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Basic MBF Blocks Properties and Rank 6 Blocks

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Abstract Considered all the Monotone Boolean Functions (MBFs) blocks of the sixth rank, it proved a series of new properties MBF blocks. The proposed methods can be used to analyze large ranks MBFs. The tables which describe all of the blocks from 4th to 6th rank are provided. On the basis of these tables are counted typical depending to these blocks.

Keywords Monotone Boolean Functions, Inequivalent MBFs, Disjunctive Complement, Conjunctive Complement, Dedekind Number, Types of MBF Blocks

1. Introduction

In 1897 R. Dedekind published the article [1], in which he found the number of free distributive lattice elements with four generators. The number of free distributive lattice ψ (*n*) with *n* generators coincides with the antichain number in a unit *n*-dimensional cube. In terms of logical algebra, *D* (*n*) = ψ (*n*) + 2 — is the number of Monotone Boolean Functions (MBFs), depending on *n* variables $x_1,..., x_n$. The ψ (*n*) calculation problem is usually called Dedekind problem. *D* (0) – *D* (5) were received by Church in 1940, *D* (6) was calculated by Ward in 1946, *D* (7) was calculated by Church in 1965 and *D*(8) was received by Wiedemann [2] in 1991. As it turned out, this problem is rather difficult and it defies solution within the frames of traditional method of generating functions.

However, the method of MBFs classification and analysis based on MBF blocks generation (MBF sets connected by three operations discussed below) has not been considered in literature, excluding the studies [3] and [4], which in a number of cases allows cancelling the MBF search due to the exclusion of all isomorphic blocks and considering only nonisomorphic ones. The MBF blocks analysis allows considering this problem from a new angle.

In [5] the first MBF blocks of various ranks were shown.

The aim of the article is conclusion the basic properties of MBFs and analysis of all the blocks from 4 to 6 ranks.

2. Results

Let us recall the main concepts in connection with MBF. Boolean function $f\left(x_1,...,x_n\right)$ is called monotone, if for any pairs of variables values sets $\left(a_1,...,a_n\right)$ and $\left(b_1,...,b_n\right)$, for which the ratio $\left(a_1,...,a_n\right) \le \left(b_1,...,b_n\right)$ is correct, the inequality $f\left(a_1,...,a_n\right) \le f\left(b_1,...,b_n\right)$ is also correct. Among elementary Boolean functions, for example, conjunction and disjunction are monotonic. Any function received from MBFs with the aid of superimposition operation is monotonic by itself. This is why the function generated with the operations of disjunction and conjunction shall be also monotonic.

Say, conjunction *B* is absorbed by conjunction *A* if conjunction *B* contains all variables that are present in conjunction *A*. We shall consider MBF as a Disjunctive Normal Form (DNF), i.e. a sum of products, in which no one is absorbed by the other one. In other words, one set of conjunctions absorbs the other set if all conjunctions of one set are absorbed by the conjunctions of the other set.

We shall designate the Boolean function of n variables (n rank) – $f_i\left(n\right)$, here *i* – is the MBF ordinal number.

We shall use a binary form to present functions. Boolean function $f\left(n\right)$ in the form of a bit vector $f\left(n\right) = a_1 a_2 ... a_t$, where $t = 2^n$, a_i can take either the value of 0 or 1. At that, we shall write the value of the function taken at the lowest set on the right side, while the value taken at the highest set – on the left side. The sets shall be arranged in the following order: the lowest variable x_1, on the right, the highest variable – x_n on the left. For example, we shall take a monotonic function from 3 variables $f\left(3\right) = x_1 x_2 = 10001000$. This function equals to 1 only subject to sets $x_3 x_2 x_1 = 011$ and $x_3 x_2 x_1 = 111$. With other sets the function equals to 0.

In [6], three unary operations were determined on set of MBFs of any ranks. Duality designated as φ^{-1}, disjunctive complement designated as $\overline{\varphi}$ and conjunctive complement designated as $\underline{\varphi}$. To obtain disjunctive complement $\overline{f_i}$ (*n*) from i-MBF $f_i(n)$ it is necessary to replace in minimal disjunctive form each conjunction of m variables with conjunction of all $n - m$ variables, beyond original conjunction. To obtain conjunctive complement $\underline{f_i}(n)$ from i-й MBF, $f_i(n)$ of m variables shall be replaced in minimal conjunctive form with disjunction of all $n - m$ variables beyond original disjunction. To obtain dual MBF $f_i^{-1}(n)$ from i-й MBF $f_i(n)$ replace all conjunctive operations in minimal disjunctive form with disjunctive operations and simultaneously replace all disjunctive with conjunctive operations. In this case dual MBF $f_i^{-1}(n)$ takes a minimal conjunctive form. To obtain dual MBF $f_i^{-1}(n)$ in minimal disjunctive form, remove the parenthesis in the obtained minimal conjunctive form and reduce similar terms.

Definition. MBF block is a set of MBFs completed as regards three operations: duality, disjunctive complement and conjunctive complement, so that any MBF of the block can be obtained from any MBF of the same block by using a certain sequence of these three operations.

For example, for ranks 0 and 1 there exists only one block which consists in the first case of two functions and in the second case – of three functions

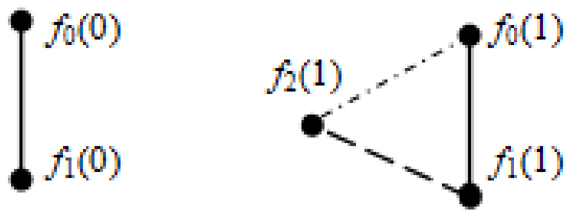

Figure 1. Blocks of zero and first ranks

The dual operation is shown here as a solid graph, disjunctive complement operation – as a dashed line and conjunctive complement operation – as a dash-dot line, $f_0(n)$ – zero MBF (equal to 0 with any variables set), $f_1(n)$ - – unit MBF (equal to 1 with any variables sets), $f_2(1) = x_1$

Lemma 1. The operation of conjunctive complement is equivalent to a sequence of dual operations, a disjunctive complement and again a dual operation. I.e. in our terms: $\underline{\varphi} = \varphi^{-1}\overline{\varphi}\varphi^{-1}$

Proof. It is known that dual function g can be obtained by replacing conjunction operations with disjunction operations in initial f operation of conjunction, subject to preserving the operations priority; and vice versa, if the same replacement of operations is carried out in a dual function, we shall obtain the initial function written down in conjunctive normal form (CDF). It follows here from that conjunctive complement of f function and disjunctive complement of g function shall be dual to one another. The lemma is proved.

Consequence. A block can be defined as a set of MBFs closed in relation to three operations: duality, disjunctive complement and conjunctive complement and such that any MBF of the block from any function of the block by using a certain sequence of only two operations φ^{-1} and $\overline{\varphi}$.

In this way, a block of functions can be constructed of one function f by using to this function successively the operation of disjunctive complement and then the duality operation. Each function can belong to only one block.

We shall introduce some notions. Block potency is the number of MBFs included. Two blocks are similar if they possess equal potency and provided the MBFs included in them are disregarded, they are identical. Two blocks are isomorphic if any MBF of one block can be obtained from a certain MBF of another block by some variables substitution. By convention, isomorphic blocks are similar.

All MBF blocks can be divided into 4 types. It follows from lemma 1 that all MBFs can be connected with a sequence of two operations – duality and disjunctive complement. The sequence of these two operations can be disconnected or cyclic. In the first case obtain three types of blocks. Blocks of first type is called a block, in which at the ends of the open-loop of sequence are 2 disjunctive self-complementary MBF. Blocks of second type are called a block, in which at one end of the open-loop of sequence are self-dual MBF, and at the other end are disjunctive self-complementary MBF. Block of third type is called a block, in which at the ends of the open-loop of sequence are 2 self-dual MBFs. In second case, self-dual and disjunctive self-complementary MBF no. We get the fourth type of block, which we call cyclic. Examples of MBF blocks are shown below:

1. Containing 2 disjunctive self-complementary MBFs.

Example of such 4 rank block:

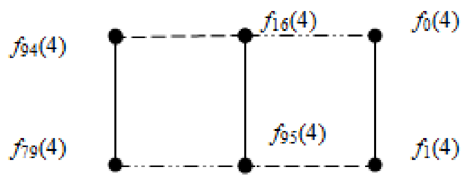

Figure 2. Block containing 2 disjunctive self-complementary MBFs

Here $f_{16}(4) = x_1 \vee x_2 \vee x_3 \vee x_4$, $f_{79}(4) = x_1x_2 \vee x_1x_3 \vee x_1x_4 \vee x_2x_3 \vee x_2x_4 \vee x_3x_4$, $f_{94}(4) = x_1x_2x_3 \vee x_1x_2x_4 \vee x_1x_3x_4 \vee x_2x_3x_4$, $f_{95}(4) = x_1x_2x_3x_4$. Functions $f_0(4)$ and $f_{79}(4)$ are disjunctive self-complementary.

From this point on the functions are taken from studies [3] and [4].

2. Containing 1 disjunctive self-complementary MBF and 1 self-dual MBF.

Example of such 4 rank block:

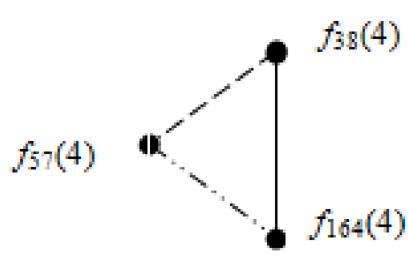

Figure 3. Block containing 1 disjunctive self-complementary MBF and 1 self-dual MBF.

Here $f_{38}(4) = x_1x_2 \vee x_1x_3 \vee x_1x_4$, $f_{57}(4) = x_2x_3 \vee x_2x_4 \vee x_3x_4$,

$f_{164}(4) = x_1 \vee x_2x_3x_4$. Function $f_{164}(4)$ disjunctive self-complementary, while $f_{57}(4)$ self-dual.

3. Containing 2 self-dual MBFs.

Example of such 4 rank block:

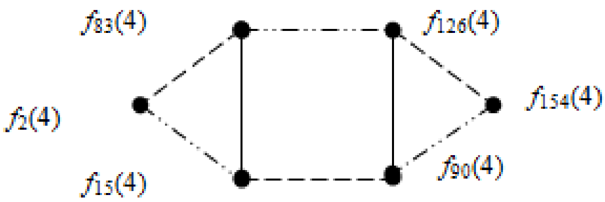

Figure 4. Block containing 2 self-dual MBFs.

Here $f_2(4) = x_1$, $f_{83}(4) = x_2x_3x_4$, $f_{15}(4) = x_2 \vee x_3 \vee x_4$, $f_{90}(4) = x_1x_2x_3 \vee x_1x_2x_4 \vee x_1x_3x_4$, $f_{126}(4) = x_1 \vee x_2x_3 \vee x_2x_4 \vee x_3x_4$ и $f_{154}(4) = x_1x_2 \vee x_1x_3 \vee x_1x_4 \vee x_2x_3x_4$. MBFs $f_2(4)$ and $f_{126}(4)$ are self-dual.

4. Cyclic blocks.

Example of such 4 rank block:

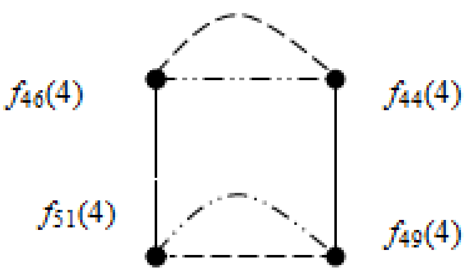

Figure 5. Cyclic block of 4 rank.

Here $f_{46}(4) = x_1x_2 \vee x_2x_3 \vee x_3x_4$, $f_{44}(4) = x_1x_2 \vee x_1x_4 \vee x_3x_4$, $f_{49}(4) = x_1x_3 \vee x_1x_4 \vee x_2x_4$, $f_{51}(4) = x_1x_3 \vee x_2x_3 \vee x_2x_4$. All functions of the block are connected by three operations.

Blocks consisting of two functions can be of two types, consisting of three functions – of one type, consisting of four functions – of three types, consisting of six functions – of three types:

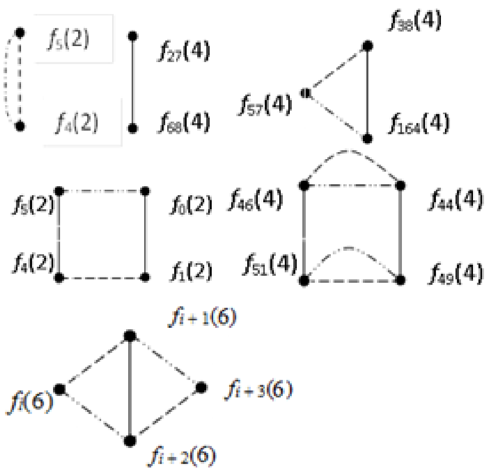

Figure 6. Types of blocks of two, three and four MBFs

Here $f_i(6) = x_1x_2 \vee x_1x_3 \vee x_1x_4 \vee x_1x_5 \vee x_1x_6 \vee x_2x_3x_4x_5x_6$,
$f_{i+1}(6) =$
$x_1 \vee x_3x_4x_5x_6 \vee x_2x_4x_5x_6 \vee x_2x_3x_5x_6 \vee x_2x_3x_4x_6 \vee x_2x_3x_4x_5$,
$f_{i+2}(6) =$
$x_1x_2x_3 \vee x_1x_2x_4 \vee x_1x_2x_5 \vee x_1x_2x_6 \vee x_1x_3x_4 \vee x_1x_3x_5 \vee x_1x_3x_6 \vee x_1x_4x_5 \vee x_1x_4x_6 \vee x_1x_5x_6$,
$f_{i+3}(6) =$
$x_4x_5x_6 \vee x_3x_5x_6 \vee x_3x_4x_6 \vee x_3x_4x_5 \vee x_2x_5x_6 \vee x_2x_4x_6 \vee x_2x_4x_5 \vee x_2x_3x_6 \vee x_2x_3x_5 \vee x_2x_3x_4$.

Other functions numbers are taken from studies [3] and [4].

The block containing functions $f_0(n)$ and $f_1(n)$ shall be called base block.

For functions of zero and first rank, there is one block, which is the base (see. Figure 1).

All MBFs of rank of 2 can be presented as 2 blocks, one of them consisting of four MBFs (base) and the other – of two MBFs:

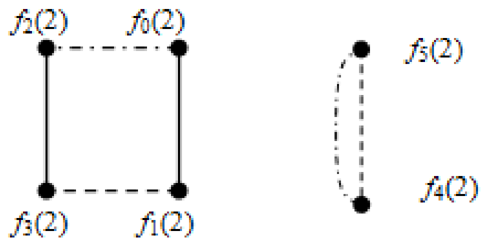

Figure 7. Blocks of rank 2.

Here $f_2(2) = x_1 \vee x_2$, $f_3(2) = x_1x_2$, $f_4(2) = x_1$, $f_5(2) = x_2$. Function $f_2(2)$ shall be disjunctively self-dual, $f_3(2)$ – conjunctively self-dual, $f_2(2)$, $f_3(2)$ – self-dual.

Base blocks can be of two types: either the first one, or the second one. To be more precise – either with one self-dual and one disjunctive self-complementary for odd ranks, or with two disjunctive self-dual and one function for even ranks. For example, base blocks of 3 and 4 ranks:

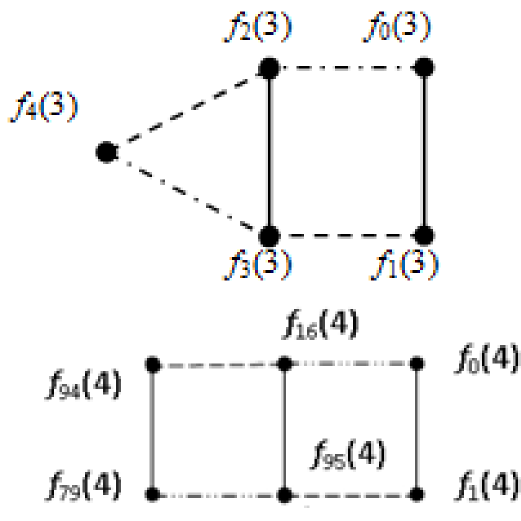

Figure 8. Base blocks of 3 and 4 ranks

Because the base block includes zero and unit MBFs, it is quite simple to show that each function of this block consists of disjunction C_n^k conjunctions of one length, here k – the number of variables in conjunction. This is why the number of base block MBFs of n- rank equals to the number of

Newton binomial raised to the n-th power factors and additionally zero MBF, i.e. $n + 2$.

Suppose MBF is given in the form of DNF: $f(x_1,\dots,x_n)=\bigvee_{j=1}^{k}(\delta_1^j \vee x_1)\cdot\ldots\cdot(\delta_n^j \vee x_n)$, here k – the number of conjunctions in DNF, $\delta_i^j=\begin{cases}1, & i\notin j\\ 0, & i\in j\end{cases}$, i – variable number, j – conjunction number. By substituting $S=(t_1,...,t_n)$ we shall obtain from this function an isomorphic function $g(x_1,\dots,x_n)=\bigvee_{j=1}^{k}(\delta_{t_1}^j \vee x_{t_1})\cdot\ldots\cdot(\delta_{t_n}^j \vee x_{t_n})$.

Lemma 2. Functions f_1 and g_1 are dual to isomorphic f and g are also isomorphic.

Proof. From MBF $f(x_1,\dots,x_n)$ of any block given in the form of DNF we shall obtain by using dual operation a dual MBF $f_1(x_1,\dots,x_n)=\bigwedge_{j=1}^{k}((1-\delta_1^j)x_1 \vee\ldots\vee(1-\delta_n^j)x_n)$.

Applying g dual operation to isomorphic MBF, we shall obtain a dual MBF $g_1(x_1,\dots,x_n)=\bigwedge_{j=1}^{k}((1-\delta_{t_1}^j)x_{t_1} \vee\ldots\vee(1-\delta_{t_n}^j)x_{t_n})$, which comes of f_1 by substituting S and, consequently, it is isomorphic in relation to f_1. Thus, we have obtained two isomorphic functions f_1 and g_1 of two isomorphic functions f and g with the aid of dual operation.

Lemma 3. Functions f_2 and g_2 are disjunctive complementary to isomorphic f and g also isomorphic.

Proof. Much as lemma 2. From MBF $f(x_1,\dots,x_n)$ using the disjunctive complement operation we obtain $f_2(x_1,\dots,x_n)=\bigvee_{j=1}^{k}(((1-\delta_1^j)\vee x_1)\cdot\ldots\cdot((1-\delta_n^j)\vee x_n))$, and from MBF $g(x_1,\dots,x_n)$ we obtain disjunctive complementary MBF $g_2(x_1,\dots,x_n)=\bigvee_{j=1}^{k}(((1-\delta_{t_1}^j)\vee x_{t_1})\cdot\ldots\cdot((1-\delta_{t_n}^j)\vee x_{t_n}))$. We can see that MBF can be obtained from f_2 by substituting S and, consequently it is isomorphic in relation to f_2

Lemma 4. Suppose function f is given, using some substitution α we shall obtain isomorphic g. If we use any succession of three operations: duality, conjunctive and disjunctive complements to these two isomorphic functions, we shall obtain two isomorphic functions f_1 and g_1. At that, one of them shall result from the other by substituting α.

Proof. Let us remark that the operation of conjunctive complement is equivalent to the sequence of operations: duality, disjunctive complement and again duality (lemma 1). Successively applying to isomorphic MBFs f and g lemmas 2 and 3 we obtain isomorphic f_1 and g_1. This shall be more than one function, because each of the three operations is unary and mutually reversible, i.e. an involution, and we cannot obtain different functions out of one function using the same sequence of operations.

From lemmas 2, 3 and 4 it follows immediately that:

Lemma 5. By applying a sequence of three above mentioned operations to any number of isomorphic functions we shall obtain the same number of isomorphic functions.

Theorem 1. Any two MBFs of one block have equal number of isomorphic.

Proof. As a consequence of lemmas we derive that out of two isomorphic MBFs f and g with the aid of any sequence of the three operations described above we also obtain isomorphic MBFs f_1 and g_1.

Let us choose arbitrary MBFs f and f_1 from block 2. Suppose that f is more isomorphic than f_1. By the definition of MBF block, f_1 can be obtained from MBF f by some sequence of operations p. Let us apply the same sequence p to all functions isomorphic to f. According to the proved above, as the result we shall obtain functions isomorphic to f_1. Consequently, we have obtained the number of isomorphic the number of isomorphic MBFs to f_1 equal to the number of isomorphic MBFs to f, which contradicts to the presumption. The theorem is proved.

Theorem 2. For each MBF in the block the number of isomorphic MBFs in the block is the same.

Proof. Suppose an MBF of block f has maximal set of isomorphic functions in the block. We shall designate this set through *A*. We shall take any other nonisomorphic to f function g. Select the sequence of operations, under which function f turns into function g. According to the block definition, such sequence can be always realized. From function g and all its isomorphic we shall obtain a set *B* functions with the same sequence of operations. All MBFs from the set *B* by definition of block are in the same block. It follows from lemma 5 that MBFs in set *B* shall be isomorphic to one another and their number shall be equal to the number in set *A*. The theorem is proved.

The number of isomorphic MBFs for one function in the block shall be called a block index. It follows from the definition of isomorphic blocks and from lemmas 2, 3, 4 and 5 that the statement is correct:

Consequence 1. Isomorphic blocks have identical index.

Consequence 2. Blocks of the first and third type have an index no more than 2. Since the function of self-dual is isomorphic must also be self-dual self-complementary and isomorphic disjunctive must also be self-complementary, and these blocks are only with two of this type of MBF. While blocks of type 2 have index 1, because they have one self-dual and one disjunctive self-complementary and isomorphic to it in the block does not exist. Blocks of the fourth type may be of any index.

Because for any nonisomorphic MBFs of one block, f and g according to theorem 2 possess equal number of isomorphic for each function f and g, consequently, the number of isomorphic for any function of the block is the divider of the number of all MBFs in the block. This is why the block index is the divider of the number of all MBFs in the block. The result of the division is the number of nonisomorphic MBFs in the block. It also follows that if we take all isomorphic MBFs to this function, we shall obtain the number of isomorphic blocks.

3. Conclusions

With this classification we can define various properties of MBFs.

For example, we shall consider partitioning all MBF blocks of 4th rank according to such classification.

Table 1. Blocks of the 4th rank containing 2 disjunctive self-complementary MBFs:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
1	6	1	1	1
2	2	3	1	1

The number of nonisomorphic blocks (or isomorphic blocks groups) of the 4th rank, i.e. the sum of values of the last columns in all 4 tables $BN(4)=\sum C_{i4}=7$, $i=1,...,4$.

The number of all blocks of rank 4, i.e. by summing the product of values in the third column by the fifth in all 4 tables $B(4)=\sum C_{i2}\cdot C_{i4}=24$, $i=1,...,4$.

Table 2. Blocks of the 4th rank containing 1 disjunctive self-complementary MBF and 1 self-dual MBF:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{20}	C_{21}	C_{22}	C_{23}	C_{24}
1	3	4	1	1

Table 3. Blocks of the 4th rank, containing 2 self-dual MBFs:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{30}	C_{31}	C_{32}	C_{33}	C_{34}
1	6	4	1	1

Table 4. Cyclic blocks of the 4th rank:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{40}	C_{41}	C_{42}	C_{43}	C_{44}
1	4	3	4	1
2	12	3	2	1
3	12	6	2	1

The number of inequivalent MBFs of the 4th rank [7] $R(4)=\sum\frac{C_{i2}\cdot C_{i4}}{C_{i3}}=30$, $i=1,...,4$ (The number of MBFs in the block to be divided by the block index and multiplied by the number of nonisomorphic blocks, then summarized in all tables).

The number of all MBFs of the 4th rank (Dedekind number) $D(4)=\sum C_{i1}\cdot C_{i2}\cdot C_{i4}=168$, $i=1,...,4$ Number of nonisomorphic disjunctive self-complementary MBFs of the 4th rank $DCN(4)=\sum(2C_{14}+C_{24})=5$, $i=1,...,4$

The number of all disjunctive self-complementary MBFs of the 4th rank $DC(4)=\sum(2C_{14}\cdot C_{12}+C_{24}\cdot C_{22})=12$, $i=1,...,4$

The number of nonisomorphic self-dual MBFs of the 4th rank $DSN(4)=\sum(2C_{34}+C_{24})=3$, $i=1,...,4$

The number of all self-dual MBFs of the 4th rank $DS(4)=\sum(2C_{34}\cdot C_{32}+C_{24}\cdot C_{22})=12$, $i=1,...,4$

The number of nonisomorphic completely non-symmetrical MBFs of the 4th rank (only the lines where $C_{42}C_{43}=4!=24$.) $RN(4)=\sum\frac{C_{i1}\cdot C_{i4}}{C_{i3}}=0$, $i=1,...,4$ are to be summarized.

The number of completely non-symmetrical MBFs of the 4th rank (only the lines where $C_{42}C_{43}=4!=24$.) $NS(4)=\sum C_{i1}\cdot C_{i2}\cdot C_{i4}=0$, $i=1,...,4$ are to be summarized

The same for the 5th rank.

Table 5. Blocks of the 5th rank containing 1 disjunctive complementary MBF and 1 self-dual MBF:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{20}	C_{21}	C_{22}	C_{23}	C_{24}
1	7	1	1	1
2	7	5	1	2
3	7	10	1	2
4	7	20	1	1
5	7	30	1	1

Table 6. Cyclic blocks of the 5th rank:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{40}	C_{41}	C_{42}	C_{43}	C_{44}
1	4	6	2	1
2	6	5	6	1
3	14	10	1	1
4	14	15	1	1
5	14	15	2	1
6	14	20	1	1
7	14	30	1	2
8	14	30	2	3
9	14	60	1	2
10	14	60	2	1
11	32	30	2	1
12	54	10	6	1

Let's calculate for the 5th rank the same expressions as for the 4th rank:

$BN(5) = \sum C_{i4} = 23$, $i = 2,4$

$B(5) = \sum C_{i2} \cdot C_{i4} = 522$, $i = 2,4$

$R(5) = \sum \frac{C_{i2} \cdot C_{i4}}{C_{i3}} = 210$, $i = 2,4$

$D(5) = \sum C_{i1} \cdot C_{i2} \cdot C_{i4} = 7581$, $i = 2,4$

$DCN(5) = \sum (2C_{14} + C_{24}) = 7$

$DC(5) = \sum (2C_{14} \cdot C_{12} + C_{24} \cdot C_{22}) = 81$

$DSN(5) = \sum (2C_{34} + C_{24}) = 7$, $i = 2,4$

$DS(5) = \sum (2C_{34} \cdot C_{32} + C_{24} \cdot C_{22}) = 81$, $i = 2,4$

Summarizing only the lines, where $C_{42}C_{43}$ = 5! = 120.

$RN(5) = \sum \frac{C_{i1} \cdot C_{i4}}{C_{i3}} = 7$, $i = 2,4$

Summarizing only the lines, where $C_{42}C_{43}$ = 5! = 120.

$NS(5) = \sum C_{i1} \cdot C_{i2} \cdot C_{i4} = 840$, $i = 2,4$

For the 6th rank 6.

Table 7. Blocks containing 2 disjunctive complementary MBFs:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
1	8	1	1	1
2	8	6	1	1
3	8	10	1	1
4	8	15	1	3
5	8	45	1	2
6	8	60	1	3
7	8	60	2	1
8	8	90	1	1
9	8	90	2	1
10	8	180	1	2
11	2	36	2	1
12	4	10	2	1
13	4	15	2	1
14	10	180	1	1
15	12	60	1	1
16	32	90	1	1

Table 8. Blocks containing 2 self-dual MBFs:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{30}	C_{31}	C_{32}	C_{33}	C_{34}
1	2	6	2	1
2	2	10	2	1
3	2	60	2	1
4	2	90	2	1
5	4	6	1	1
6	6	45	2	3
7	8	6	1	1
8	8	20	1	1
9	8	30	1	1
10	8	60	1	4
11	8	180	1	2
12	8	180	2	1
13	10	180	1	1

Table 9. Cyclic blocks of the 6th rank:

Nos	Number of MBFs in the block	Number of isomorphic blocks	Block index	Number of nonisomorphic blocks
C_{40}	C_{41}	C_{42}	C_{43}	C_{44}
1	4	15	2	1
2	4	45	4	2
3	4	60	2	4
4	4	60	4	2
5	4	90	2	2
6	4	90	4	6
7	4	120	2	2
8	4	180	2	10
9	4	180	4	4
10	4	360	1	1
11	4	360	2	22
12	4	720	1	5
13	6	20	6	2
14	8	15	2	1
15	8	45	8	1
16	8	60	1	1
17	8	90	2	1
18	12	30	6	2
19	12	45	4	3
20	12	60	2	2
21	12	90	2	1
22	12	180	2	3
23	12	360	1	1
24	12	360	2	3
25	12	720	1	1
26	16	10	2	2
27	16	15	1	2
28	16	15	2	1
29	16	30	1	2
30	16	45	1	2
31	16	45	2	12
32	16	45	4	4
33	16	60	1	18
34	16	60	2	11
35	16	90	1	9
36	16	90	2	35
37	16	90	4	1
38	16	90	8	1
39	16	120	1	11
40	16	180	1	53
41	16	180	2	48
42	16	180	4	3
43	16	360	1	118
44	16	360	2	47
45	16	720	1	54
46	20	45	2	2
47	20	90	2	2
48	20	180	2	1
49	20	180	4	2
50	20	360	2	5
51	24	30	6	2
52	24	60	12	2
53	24	90	2	1
54	24	90	4	3
55	24	120	6	2
56	24	180	2	1
57	24	180	4	4
58	24	360	2	2
59	28	90	2	2
60	28	180	2	5
61	28	180	4	1
62	28	360	2	7

63	28	720	1	1
64	32	36	2	1
65	32	90	2	1
66	32	180	2	2
67	32	360	1	2
68	32	360	2	9
69	36	45	4	2
70	36	120	6	1
71	36	180	2	2
72	36	360	2	6
73	40	180	2	1
74	40	360	2	1
75	44	180	2	1
76	48	15	6	1
77	48	30	6	1
78	48	60	6	1
79	48	90	8	2
80	48	120	6	1
81	48	180	2	5
82	48	180	4	2
83	48	240	3	2
84	48	360	2	1
85	52	180	2	1
86	56	180	2	1
87	56	180	4	2
88	64	360	2	1
89	68	360	2	4
90	70	360	1	1
91	72	60	12	2
92	78	90	2	2
93	80	180	2	2
94	80	180	4	1
95	80	360	2	3
96	84	120	6	2
97	88	180	4	1
98	92	180	2	2
99	96	60	6	1
100	96	120	6	2
101	96	180	2	1
102	104	180	2	1
103	108	120	6	4
104	108	180	2	2
105	112	180	4	1
106	112	360	2	1
107	116	360	2	1
108	120	180	4	2
109	120	360	2	1
110	128	90	4	2
111	128	360	2	1
112	132	360	2	1
113	136	180	4	4
114	140	360	2	2
115	144	60	6	1
116	152	180	4	2
117	152	360	1	1
118	156	60	12	2
119	156	360	2	1
120	162	60	6	2
121	164	180	2	1
122	164	180	4	2
123	172	180	2	1
124	174	120	6	2
125	180	180	2	1
126	184	180	2	2
127	184	360	1	1
128	188	180	2	1
129	196	360	2	1

130	204	60	12	2
131	204	120	6	1
132	204	180	2	1
133	204	360	2	1
134	208	180	2	2
135	212	180	2	1
136	216	180	4	1
137	240	120	3	1
138	244	360	2	1
139	248	90	4	2
140	248	90	8	2
141	256	90	2	1
142	256	180	2	1
143	256	360	1	1
144	264	120	6	1
145	264	360	2	1
146	268	180	2	1
147	288	120	6	1
148	296	180	4	1
149	336	90	4	2
150	376	90	4	1
151	376	180	2	1
152	396	120	6	1
153	408	180	4	1
154	432	20	6	1
155	432	60	6	2
156	432	90	2	1
157	438	120	6	2
158	444	120	6	1
159	456	120	6	1
160	468	120	6	1
161	504	120	6	1
162	512	90	8	2
163	540	120	6	1
164	576	120	6	1
165	696	120	6	1
166	732	180	4	2
167	760	180	4	1
168	768	120	6	1
169	792	120	6	1
170	828	120	6	1
171	1232	90	8	1
172	2052	120	6	1
173	2064	120	6	1

Let's calculate for the 6th rank the same expressions as for the 4th one:

$$BN(6) = \sum C_{i4} = 775, \quad i = 1,3,4$$

$$B(6) = \sum C_{i2} \cdot C_{i4} = 189182, \quad i = 1,3,4$$

$$R(6) = \sum \frac{C_{i2} \cdot C_{i4}}{C_{i3}} = 16353, \quad i = 1,3,4$$

$$D(6) = \sum C_{i1} \cdot C_{i2} \cdot C_{i4} = 7828354, \quad i = 1,3,4$$

$$DCN(6) = \sum (2C_{14} + C_{24}) = 39$$

$$DC(6) = \sum (2C_{14} \cdot C_{12} + C_{24} \cdot C_{22}) = 2646$$

$$DSN(6) = \sum (2C_{34} + C_{24}) = 30, \quad i = 1,3,4$$

$$DS(6) = \sum (2C_{34} \cdot C_{32} + C_{24} \cdot C_{22}) = 2646, \quad i = 1,3,4$$

Summarizing only the lines, where C42C43 = 6! = 720.

$$RN(6) = \sum \frac{C_{i1} \cdot C_{i4}}{C_{i3}} = 7281, \quad i = 1,3,4$$

Summarizing only the lines where C42C43 = 6! = 720.

$$NS(6) = \sum C_{i1} \cdot C_{i2} \cdot C_{i4} = 5242320, \quad i = 1,3,4$$

This method is not limited by the number of variables because for each MBF with any number of variables n we can build a block containing this MBF using operations of disjunctive complement and duality. In this case, it is sufficient to build only nonisomorphic blocks to describe all MBFs of n variables. This significantly facilitates the description of all MBFs of n variables. For example, all MBFs of 6 variables can be split into 189182 blocks but only 775 of them can be selected as mutually nonisomorphic. Classes of equivalence and nonisomorphic blocks are directly connected with the Dedekind numbers.

REFERENCES

[1] Dedekind R. Über Zerlegungen von Zahlen durch ihre grössten gemeinsamen Teiler. Festschrift Hoch. Braunschweig u. ges. Werke. II, 1897, 103–148.

[2] Wiedemann, Doug A computation of the eighth Dedekind number. Order 8 (1991), no. 1, 5–6.

[3] Tkachenco V.G., Sinyavsky O.V. (2016). Blocks of Monotone Boolean Functions, Computer Science and Information Technology Vol. 4(2), pp. 72 – 78. DOI: 10.13189/csit.2016.040203.

[4] Tkachenco V.G., Sinyavsky O.V. (2016). Blocks of Monotone Boolean Functions of Rank 5, Computer Science and Information Technology 4(4): pp.139-146, 2016, DOI: 10.13189/csit.2016.040402

[5] Ткаченко В.Г. Метод построения блоков монотонных булевых функций. Наукові праці ОНАЗ ім. О.С. Попова. 2011. № 1. С.79 – 83.

[6] Ткаченко В.Г, Иваницкий А.М. Взаимосвязь между матроидами и монотонными булевыми функциями электрических цепей. Наукові праці ОНАЗ ім. О.С. Попова. 2009. № 1. С.18 – 26.

[7] T. Stephen, T. Yusun, Counting inequivalent monotone boolean functions, Discrete Appl. Math. 167 (2014) 15–24. doi:10.1016/j.dam.2013.11.015. URL http://dx.doi.org/10.1016/j.dam.2013.11.015