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A Numerical Algorithm for the Analysis of the Thermal Stress-Stain State of a Rod

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Abstract A universal algorithm for solving applied tasks of thermo elastic state of a partially heat insulated rod of limited length, in the presence of axial force, temperature, heat flow and heat exchange, was developed. The corresponding numerical calculation of efforts of the rod partially thermally insulated and clamped by two ends, in the presence of heat flow was elaborated. The numerical algorithm allowed calculate the lengthening value of the partially thermally insulated rod or the compressive stress and strain in the rods of limited length with the heat flow, the heat exchange, heat insulation and the axial tensile force.

Keywords Universal Algorithm, Thermo Elastic State, Limited Length; Axial Force, Heat Flow, Efforts of the Rod, Axial Tensile Force

1. Introduction

Construction of numerical algorithms and complex of programs, as well as the solution of the task of the steady thermal elastic state of a partially isolated heat rod while the simultaneous effects of axial force, temperature and heat flow represent the appropriate mathematical and computational difficulties. In view of the importance of ensuring stable work of construction elements, while the simultaneous thermal and mechanical action, we are really convinced that the development of universal computing algorithms of established processes, of the field temperature distribution, of elastic displacements and thermo elastic condition of construction elements with mechanical forces, heat flow, partial insulation and heat exchange is the actual problem of modern numerical methods.

With simultaneous consideration of the above factors, the analytical solution of the steady thermo elastic compression and stretching of the rods becomes very complicated. In this connection, the need naturally arises to develop appropriate universal computational algorithms oriented to modern computer facilities and object-oriented high-level programming languages that allow one to investigate the thermo elastic state of partially insulated one-dimensional structural elements, in the presence of heat exchange with the environment, the temperature field, thermal and axial tensile forces.

On the basis of the energy principles and finite element method with quadratic elements, the universal numerical algorithm for solving applied tasks of thermo elastic state of a partially heat insulated rod of limited length, in the presence of axial force, temperature, heat flow and heat exchange, was developed.

The field regularities of distribution of the temperature, displacement, deformation and strain along the length of the rod, depending on the area of the lateral heat-isolated surface, on the applied temperature, heat flow, heat exchange and heat exchange coefficients with the surrounding parts of the lateral surface, were revealed.

2. Calculation Scheme of the Task

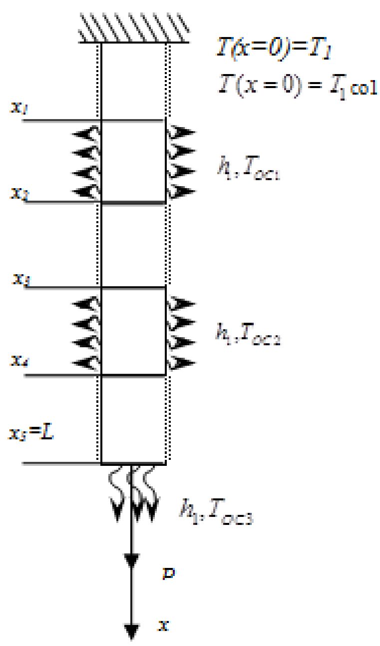

Figure 1. Calculation scheme of the task

Earlier the authors made the numerical study of lengthening of partially heat-isolated rod in the presence of a heat sources and tensile strength is given. The numerical study with the different source data was conducted [1].

To do it, the rod L with length will be to discrete quadratic finite elements with three nodes.

Then, for the final elements in the heat isolated lateral surface parts, the view o the functional, expressing conservation and change of the total thermal energy is as follows

$$I_i = \int_{V_i} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV \quad (1)$$

For the finite element heat isolated by the lateral surface, the functional type is as follows

$$I_n = \int_{V_n} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int_{S_{ss}^s} \frac{h_3}{2} (T - T_{OC3})^2 dS \quad (2)$$

On the lateral surface $x_1 \le x \le x_2$ of the rod, the heat exchange with the environment takes place.

$$I_j = \int_{V_j} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int_{S_{sls}^j} \frac{h_1}{2} (T - T_{oc1})^2 dS \quad (3)$$

On the lateral surface $x_3 \le x \le x_4$ of the rod, the heat exchange with the environment takes place as well.

$$I_k = \int_{V_k} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int_{S_{sls}^k} \frac{h_2}{2} (T - T_{OC2})^2 dS \quad (4)$$

Then, for the rod in general, the expression of the corresponding functional is as follows

$$I = I_i + I_j + I_k + I_n . \quad (5)$$

The influence of temperature and tensile strength on the lengthening of the rod was analyzed [1].

Moreover, the lengthening of considered rod is affected the by heat transfer coefficient $h, \left(Vt/cM^2 \cdot {}^\circ C \right)$ between the rod material and the cross-sectional area of the clamped rod ends [2].

3. The Payment Scheme

The numerical study of thermal strain deformed of partially heat isolated and clamped by two rod ends of limited length in the presence of different heat sources was carried out [3].

The field of temperature distribution along the length of the rod, the law of distribution of elastic displacements, strain and stress components along the length of the rod was defined. For this, the length of the rod is divided into equal n - parts. Then the length of one side will be $l = L/n$. The number of nodes of the discrete elements will be $2n+1$. The field of distribution of temperature and elastic displacement within each element is approximated by a quadratic curve passing through three points.

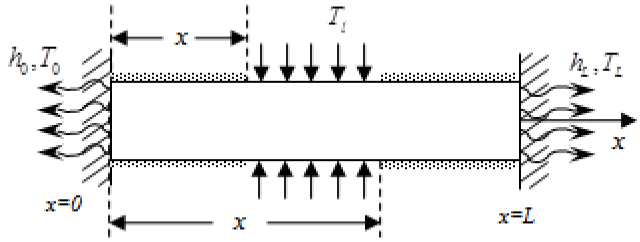

Figure 2. The payment scheme

$$\left. \begin{aligned} T(x) &= \varphi_i(x) T_i + \varphi_j(x) T_j + \varphi_k(x) T_j \\ u(x) &= \varphi_i(x) u_i + \varphi_j(x) u_j + \varphi_k(x) u_k \end{aligned} \right\} \quad (6)$$

For the first element the form of the functional that expresses the full heat energy is as follows

$$I_1 = \frac{FK_{xx}}{2l} \begin{bmatrix} \frac{7}{3} T_1^2 - \frac{16}{3} T_1 T_2 + \frac{2}{3} T_1 T_3 - \\ -\frac{16}{3} T_2 T_3 + \frac{16}{3} T_2^2 + \frac{7}{3} T_3^2 \end{bmatrix} + \frac{Fh_0}{2} (T_1 - T_0)^2 \quad (7)$$

And for all other elements of the portion $0 \le x \le x_1$ of the rod, the analogous functional form will be as follows

$$I_{2i} = \int_{V_i} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV = \frac{FK_{xx}}{2l} \begin{bmatrix} \frac{7}{3} T_i^2 - \frac{16}{3} T_i T_j + \frac{2}{3} T_i T_k - \\ -\frac{16}{3} T_j T_k + \frac{16}{3} T_j^2 + \frac{7}{3} T_k^2 \end{bmatrix} \quad (8)$$

The last n element was considered. The lateral surface of this element was thermally insulated. But through cross sectional area of the right end, which corresponds to the section $x = L$, a heat exchange with the environment takes place. Then this element is characterized by full expression of functional thermal energy has the following form

$$\begin{aligned} I_n &= \int_{v_n} \frac{K_{xx}}{2} \left(\frac{\partial T}{\partial x} \right)^2 dV + \int_{s_{n+1}} \frac{h_L}{2} (T - T_L)^2 dS = \\ &= \frac{FK_{xx}}{2l} \left[\frac{7}{3} T^2{}_{n-1} - \frac{16}{3} T_{n-1} T_n + \right. \\ &\left. + \frac{2}{3} T_{n-1} T_{n+1} - \frac{16}{3} T_n T_{n+1} + \frac{16}{3} T_n^2 + \frac{7}{3} T_{n+1}^2 \right] + \\ &+ \frac{Fh_L}{2} (T_{n+1} - T_L). \end{aligned} \qquad I = \sum_{i=1}^{n} I_i \quad (9)$$

By minimizing, I by T_i we get the resolution system of algebraic equations regarding the given parameters, i.e.

$$\frac{\partial I}{\partial T_i} = 0 \quad (10)$$

Solving the resulting system of linear algebraic equations by Gauss method, we determine the nodal temperatures T_i.

Then we present the second part of the considered problem. For each quadratic finite element with the length l_1 the expression of the potential energy of elastic deformations, taking into account the available steady temperature field, is

$$\Pi_i = \int_{V_{i\kappa}} \frac{\sigma_x \varepsilon_x}{2} dV - \alpha E \int_{V_i} T(x)\varepsilon_x dV \qquad (11)$$

Within this element, the expression of the elastic component of deformations is as follows

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{1}{l_1^2}\left[\begin{matrix}(4x-3l_1)u_i + \\ +(4l_1-8x)u_j + (4x-l_1)u_k\end{matrix}\right]. \qquad 2)$$

After a little simplification for i-quadratic finite element, the expression of the potential energy of elastic deformation (11) has the following form

$$\Pi_i = \frac{E}{2}\int_{V_i}\left(\frac{\partial u}{\partial x}\right)^2 dV - \alpha E \int_{V_i} T(x)\frac{\partial u}{\partial x} dV =$$

$$= \frac{EF}{2}\int_0^{l_1} \varepsilon_x^2 dx - \alpha EF \int_0^{l_1} T(x)\varepsilon_x dx \qquad (13)$$

Then, the form of the functional expressing the potential energy of the considered rod clamped at both ends will be as follows

$$\Pi = \sum_{i=1}^{ЧКЭ} \Pi_i \qquad (14)$$

Next we present the minimization of the total potential energy of elastic deformation of the considered rod by the nodal values of displacements

$$\frac{\partial \Pi}{\partial u_i} = 0, \; i = 2,3,\ldots,nn \qquad (15)$$

We determine the value of the elastic displacements of the rod sections that correspond to the element nodes. After that, we propose the algorithms for computing the components of the elastic and thermal components of deformations and strains for half of each finite element [4].

$$\left.\begin{aligned} &\varepsilon_x\left(x = \frac{x_j - x_i}{2}\right) = \frac{\partial u}{\partial x}\left(x = \frac{x_j - x_i}{2}\right); \\ &\sigma_x\left(x = \frac{x_j - x_i}{2}\right) = E\varepsilon_x\left(x = \frac{x_j - x_i}{2}\right); \\ &\sigma_T\left(x = \frac{x_j - x_i}{2}\right) = -\alpha ET\left(x = \frac{x_j - x_i}{2}\right). \end{aligned}\right\} \qquad (16)$$

4. The Analysis of the Effect

After testing of the developed computational algorithm, the influence of local temperature and heat exchange on the thermal-strained and deformed state of the test rod is analyzed [5]. To do this, the values of the compressive force and the true stress on different rod parts are calculated $R,(\kappa G)$. The influence of the heat flow on the thermal strained deformed and state of the test rod was analyzed $\sigma,(\kappa G/см^2)$.

The analysis of the effect of heat flow on the cross-sectional area and analyses of the effect of the heat exchange with the environment through a portion of the lateral surface on the thermal-strained and deformed state of the test rod were carried out. These results are shown in Fig. and 3-6 in Table 1.

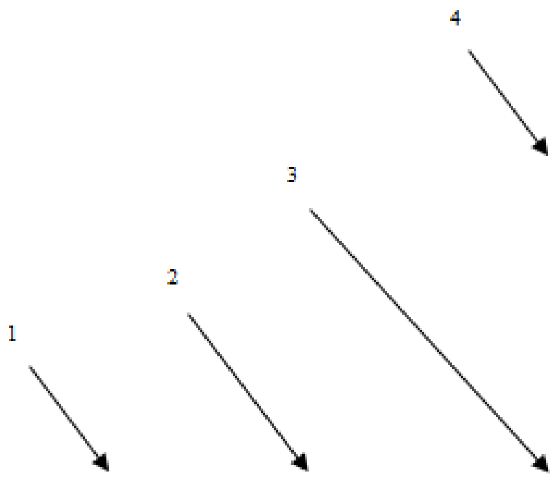

Figure 3. The field of temperature distribution along the length of the rod

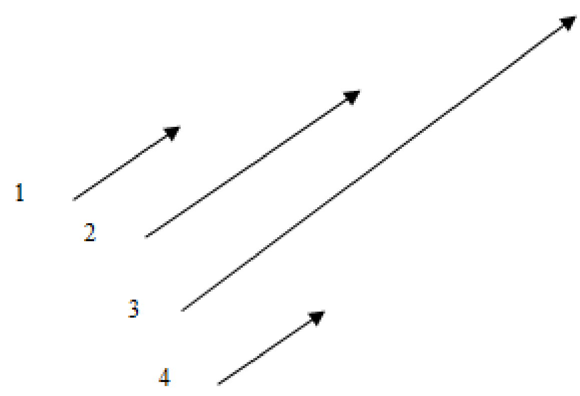

Figure 4. The law on the distribution of displacements of nodal points along the rod length

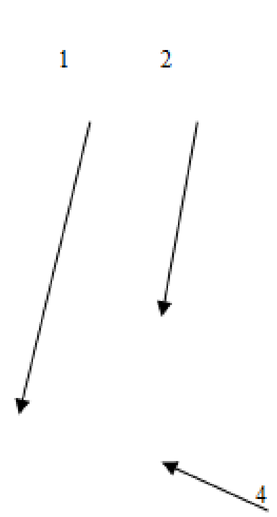

Figure 5. The field of distribution ε_x of along the length of the rod

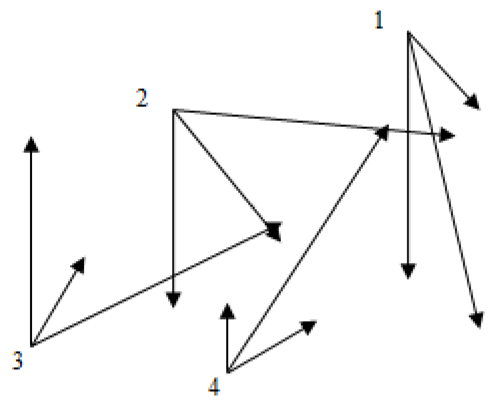

Figure 6. The field of distribution $\sigma_x, \sigma_T, \sigma$ along the length of the rod

The influence of heat flow on the thermal strained and deformed state of the test rod.

№	Area of rod	$R,(\kappa G)$	$\sigma,(\kappa G/c\text{м}^2)$	%
1	$0 \le x \le 16(c\text{м})$	-20137	-1006,85	100
2	$16 \le x \le 32(c\text{м})$	-16804	-840,18	83,44
3	$32 \le x \le 48(c\text{м})$	-15693	-784,63	77,92

5. Conclusions

1. The corresponding calculation algorithm of the numerical investigation of efforts of the rod partially thermally insulated and clamped by two ends, in the presence of heat flow was elaborated. At the same time it was revealed that under the heat flow in the areas $0 \le x \le 16(c\text{м})$, $16 \le x \le 32(c\text{м})$, $32 \le x \le 48(c\text{м})$, $48 \le x \le 64(c\text{м})$ and $64 \le x \le 80(c\text{м})$ of the rod, the component values of deformations and stress components will vary accordingly ε_x = 100; 138.18; 148.18, 130.08 and 83.81%, σ_T = 100; 141.64; 152.93, 133.85 and 84.40%, σ_x = 100; 138.20; 148.18; 130 and 83.80%, σ = 100; 143.53; 155.50, 135.89 and 84.72%.
2. On the basis of the developed mathematical model, corresponding to the computational algorithm, the set of applications for solving the class of problems in order to determine the temperature distribution field, displacements, deformations, strains was developed. This complex will make it possible to calculate the lengthening value of the partially thermally insulated rod or the compressive stress and strain in the rods of limited length with the heat flow, the heat exchange, heat insulation and the axial tensile force.

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