing the field intensity inside the cavity results lowering the absorption that the field experiences. Because of reducing the absorption of the material, the field inside the cavity tends to remain large if the intensity of the incident field is subsequently lowered. This give us within some range of input intensities more than one output intensity. This is called optical induced bistability. Optical induced bistability is an optical effect of a system that a single value of the input field gives two different values of the local field intensity. From the nonlinear form of the inclusion dielectric core (eq.18) the local field in the presence of weak incident field ($\chi|\mathbf{E}|^2 \ll \varepsilon_{d0}$) becomes

$$\mathbf{E} = A\mathbf{E}_h \tag{20}$$

with $\varepsilon_d = \varepsilon_{d0} + \chi|\mathbf{E}|^2$ and $\varepsilon_m = \varepsilon'_m + i\varepsilon''_m$ in A.

Since A is a complex value, then it is convenient to use $|A|^2$. So, squaring and multiplying both sides of the above equation by χ gives

$$\chi|\mathbf{E}|^2 = \chi|A|^2|\mathbf{E}_h|^2 \tag{21}$$

By letting $\mathbf{X} = \chi|\mathbf{E}|^2$ and $\mathbf{Y} = \chi\mathbf{E}_h|^2$, we obtain the following cubic equation for $\mathbf{X}$:

$$\alpha\mathbf{X}^3 + \beta\mathbf{X}^2 + \delta\mathbf{X} = \eta\mathbf{Y} \tag{22}$$

where

$$\alpha = e_2^2 + e_4^2 \tag{23}$$

$$\beta = 2(e_1e_2 + e_3e_4) \tag{24}$$

$$\mathbf{Y} = \chi|E_h|^2 \tag{25}$$

$$\delta = e_1^2 + e_3^2 \tag{26}$$

$$\eta = \frac{81\varepsilon_h^2(\varepsilon'^2_m + \varepsilon''^2_m)}{4p^2} \tag{27}$$

with $a = \frac{3}{p} - 1$, $b = \frac{3}{2p} - 1$, $c = \varepsilon_{d0} + \frac{2I_1}{r_1}$, $d = \varepsilon_h + \frac{I_2}{r_2}$, $e_1 = ((ad + bc)\varepsilon'_m + dc + \varepsilon'^2_m - \varepsilon''^2_m)$, $e_2 = b\varepsilon'_m + d$, $e_3 = \varepsilon''_m(2\varepsilon'_m + ad + bc)$, and $e_4 = \varepsilon''_m b$.

The cubic equation (Eq. 22) have either one real positive root or three real positive roots depending up on the applied field. If it has one real positive root then, the local field in the nano-particle is a single-valued function of the applied field. Otherwise the local field is not a single-valued function and have three real positive roots of the applied field and the system becomes unstable. And this is called optical induced bistability (OIB). Usually OIB illustrated in the $\mathbf{Y} - \mathbf{X}$ plane and observed S-like curves.

Increasing of $|E_h|^2$ from zero gives increasing of $|E|^2$ monotonically. However, after a certain value of $|E_h|^2$, decreasing the value of $|E_h|^2$ gives increasing of the local field $|E|^2$. Using linear stability analysis this branch of solution is unstable. That means if the system is initially in this state, it will rapidly switch to one of the stable solutions through the growth of small perturbations. So, with increasing of $|E_h|^2$ from zero to and when it passes the turning point in the lower branch then

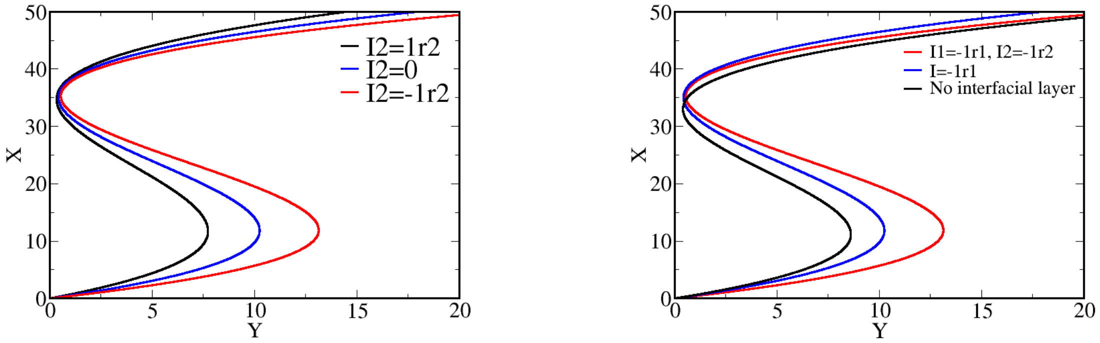

Figure 4. (a) (Color online) (a) Optical induced bistability (IOB)in a small spherical metal/dielectric composite with $I_2 = 1r_2$ (black line), $I_2 = 0$ (blue line) and $I_2 = -1r_2$ (red line) in linear host matrix. In both plots of a, $I_1 = -1r_1$, (b) Optical induced bistability (IOB)in a small spherical metal/dielectric composite without interfacial layer (black line), with single interfacial layer of $I = -1r_1$ (blue line) and double interfacial layer of $I_1 = -1r_2$ (black line) and $I_2 = -1r_2$ (red line) in linear host matrix. Other parameter fixed as $p = 0.99$ and $z = 0.2$

immediately it switching up to the upper branch. On the other hand, if the input intensity is slowly decreased, the system will remain on the upper branch and the output intensity will continue and at the turning point switching down to the lower branch. As a result we observe optical induced bistability within a certain domain of input intensity. We plots the numerical results of optical induced bistability for three different values of the second interfacial factor with $I_2 = 2r_2$ (black line), $I_2 = 0$ (blue line), and $I_2 = -2r_2$ (red line), by taking the same value of the first interfacial factor $I_1 = -2r_1$ as shown in fig. 4a. Regardless of the nature of the first interfacial layer, the input domain threshold of optical induced bistability increase for the transition of the second interfacial layer from dielectric property to metallic property. The switching up and switching down input intensity for different interfacial layer factors are listed out in table 3 3. For the transition of the second interfacial layer from dielectric property to metallic property both switching up and switching down input intensities increase. Not only that but also the input intensity gap between switching up and switching down input intensities increases. Further more, the switching up input intensity strength for changing of the second interfacial factor to metallic increases in a large rate in comparison to the switching down input intensity. This wide range of optical induced bistability state can make such system to be preferable in optical memory device and optical switch. . In fig. 4b, we compare the input domain threshold of optical induced bistability for a composite without interfacial layer, having single interfacial layer, and with double interfacial layer of the same composite. It is understood that a composite without interfacial layer is the same as composite having double interfacial layer with $I_1 = 0$ and $I_2 = 0$. Similarly a composite having single interfacial layer is the same as composite having double interfacial layer with $I_1 \neq 0$ and $(I_2 = 0)$ interfacial factors. According to fig. 4b, our result showed that, the input domain threshold values of the optical induced bistability increased in the presence of double interfacial layers with metallic property (red line). However, the rang of switching up and switching down input intensity decreases with increasing dielectric property of the second interfacial layer.

Table 3. **Y**at different values of I_2. Rest of the parameters of the composite are specified in Fig. 3a.

I_2	Switch up **Y**	Switch down **Y**
r_2	7.73160	0.33354
0	10.25287	0.42971
-1r_2	13.1341	0.54016

In general, depending on the nature of the second interfacial layer we can increase the switching up input intensity and the range of the local field that optical induced bistability observed. Optical induced bistability observed in a wide domain of the input field for the transition of the second interfacial layer from dielectric property to metallic property.

4 Summary and Conclusion

In this we proposed a metallic/dielectric composite with double interfacial layers. The interfacial layers are between the dielectric core and the metal shell and between the metal shell and the host matrix. The enhancement factor with an input electric field shows two maxima at two different resonant frequencies. Depending on the nature of the second interfacial layer, the enhancement factor becomes stronger in comparison with a composite without interfacial layer and a composite having single interfacial layer. The enhancement factor grows with increasing the metal fraction in the inclusion and metallic nature of the interfacial layer. Furthermore, depending on the nature of the second interfacial layer, the input domain threshold of optical induced bistability is stronger for metallic property. Also, within a small domain of the input domain, optical induced bistability is observed for dielectric property of the second interfacial layer.

In conclusion, this work suggested an alternative ways of enhancing the enhancement factor and increasing the threshold of optical induced bistability in a metal/dielectric composite. It helps for further study of memory device and optical switch.

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