

SETAR (Self-exciting Threshold Autoregressive) Non-linear Currency Modelling in EUR/USD, EUR/TRY and USD/TRY Parities

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Abstract In economies that are open to foreign markets the numerical value of the currencies as a macroeconomic variable is of great importance especially when the mutual dependency among the economies is concerned. When it is considered in terms of political economy, the targeted level of the currencies have vital importance especially in economies that have the characteristics of export-driven growth and in economies that struggle not to disrupt the picture in macroeconomic design. When it is considered that each time series has a structure that is sensitive to its own internal dynamics (sometimes these dynamics are expressed as the time series components), these dynamics provide us with coordinates for estimations and may eliminate the compulsory dependency on the outsourced variables at a serious level. This is exactly what has been done in this study. First of all, the non-linear time series analyses are examined in terms of linearity tests, and the linearity tests are applied for all parties and for different time periods. Then, the SETAR Modelling, which is the title of the study, has been applied in order to explain the non-linear pattern in detail. The SETAR Modelling process and other definitions statistical analyses of this model have been applied in relevant parities for separate time periods. The SETAR model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling. In this study, the secondary purpose is to express that the SETAR model performance is superior to the other models by considering the observation values of the parities.

Keywords Currency, Linearity Tests, Non-linear Time Series Analysis, SETAR Model

because of being more practical, keeps its popularity in academic literature. The basic logic of TAR Modelling is allowing different regimes that allow autoregressive analysis of different levels. TAR Modelling was first handled by Tong (1978) in [21] in the literature. Then, parallel studies were conducted by Tong and Lim (1980) in [19]. Again, [23] presented the SETAR (Self Excited Autoregressive Models), which has a popular usage area, to the literature, and with the help of this a light was cast on many non-linear analyses. For example, the SETAR Modelling was used by Tong and Yeung (1991)¹ in [20], who examined the data on the assets market prices; by Tahir İsmail and Zaidi (2006) in [12], who examined the currency incomes by using the currencies of Malaysia, Singapore and Thailand; by Krager and Kugler (1993) in [11], who studied currencies, by Chappell, Mistry, Padmore, Ellis (1996) in [7], who examined the real currencies; by Feng and Liu (2002) [8] in a study conducted on the GDP of Canada covering the period between 1965 and 2000 period; and by P. Clements and Smith (1997) [14] in a study that was based on currencies, GNP and on the estimation performance among the other variables. Chan and Tsay (1998) in [6], Potter (1995) in [16], Tong (1990) in [23], Tiao and Tsay (1994) in [17], Hansen (1997) in [9] also have studies conducted on these group modellings. The definition, estimation and interpretation in TAR models are simpler when compared with the other non-linear models. Bratickoviene (2012) in [4] conducted a study on Real Harmonized Consumer Price Index in Lithuania, and used the adapted SETAR models for January/1996 - December/2009 period. He reported that the properties or the characteristics of the adapted SETAR of time series with complex nature were observed. Aydın and Güneri (2015) in [1] conducted a comparative study for Export Volume Index and Domestic Producer Price Index Series in Turkey, and

1. Introduction and the Literature

The TAR (Threshold Autoregressive) Model Family, which is important in non-linear time series modelling

¹ SETAR-type non-linearity tests were included in the articles of the same authors and the article was published in Applied Statistics Magazine in 1991.

evaluated the performances of Hybrid AR, SETAR and ARM models. As a result of the evaluations, they reported that the best modeling for the time series in question was the AAR-SETAR modeling. Boero and Lampis (2016) in [3] conducted a study for 4 major European countries and applied SETAR modeling by using Industrial Production Index (IPI) values, and investigated the belief that claimed that the dynamic determination process increased the prediction performance (Teräsvirta et al. (2005)). However, the literature is not rich in terms of the properties of the sampling on this model group and in terms of test statistics (Hansen, 1997).

2. TAR Group Models

It is possible to show a TAR Model with k regime as follows; (Montgomery and others (1998) in [13])

$$Y_t = \beta_0^{(i)} + \beta_1^{(i)}Y_{t-1} + \dots + \beta_{p_i}^{(i)}Y_{t-p} + \varepsilon_{it}$$

$$\tau_{i-1} < x_{t-d} < \tau_i, \quad i = 1, \dots, k \quad (1)$$

Here, $\beta_1^{(i)}, \beta_2^{(i)} \dots \beta_p^{(i)}$ are the autoregressive parameters of i^{th} regime, $Y_{t-1}, Y_{t-2} \dots Y_{t-p}$ variables are the ones that are believed to influence the Y_t dependent variable in each regime, and ε_t is the error term. d and p_i denotes a positive integer (d shows the delay parameter here), k denotes the number of the regimes in the model; x_{t-d} denotes the threshold variable, and τ denotes a real number ($-\infty < \tau < +\infty$). The model may be determined as based on the position of the observance values according to the x_{t-d} variable.

A two-variable TAR Model, on the other hand, is expressed as follows in Hansen (1997);

$$Y_t = (\phi_{1,0} + \phi_{1,1}Y_{t-1} + \dots + \phi_{1,p}Y_{t-p})I(q_{t-1} \leq \gamma) + (\phi_{2,0} + \phi_{2,1}Y_{t-1} + \dots + \phi_{2,p}Y_{t-p})I(q_{t-1} > \gamma) + e_t \quad (2)$$

Here, q_{t-1} denotes the threshold variable. $I(\cdot)$ denotes the indicator function, $q_{t-1} = q(Y_{t-1}, \dots, Y_{t-p})$ denotes the functional structure, and γ denotes the threshold value (or the threshold parameter). e_t is the martingale difference series. [2]

Tsay (1989) expressed the following operation flow on the estimation of the model.

1. Determining the level of AR process (p),
2. Selecting the d delay parameter,
3. Determining the level of the AR for each regime with the determination of the threshold value in the direction of the obtained p and d values.

According to the different forms of the x_{t-d} threshold variable, the TAR group models also show differences. In this sense, the SETAR Modelling will be examined in the next title.

2.1. Self-excited Autoregressive Models (SETAR)

Although there are few origin studies in the field of threshold autoregressive models, several studies attract attention. [2] and [21] conducted studies and expressed the following procedure about the SETAR models and parameter estimations, which we will explain.

2.1.1. Hansen's Approach

The Structure of the Model

These are the models that are valid in case the threshold variable in the TAR modelling is determined as the delayed value of the relevant time series.

A Double-Regime SETAR Model may be shown in a simple way as follows. (Yilanci, (2007), in [26])

$$Y_t = \begin{cases} \beta^{(1)}Y_{t-1} + \varepsilon_t^{(1)} & Y_{t-1} < \gamma \\ \beta^{(2)}Y_{t-1} + \varepsilon_t^{(2)} & Y_{t-1} \geq \gamma \end{cases} \quad (3)$$

Or it may be shown as follows based on the notation given in the study of Hansen (1997) in [9];

$$Y_t = (\alpha_0 + \alpha_1Y_{t-1} + \dots + \alpha_pY_{t-p})I(Y_{t-d} \leq \gamma) + (\beta_0 + \beta_1Y_{t-1} + \dots + \beta_pY_{t-p})I(Y_{t-d} > \gamma) + e_t \quad (4)$$

Here, again p denotes the autoregressive level; γ denotes the threshold parameter or the threshold value. The threshold variable is expressed as Y_{t-d} . (d is an integer)²

When expressed in a different way, the following model may be considered; (Hansen (1997))

$$Y_t = X_t(\gamma)' \omega + e_t \quad (5)$$

Here, if the relevant model is described by separating it into terms, the ω parameter will be as follows;

$$\omega = (\alpha' \beta')', \quad (\alpha = (\alpha_0 \alpha_1 \dots \alpha_p)') \quad \text{and} \\ \beta = (\beta_0 \beta_1 \dots \beta_p)' \quad (6)$$

The expression $X_t(\gamma)$, on the other hand, will be expressed as follows in Hansen (1997)

$$X_t(\gamma) = (X_t' I(Y_{t-d} \leq \gamma) X_t' I(Y_{t-d} > \gamma))' \quad (7)$$

As a consequence, the relevant model will be shown as follows, as it is expressed in the study of Hansen (1997) in [9],

² The problem of the level of the delay here will be solved in further parts.

$$Y_t = x_t' \alpha I(Y_{t-d} \leq \gamma) + x_t' \beta I(Y_{t-d} > \gamma) + e_t \quad (8)$$

When the (5) model is considered, the method for estimating the ω parameter is the Successive Least Squares Method because the model parameters are non-linear (Hansen, 1997)

The Estimation of the Parameters

The following formulas are given by Hansen (1997) in [9] for the estimation of the ω parameter and for the inclusion.

$$\hat{\omega} = \left(\sum_{t=1}^n X_t(\gamma) X_t(\gamma)' \right)^{-1} \left(\sum_{t=1}^n X_t(\gamma) Y_t \right) \quad (9)$$

$$\hat{e}_t(\gamma) = Y_t - X_t(\gamma)' \hat{\omega}(\gamma) \quad (10)$$

The inclusion variance that is dealt with in the Successive Least Squares Method is as follows; (Hansen, (1997))

$$\sigma_n^2(\gamma) = \frac{1}{n} \sum_{t=1}^n \hat{e}_t(\gamma)^2 \quad (11)$$

In the successive least squares estimation of the threshold parameter the minimization of equality (11) is the basic principle. The situation has been expressed as follows, again in Hansen (1997);

$$\text{While } \Gamma = [\underline{\gamma}, \bar{\gamma}], \hat{\gamma} = \arg \min_{\gamma \in \Gamma} \hat{\sigma}_n^2(\gamma) \quad (12)$$

The search in (12) is important for the selection of the threshold parameter. The possible threshold parameter in the time series being more in number makes the error variance to be obtained become various. For this reason, it would be proper to select the threshold parameter (γ) that will minimize the inclusion variance (12) as shown. Right at this point, the process will follow an algorithmic system for selecting the threshold variable Y_{t-d} and the threshold parameter (γ), which will minimize the error variance.³

2.1.2. Tong’s Approach

The Structure of the Model

$$Y_t = \begin{cases} \beta_0^{(1)} + \sum_{i=1}^{p_1} \beta_1^{(1)} Y_{t-1} + e_t^{(1)} & Y_{t-d} \geq \gamma \\ \beta_0^{(2)} + \sum_{i=1}^{p_1} \beta_1^{(2)} Y_{t-1} + e_t^{(2)} & Y_{t-d} < \gamma \end{cases} \quad (13)$$

³ When Y_{t-d} variable is considered, it is important to select the delay length. Hansen (1997) expressed in his study that the d delay length might not be known, and for this reason, the iterations would be followed as if the d numerical value were known.

Tong (1983) in [22] took the above model as the basis and recommended the 3 steps explained as follows.

The Estimation of the Parameters

Step 1

It is assumed that the d and γ values are known as the first step. Based on these assumptions, the observation values are separated into small sub-groups, and the AIC data criterion for each sub-group is calculated with the p_i ($i = 1, \dots, k$) level.⁴

This is shown as follows;

$$AIC(\hat{p}_i) = \min[AIC(k_i)], \quad i = 1, 2 \quad (14)$$

In this situation, firstly the p_i value of each regime is obtained (by using $\min[AIC(k_i)]$ in return for the constant d and γ values.

Step 2

In the second step, the d value is kept constant (it is assumed that it corresponds to a certain value; in other words, it is known), this time, the threshold parameters that will minimize the AIC data criterion value are tested. In other words, the γ value that minimizes the $AIC(d, \hat{\gamma})$ value is selected from among the other threshold parameters. This situation is shown in Tong (1983) in [22] as follows.

$$AIC(d_0, \hat{\gamma}) = \min[AIC(d_0, \gamma)] \quad (15)$$

Step 3

In the first two steps, the p_i and γ values are determined. In the remaining third step, the d value will be determined. The d value that minimizes the $NAIC(d)$ value will be found from the d choice in k number.⁵

After the 3 steps that are evaluated by using the data criterion, the model will be estimated by depending on the abovementioned parameters.

2.1.3. Tsay’s Approach

Tsay (1989) presented the successive regression and local estimation as a different method to the SETAR Modelling. Tsay explained this estimation procedure both in his own study in 1989 and in the study about the assets volatility he conducted with [2]. The modelling steps in the study of Tsay (1989) were presented as follows;

⁴ Although Tong suggested the maximum level in the regimes as $L = n^\alpha$ ($\alpha < (1/2)$), the selection of this value is optional.

⁵ Here, since the values of the variable d will influence the number of the observations (n) in different regimes, the NAIC value will be used instead of the usual AIC. The NAIC here is shown as $NAIC(d) = \frac{AIC(d)}{n - n_d}$.

1. **Determining the AR level (p)** (Here, two possible methods are mentioned. The first one is using the AIC data criterion, and the second is making use of the PACF.)⁶
2. **Applying the threshold linearity test corresponding to the p and possible d , which are data.** (If the process is decided to be non-linear, the delay value that gives the highest F test value for the relevant p value will be selected from among the threshold variables. Actually, the linearity test recommended here has been expressed as the combination of several linearity tests, as it is also expressed in Tsay (1989) in [25]. The linearity tests mentioned here are the linearity tests belonging to Petrucci and Davies (1986) in [15], Keenan (1985) in [10], and Tsay (1986) in [24])^{7,8}
3. **Determining the threshold values by using the distribution graphics.** (Here, the purpose is determining the place of the threshold parameter by making use of distribution graphics visually for the threshold variable of some statistical parameters.) The statistical parameters in question are as follows:
 - Inclusion estimations
 - Standardized inclusion estimations
 - The t value of the successive estimations of the autoregressive coefficients

In practice, mostly the t values of the recursive estimations of the autoregressive (AR) coefficients in distribution regressions are the subject matters. The significant t values converge towards a certain value as the repetitive (successive) regressions are established. When the unknown threshold parameter is encountered, the parameter estimations and therefore the t values begin to be distracted. When the distribution graphics of the inclusion estimations are considered, it is necessary that the dependency relation between the internal variables and the inclusion estimations from the graphics. When the graphical verticalization between the dependent variables and the inclusion estimations disappears, the model starts to alternate. This situation in return changes the regime in the location where the change happens. Eventually, the observation value at which changes the regime will be

⁶ Tsay suggested the partial autocorrelation coefficients function (*PACF*) instead of the AIC data criterion in determining the autoregressive level because the process included a non-linear dynamism.

⁷ Tsay expressed the selection of the delay parameter with

$$\hat{F}(p, d_p) = \max \left[\hat{F}(p, v) \right] \quad , \quad (v \in S) \quad \text{Here,}$$

$$S = 1, 2, \dots, p.$$

The formula which is the subject matter of F test is

$$\hat{F}(p, d) = \frac{\left[\sum \hat{\varepsilon}_i^2 - \sum \hat{\varepsilon}_i^2 \right] / (p+1)}{\sum \hat{\varepsilon}_i^2 / (n-d-b-p-h)}$$

⁸ In Tsay (1989) in [25], the inclusion formula that is the subject matter of

the least square is shown as; $\hat{\varepsilon}_{\pi_{t+d}} = \phi_0 + \sum_{v=1}^p \phi_v Y_{\pi_{t+d-v}} + \varepsilon_{\pi_{t+d}}$

selected as the threshold parameter.)

4. **Determining the autoregressive level of each regime with linear auto regression techniques.**

In summary, Tsay (1989) conducted a study and determined firstly the AR level in modeling process, and then applied threshold linearity test for predefined p and probable d value. Through this Linearity Test, he decided whether the process was linear or not, and selected the delay value (d) that gave the highest F test value for the relevant p value, which was selected as the threshold value. After the (Y_{t-d}) value of the threshold variable was determined, the threshold value was determined by making use of the distribution graphics. The distribution graphics residual estimations mentioned here are the t values of the standardized residual estimations and consecutive estimations of the model coefficients. After this stage, the autoregressive level of each regime was determined with the linear modeling approaches.

3. Method and Findings

3.1. Application on Currencies

In our study, the parities that are expressed as major parties, which are many in number among the currency parities, and the EUR/USD parity, which is active in operations in spot currency market, and the USD/TRY, EUR/TRY parities, which are convertible into TL will be taken as the basis. The parities will be examined for descriptive statistics in 3-period intervals as D1 (1-day), W1 (1-week), MN (1-month); and for linearity tests, the examination will be made as D1 (1-day), W1 (1-week), MN (1-month). Since the time intervals of the observation values were taken from the history section of the Forex platform that was provided to the users by intermediary institution acting in Forex market, and since the maximum observation number was limited with the data given here, they were expressed as given in the table. In addition, since investors are inclined to take long-term position, daily (D1), weekly (W1) and monthly (MN) data were taken. The parities are given as follows with the relevant intervals and the start and end dates. If we express the justification of using the EUR/USD, USD/TRY and EUR/TRY parities in this paper; firstly, the EUR/USD parity was used because it is the most frequently processed parity in Forex Market. The USD/TRY and EUR/TRY parities, on the other hand, were used because of the volatility experienced in the Turkish Lira in recent years. Especially the exchange policy and monetary policy of the economy administrator in Turkey require more sensitive investigation in terms of mathematical modeling, and therefore SETAR, which is a more sensitive modeling tool, was used.

Table 1. Parities and the Number of the Observation of the Parities (n)

Parities		Time Period	Start Date	End Date	Number of Obsrv. (n)
EUR/USD	Euro-US Dollar	DI	31.12.1999 00:00	26.02.2015 00:00	3944
		WI	26.12.1999 00:00	22.02.2015 00:00	792
		MN	1.12.1999 00:00	1.02.2015 00:00	183
EUR/TRY	Euro- Turkish Lira	DI	31.05.2010 00:00	26.02.2015 00:00	1171
		WI	30.05.2010 00:00	22.02.2015 00:00	247
		MN	1.05.2010 00:00	1.02.2015 00:00	58
USD/TRY	US Dollar-Turkish	DI	31.05.2010 00:00	26.02.2015 00:00	1202
		WI	30.05.2010 00:00	22.02.2015 00:00	248
		MN	1.05.2010 00:00	1.02.2015 00:00	58

In the light of the abovementioned currency categories, the relevant *observation values* of the given intervals are obtained from Integral Trader Version: 4.00 Build 765 (12 Dec 2014) (Copyright 2001-2014, MetaQuotes Software Corp.) platform.

The descriptive statistics of linear time series, modellings and diagnostic methods will be applied separately to the observation values mentioned in this study.

It is important that before the SETAR modelling is applied to the datasets, the issue of whether they are linear or not must be tested. For this purpose, firstly, the linearity tests will be dealt with, and then the SETAR modellings will be given together with parameter estimations.

4. Linearity Tests

Although the linearity tests for non-linear time series are various, they show parallelism with the method used in relevant studies. The most popular and qualified linearity tests will be explained in this study; and eventually, the Likelihood Ratio Linearity Test, which tests the threshold linearity hypotheses, will especially be dealt with.

4.1. Keenan’s One-Degree Test for Nonlinearity

It denotes the linearity test against Second-Level Volterra Expansion. In this context, the Keenan Test examines and tests the Quadratic Nonlinearity Hypothesis and provides information on threshold nonlinearity (Keenan, 1985 in [10]). This situation refers to the F test in the following model:

$$Y_t = \mu + \sum_{u=-\infty}^{\infty} a_u \varepsilon_{t-u} + \sum_{u,v=-\infty}^{\infty} a_{uv} \varepsilon_{t-u} \varepsilon_{t-v} \quad (16)$$

In case the 3rd Term of the model is zero, it gives proof on linearity. In this situation, the null hypothesis will express the situation in which the 3rd Term in Model (16) is equal to

zero.

In [10]’s test, first of all a regression is formed between Y_t and $(1, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p})$ at p level; and based on this, the $\hat{\varepsilon}_t$ inclusions are obtained. Then, a regression is established between Y_t^2 and $(1, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p})$, and the following is formed with the $\hat{\zeta}_t$ inclusions (Keenan, (1985)),

$$\hat{\eta}_t = \frac{\sum_{t=p+1}^n \hat{\varepsilon}_t \hat{\zeta}_t}{\sum_{t=p+1}^n \hat{\zeta}_t^2} \quad (17)$$

and then the F test statistics will be expressed as follows in Keenan (1985):

$$F = \frac{(n - 2p - 2)\hat{\eta}^2}{(HKT - \hat{\eta}^2)} \quad (18)$$

Here, HKT denotes the sum of the error squares expressed in the first model. ($F \sim F_{1, n-2, p-2}$)

4.2. Tsay’s Test for Nonlinearity

Tsay’s (Tsay, 1986) linearity test is based on recursive auto regression and destructive term estimators, and firstly, the recursive auto regressions are established starting from b . observation value in return for the p and the relevant d values with AR level, and then the model is established between the \hat{e}_t values and $(1, Y_{t-1}, Y_{t-2}, \dots, Y_{t-p})$.⁹ Then the following test is obtained among the inclusions of the model formed

⁹ Here, b is expressed as $b = (n/10) + p$.

with $\hat{\epsilon}_i$:

$$\hat{F}(p, d) = \frac{[\sum \hat{\epsilon}_i^2 - \sum \hat{\epsilon}_i^2] / (p+1)}{\sum \hat{\epsilon}_i^2 / (n-d-b-p-h)} \quad (19)$$

Here, it is expressed that $h = \max(1, p+1-d)$.

In the F test of Tsay, the null hypothesis claiming that the relevant time series is $AR(p)$.

The recursive regression mentioned here is the modeling approach that was focused on the change in the model parameters. The sharp changes in the numerical values of the parameters in the model that occur as a result of each additional iteration, degenerations or refractions are considered in recursive regression.

4.3. Likelihood Ratio Test for Threshold Nonlinearity

Keenan's and Tsay's quadratic nonlinear tests above are insufficient in dealing with the threshold non-linearity hypothesis. Therefore, Tong (1986) in [18] developed a different approach to test the opposite hypotheses.

- The null hypothesis: "Model is an autoregressive process. In other words $AR(p)$ ", and
- The opposite hypothesis "Model is the TAR model with constant variance at p level with two regimes".

In their studies, Tong (1986) dealt with the probability ratio that focused the numerical approach.

When the excessiveness of the observation value of the dataset in question is considered, it is inevitable that the threshold autoregressive processes and the iterative mathematical processes that must be performed about the modelling of these processes are done in a package program that includes statistical notations. Especially the "R" statistical package program has been used because of its having open-source systematic and its being used in a great extent that cannot be underestimated in the field of statistics in the literature. This program provides vast opportunities in terms of data and graphics performance. It was necessary to develop the commands or the codes that were dealt with in time series to be able to form the threshold autoregressive

model with R, which is the subject matter of this study. These codes include both the time series analysis commands and the more up-to-date structures in the developing academic literature.

The linearity tests that will be dealt with in the further parts of the study were found by finding the model hyper parameters; and the descriptive statistics part and the SETAR modelling results were found with R Project 3.1.2 Version.

5. Applying the Linearity Tests to the Relevant Parameters

When the significance values in Table 2 are examined, it is observed that the null hypothesis in the 3 parities in the Keenan's (1985) test and Tsay's (1986) test are not denied. The null hypothesis was denied in the three parities that were examined in the Likelihood Ratio test except for the EURUSD D1, EURTRY D1 and EURTRY MN parities. Therefore, the expression that may be uttered about linearity is as follows; "Some of the parities in different time periods show linear characteristics, while some others show non-linear pattern. The linearity tests of Keenan (1985) and Tsay (1986) give similar results in relevant parities. It is necessary to point out that, here, while the null hypothesis is tested against a non-alternative hypothesis in the linearity tests of Keenan and Tsay, the null hypothesis is tested against a specific alternative hypothesis in the Likelihood Ratio test".

5.1. Applying the SETAR Modelling to Parities

In this part of the study, the SETAR modelling in Tong's approach will be applied separately in the relevant time period for each parity, and the findings will be discussed in the light of descriptive statistical notations. For this purpose, firstly, the hyper parameters of the SETAR modelling (SETAR hyper parameters) of the relevant parities will be given with the R Project (version 3.1.2) printout.

Table 2. Keenan (1985), Tsay (1986) and Likelihood Ratio linearity tests

Linearity Tests		Keenan's (1985) One-Degree Test For Nonlinearity		Tsay's (1986) Test For Nonlinearity		Likelihood Ratio Test for Threshold Nonlinearity	
Parity		Test Statistics	P value	Test Statistics	P value	Test Statistics	P value
EURUSD	D1	3.424	0.064	2.513	0.113	5.016	0.307
	W1	3.448	0.063	2.698	0.100	2421.08	0*
	MN	3.655	0.057	3.015	0.084	528.438	3.638934e-114*
EURTRY	D1	2.792	0.094	0.679	0.410	9.744	0.160
	W1	2.666	0.103	0.381	0.537	690.584	2.55868e-149*
	MN	2.513	0.118	0.336	0.564	8.908	0.199
USDTRY	D1	0.023	0.877	0.010	0.919	2989.552	0*
	W1	0.110	0.739	0.070	0.790	606.222	5.021763e-131*
	MN	0.104	0.747	0.243	0.623	145.833	2.324389e-31*

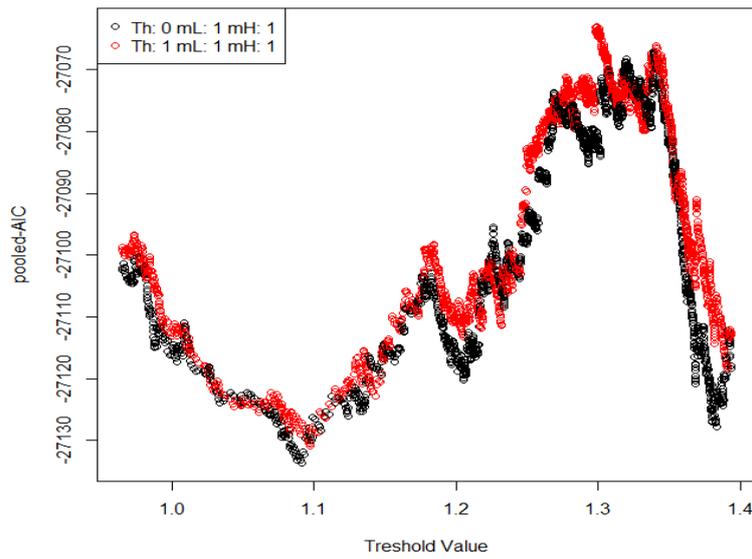
* The null hypothesis may be rejected at 5% significance level. (Since many statistical analyses take the analysis significance value as 5%, the significance value (α) was taken as 0.05 in this study.)

Table 3. EUR/USD - SETAR Model Hyper parameters

Parity		SETAR Hyper parameters						Number of Possible Threshold Value	Number of Threshold Values tested with Hyper Parameters
		m	Threshold Delay	mL	mH	Threshold Value	Pooled AIC		
EURUSD	D1	2	0	1	1	1.0915	-27133.74	1954	3908
	W1	2	0	1	1	1.32480	-4183.613	498	996
	MN	2	0	1	1	1.38060	-678.8609	124	248

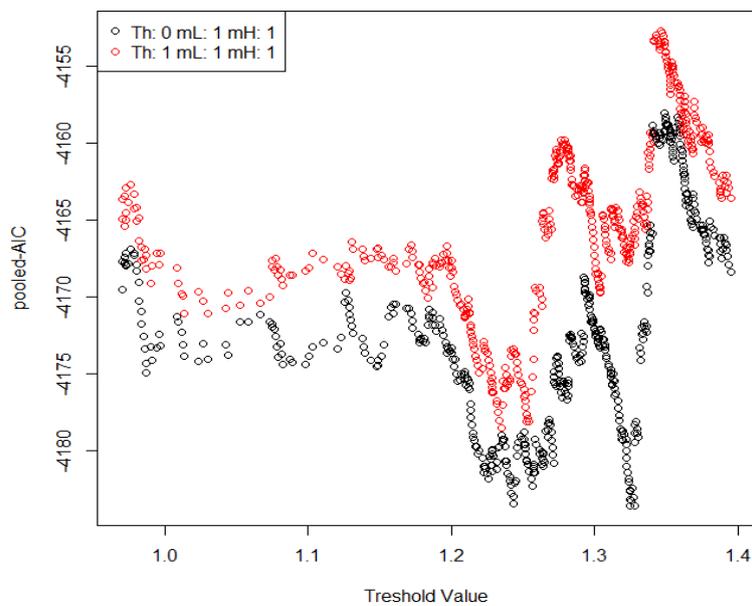
*m: The autoregressive level of the whole model
 **Threshold Delay: The delay level of the Self-excited model
 ***mL: Low regime level, mH: High regime level

Results of the grid search

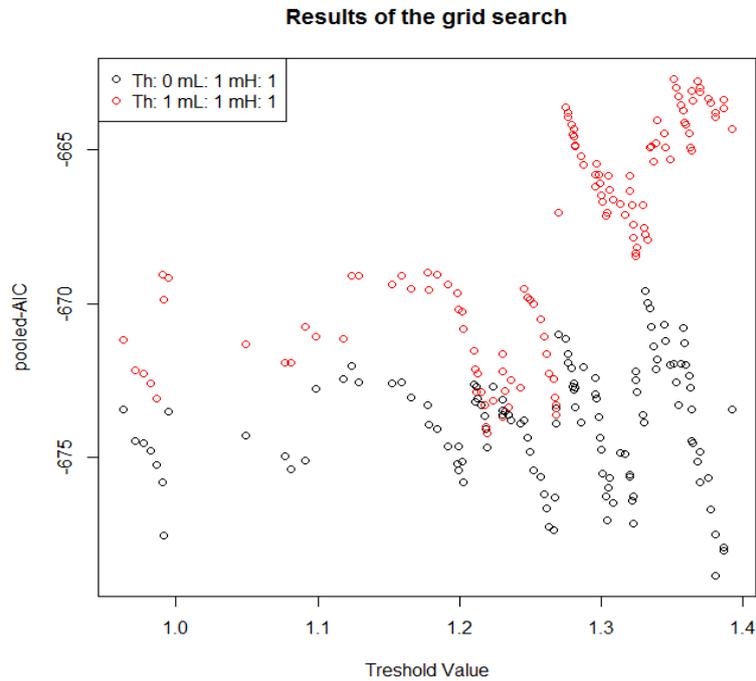


Graphic 1. EUR/USD, D1 threshold and AIC values

Results of the grid search



Graphic 2. EUR/USD, W1 threshold and AIC values



Graphic 3. EUR/USD, MN threshold and AIC values

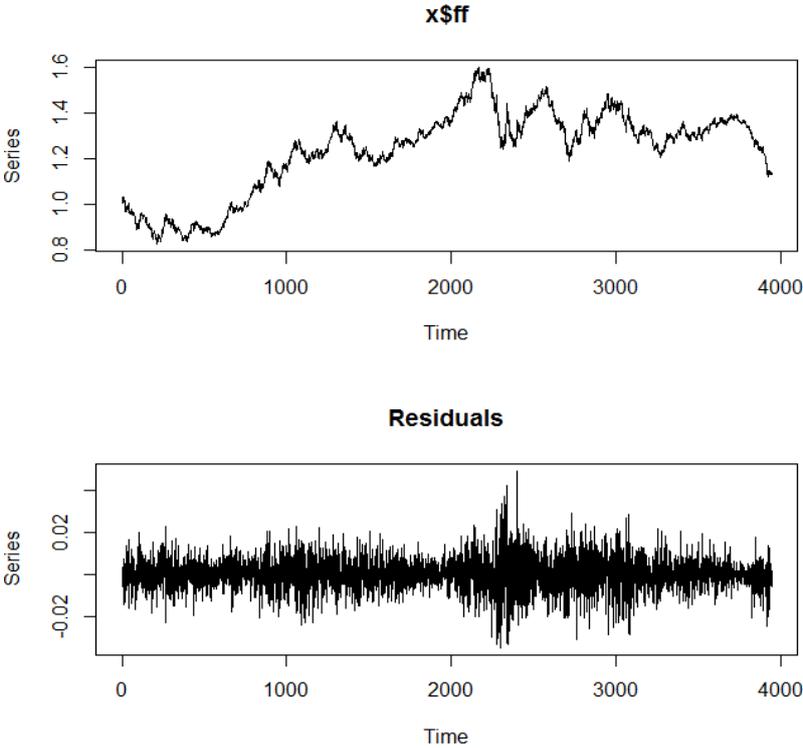
The delay (d) of the threshold value of the model and the threshold value are defined in Graphic 1-2-3 (within relevant time range) where the pooled-AIC value is at the minimum level.

Table 4. EUR/USD - SETAR Model Estimation and Coefficient Statistics

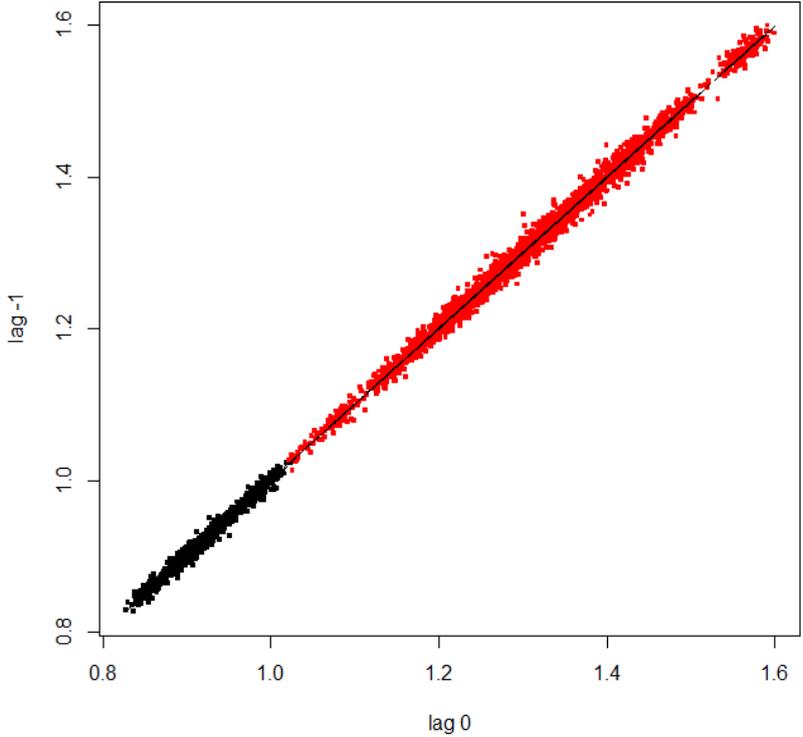
Parity		SETAR Model						Threshold Value	Residuals Variance
		Low Regime			High Regime				
		Constant	phiL.1	phiL.2	Constant	phiL.1	phiL.2		
EUR/USD	D1	0.008094	0.991197	-	0.004810	0.996353	-	1.017	6.125e-05
	W1	0.03896	0.957703	-	0.02339	0.98226	-	1.013	0.0002993
	MN	0.15693	0.829344	-	0.11794	0.91075	-	0.9912	0.00135
Coefficient Statistics		Estimation	Standard Error	p Value	Estimation	Standard Error	p Value		
D1	Constant	0.00809	0.00566	0.1528	0.00481	0.00172	0.005425**		
	phiL.1	0.99119	0.00615	2.2e-16***	0.99635	0.00131	2.2e-16***		
	phiL.2	-	-	-	-	-	-		
W1	Constant	0.03896	0.028179	0.1670	0.02339	0.00859	0.006628**		
	phiL.1	0.95770	0.03060	2.2e-16***	0.98226	0.00653	2.2e-16***		
	phiL.2	-	-	-	-	-	-		
MN	Constant	0.15693	0.13341	0.24103	0.11794	0.03743	0.00191**		
	phiL.1	0.82934	0.14555	4.9e-08***	0.91075	0.0284	2.2e-16***		
	phiL.2	-	-	-	-	-	-		

***: Very close to zero, **: Below 0,001, *: Below 0,01, (.) : Below 0,05, () : Very close to 1

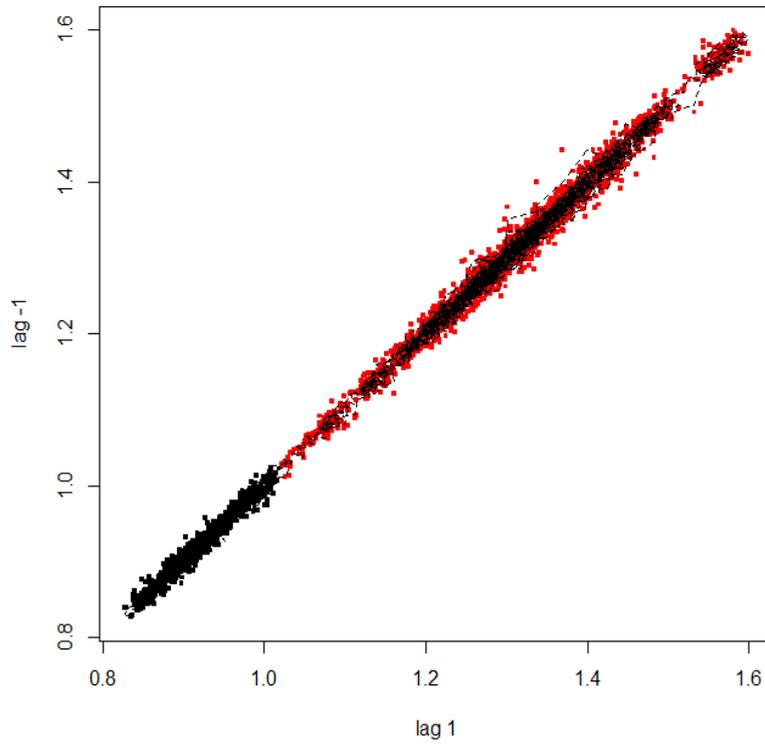
The descriptive properties of the SETAR model of the relevant parities are given in D1 time periods in the following tables. The “x\$ff” expression in the graphics is written for ease of encoding, and denotes the relevant time series.



Graphic 4. EUR/USD, D1 data and inclusions

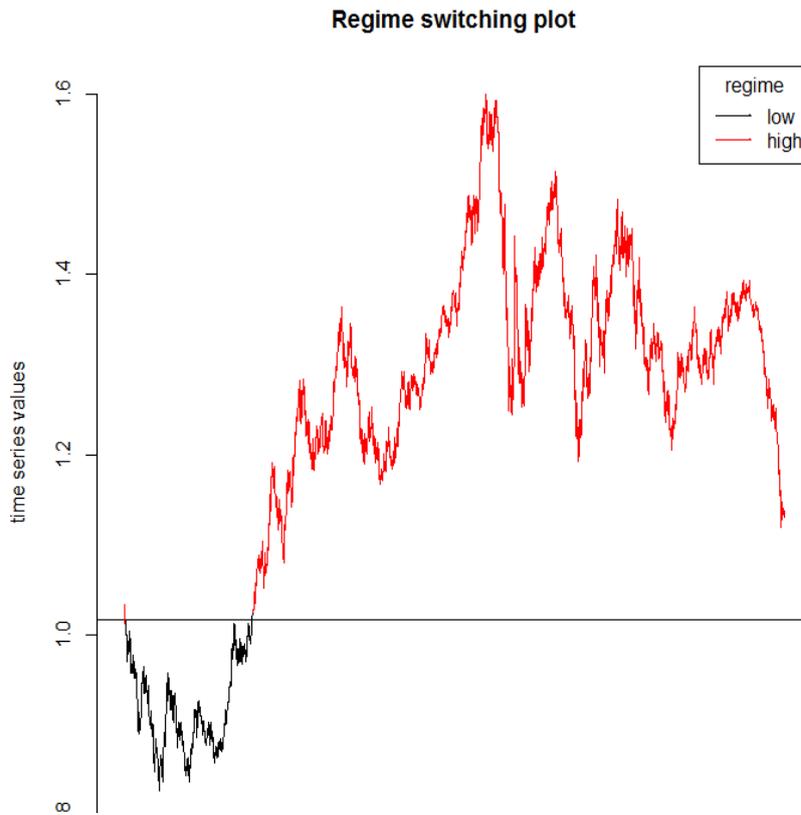


Graphic 5. EUR/USD, D1- Regime-sensitive Scatter Diag

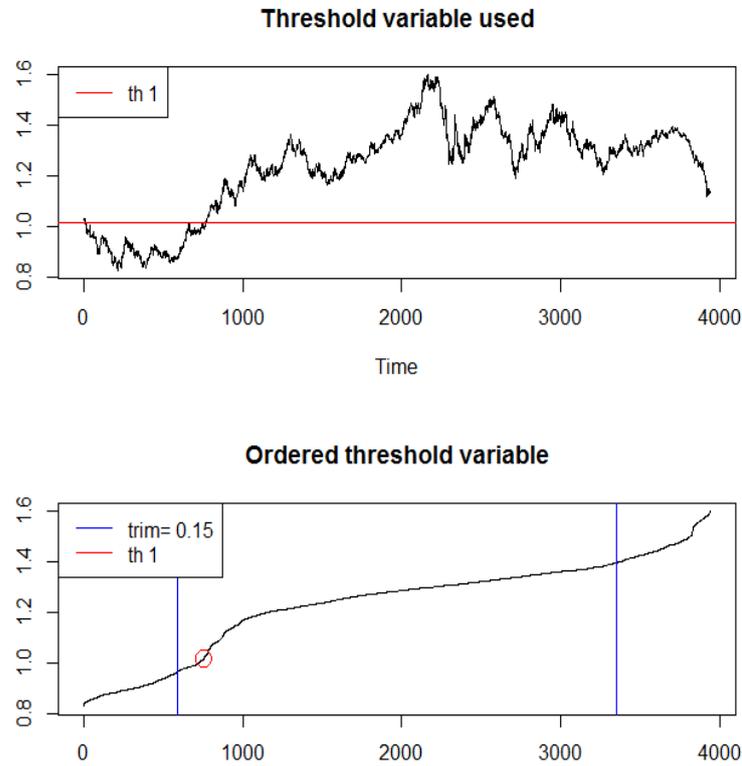


Graphic 6. EUR/USD, D1- Regime-sensitive Scatter Diag

When especially Graphic 5 and Graphic 6 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.



Graphic 7. EUR/USD, D1- Regime change graphics



Graphic 8. EUR/USD, D1- Threshold value graphics

Graphic 7 expresses the red pattern which is over the threshold value and the black patterns which is over the threshold value. When Graphic 8 is examined the first graphic threshold value indicates the observation values. Again, when the second graphic is examined in Graphic 8, it is observed that the position of the threshold value may be determined provided that 15% of the observation values from both ends are kept hidden.

Table 5. EUR/TRY - SETAR Model Hyper parameters.

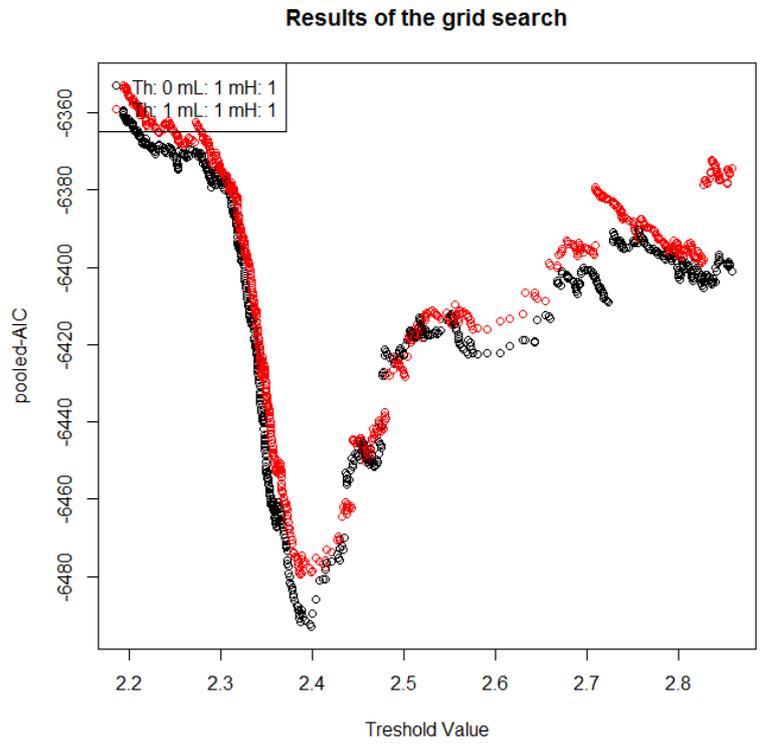
EUR/TRY (Euro - Turkish Lira) Parity

Parity		SETAR Hyper parameters						Number of Possible Threshold values	Number of Threshold Values Tested with Hyper parameters
		m	Threshold Delay	mL	mH	Threshold Value	Pooled AIC		
EURTRY	D1	2	0	1	1	2.3985	-6493.028	738	1476
	W1	2	0	1	1	2.5729	-977.2308	168	336
	MN	2	0	1	1	2.4898	-154.7870	38	76

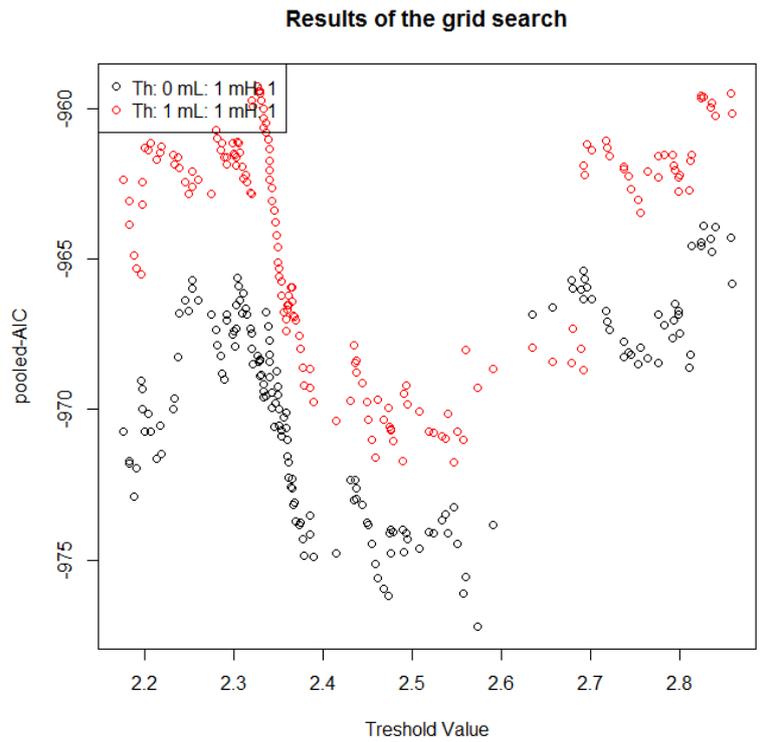
*m: The autoregressive level of the whole model

**Threshold Delay: The delay level of the Self-excited model

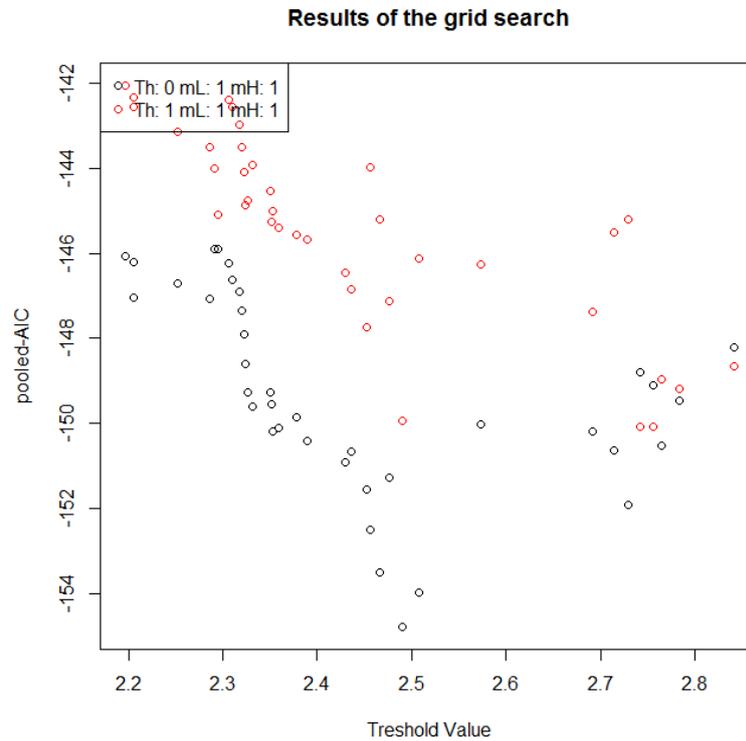
***mL: Low regime level, mH: High regime level



Graphic 9. EUR/TRY, D1 threshold and AIC values



Graphic 10. EUR/TRY, W1 threshold and AIC values



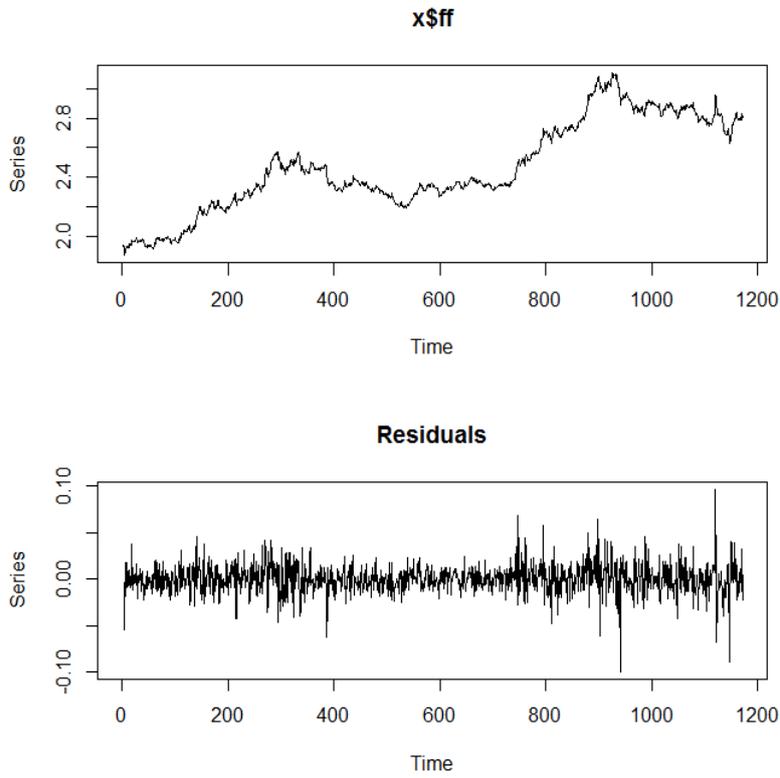
Graphic 11. EUR/TRY, MN threshold and AIC values

The delay (d) of the threshold value of the model and the threshold value are defined in Graphic 9-10-11 (within relevant time range) where the pooled-AIC value is at the minimum level.

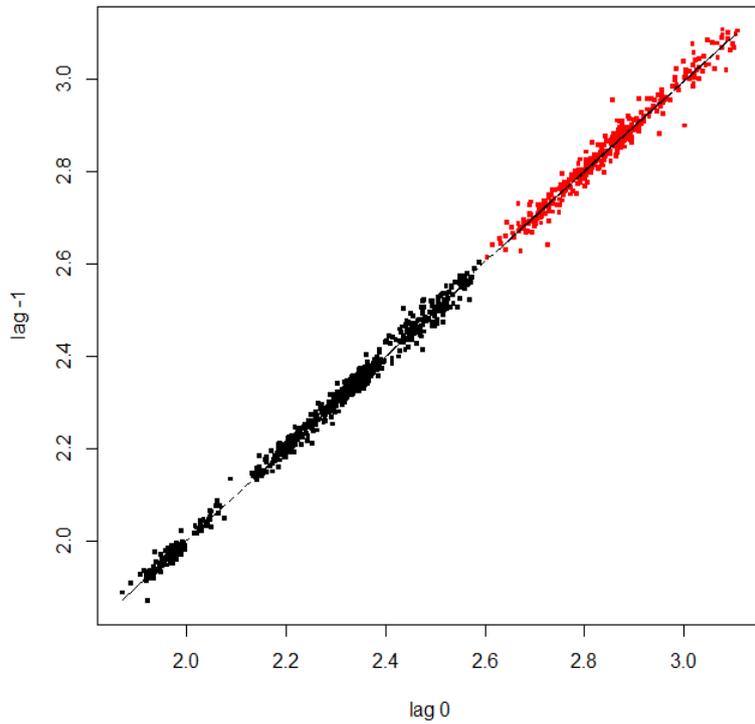
Table 6. EUR/TRY - SETAR Model Estimation and Coefficient Statistics

Parity		SETAR Model						Threshold Value	Residuals Variance
		Low Regime			High Regime				
		Constant	phiL.1	phiL.2	Constant	phiL.1	phiL.2		
EUR TRY	D1	0.01015	0.99591	-	0.06689	0.97666	-	2.575	0.0002534
	W1	0.04638	0.98144	-	0.27188	0.9052	-	2.573	0.00107
	MN	0.192976	0.92151	-	0.78153	0.72827	-	2.49	0.00313
Coefficient Statistics		Estimation	Standard Error	p Value	Estimation	Standard Error	p Value		
D1	Constant	0.01015	0.00765	0.18488	0.06689	0.021021	0.001499 **		
	phiL.1	0.99591	0.00334	2.2e-16 ***	0.97666	0.00739	2.2e-16 ***		
	phiL.2	-	-	-	-	-	-		
W1	Constant	0.0463	0.03352	0.16778	0.27188	0.09364	0.004031 **		
	phiL.1	0.9814	0.0146	2.2e-16 ***	0.90529	0.03292	2.2e-16 ***		
	phiL.2	-	-	-	-	-	-		
MN	Constant	0.19297	0.1370	0.1649	0.78153	0.26237	0.00433 **		
	phiL.1	0.92151	0.06021	2.2e-16 ***	0.72827	0.092895	1.79e-10 ***		
	phiL.2	-	-	-	-	-	-		

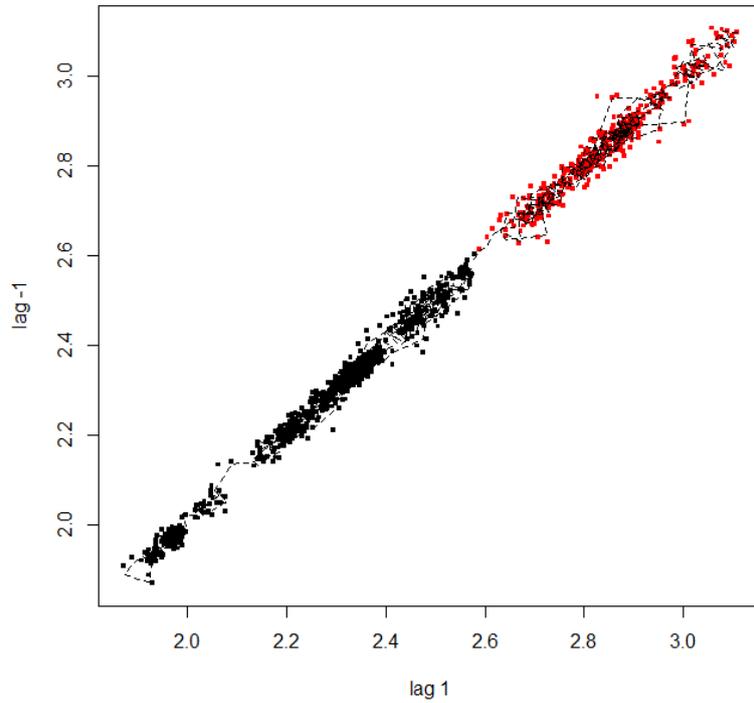
***: Very close to zero, **: Below 0,001, *: Below 0,01, (.) : Below 0,05, () : Very close to 1



Graphic 12. EUR/TRY, D1 data and inclusions

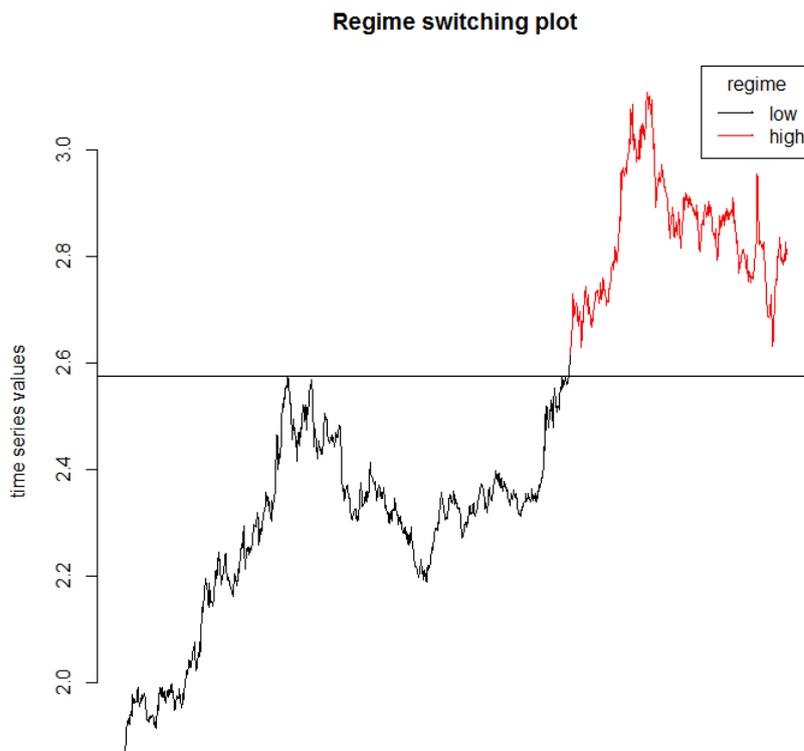


Graphic 13. EUR/TRY, D1- Regime-sensitive Scatter Diag

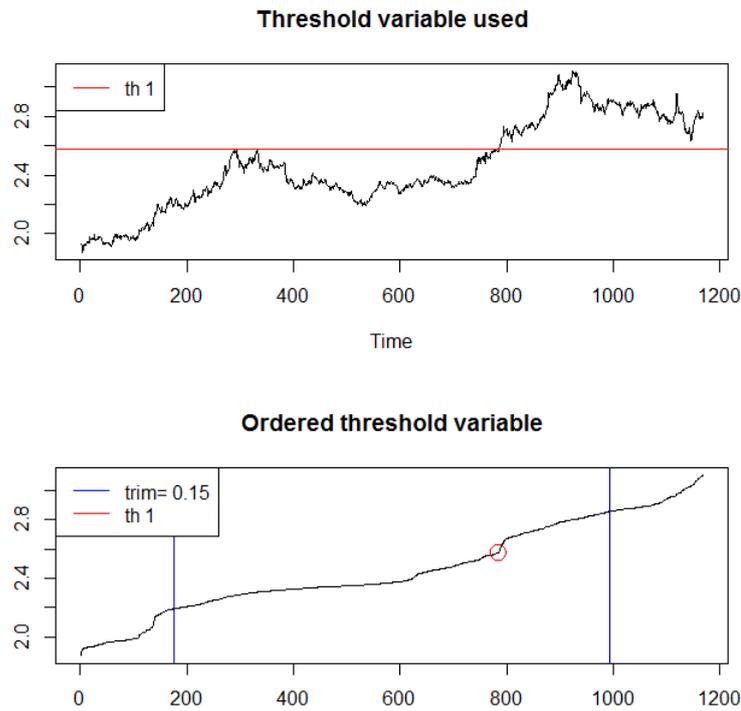


Graphic 14. EUR/TRY, D1- Regime-sensitive Scatter Diag.

When especially Graphic 13 and Graphic 14 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.



Graphic 15. EUR/TRY, D1- Regime change graphics



Graphic 16. EUR/TRY, D1-Threshold value graphics

Graphic 15 expresses the red pattern which is over the threshold value and the black patterns which is over the threshold value. When Graphic 16 is examined the first graphic threshold value indicates the observation values. Again, when the second graphic is examined in Graphic 16, it is observed that the position of the threshold value may be determined provided that 15% of the observation values from both ends are kept hidden.

Table 7. USD/TRY - SETAR Model Hyper parameters

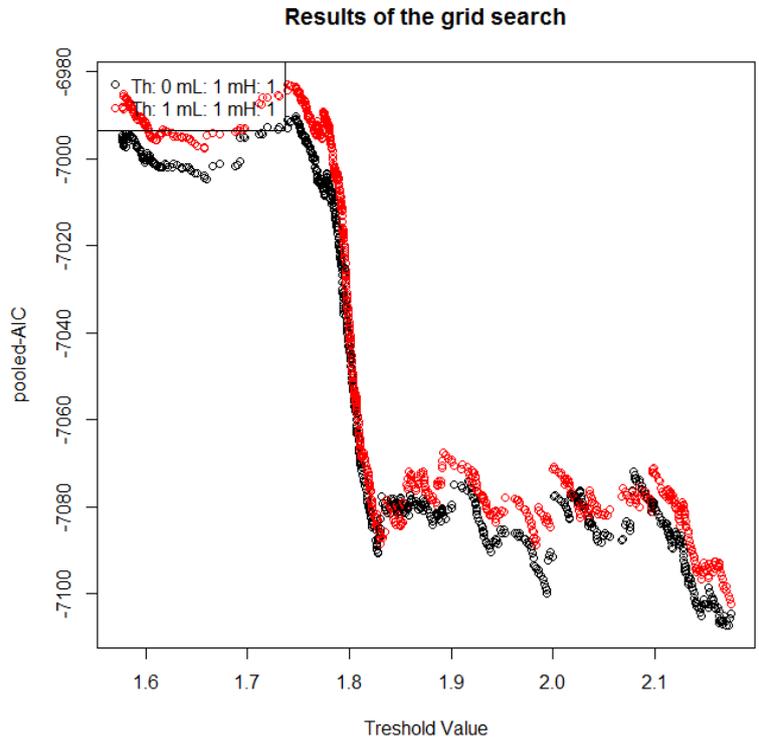
USD/TRY (USD Dollar – Turkish Lira) Parity

Parity		SETAR Hyper parameters						Number of Possible Threshold Values	Number of Threshold Values Tested with Hyper Parameters
		m	Threshold Delay	mL	mH	Threshold Value	Pooled AIC		
USDTRY	D1	2	0	1	1	2.1722	-7107.389	719	1438
	W1	2	0	1	1	1.8269	-1091.849	162	324
	MN	2	0	1	1	1.7685	-156.5893	38	76

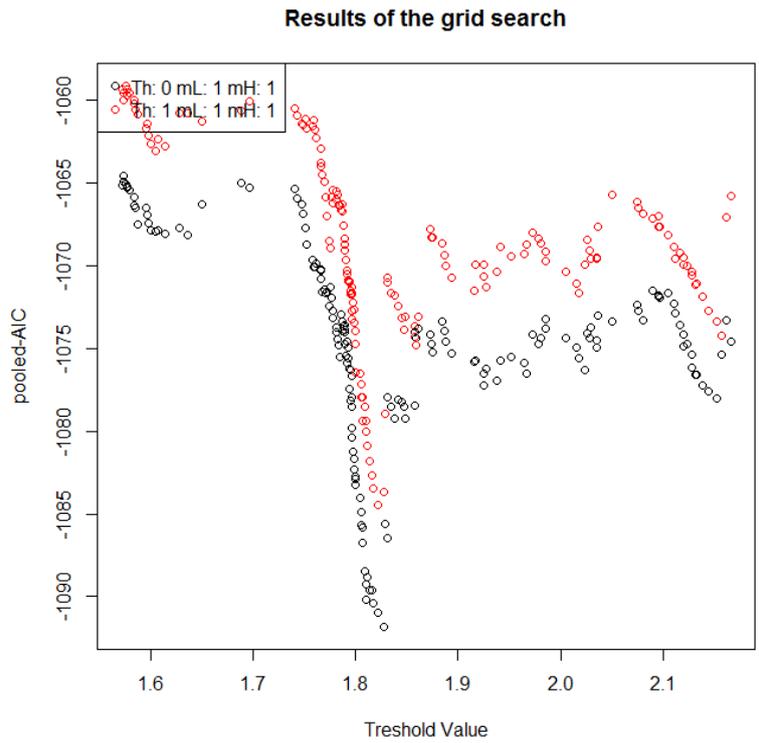
*m: The autoregressive level of the whole model

**Threshold Delay: The delay level of the Self-excited model

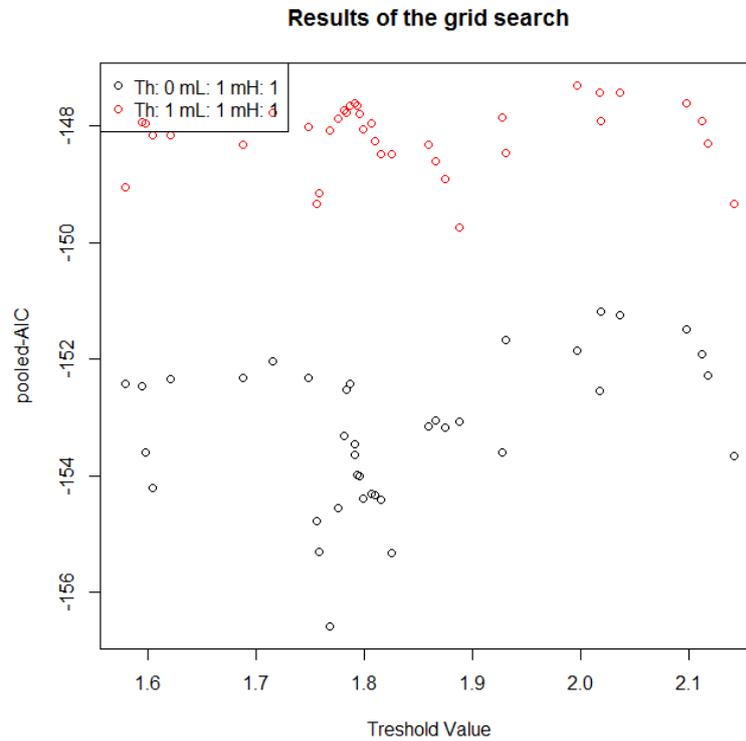
***mL: Low regime level, mH: High regime level



Graphic 17. USD/TRY, D1 threshold and AIC values



Graphic 18. USD/TRY, W1 threshold and AIC values



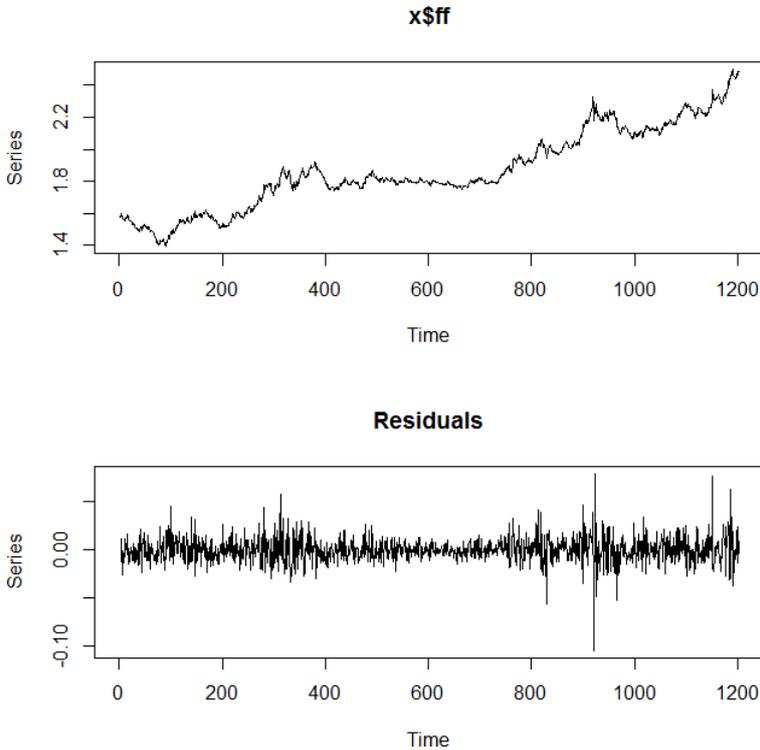
Graphic 19. USD/TRY, MN threshold and AIC values

The delay (d) of the threshold value of the model and the threshold value are defined in Graphic 17-18-19 (within relevant time range) where the pooled-AIC value is at the minimum level.

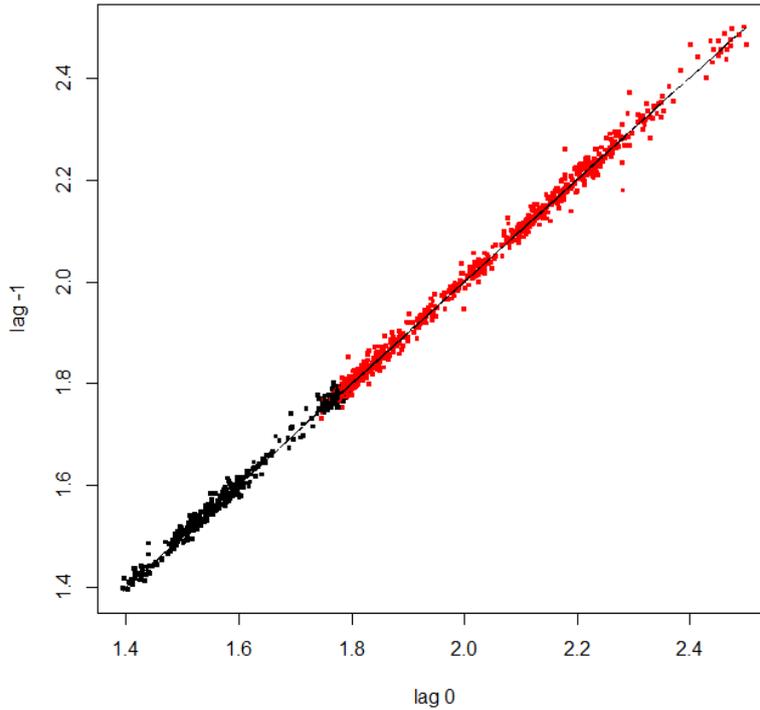
Table 8. USD/TRY - SETAR Model Estimation and Coefficient Statistics

Parity		SETAR Model						Threshold Value	Residuals Variance
		Low Regime			High Regime				
		Constant	phiL.1	phiL.2	Constant	phiL.1	phiL.2		
USD TRY	D1	-0.01599	1.01086	-	-0.0052	1.002866	-	1.775	0.0001724
	W1	-0.0509	1.0351	-	-0.0291	1.015810	-	1.785	0.000739
	MN	-0.28829	1.19714	-	-0.12986	1.07049	-	1.768	0.002994
Coefficient Statistics		Estimation	Standard Error	p Value	Estimation	Standard Error	p Value		
D1	Constant	-0.01599	0.01025	0.1191	-0.00521	0.00474	0.2713		
	phiL.1	1.01086	0.00639	2e-16 ***	1.0028	0.00237	2e-16 ***		
	phiL.2	-	-	-	-	-	-		
W1	Constant	-0.05090	0.03967	0.2007	-0.0291	0.0229	0.2046		
	phiL.1	1.03519	0.0245	2e-16 ***	1.0158	0.0114	2e-16 ***		
	phiL.2	-	-	-	-	-	-		
MN	Constant	-0.2882	0.19580	0.1467	-0.1298	0.098387	0.1924		
	phiL.1	1.1971	0.12202	1.3e-13 ***	1.0704	0.04921	2.2e-16 ***		
	phiL.2	-	-	-	-	-	-		

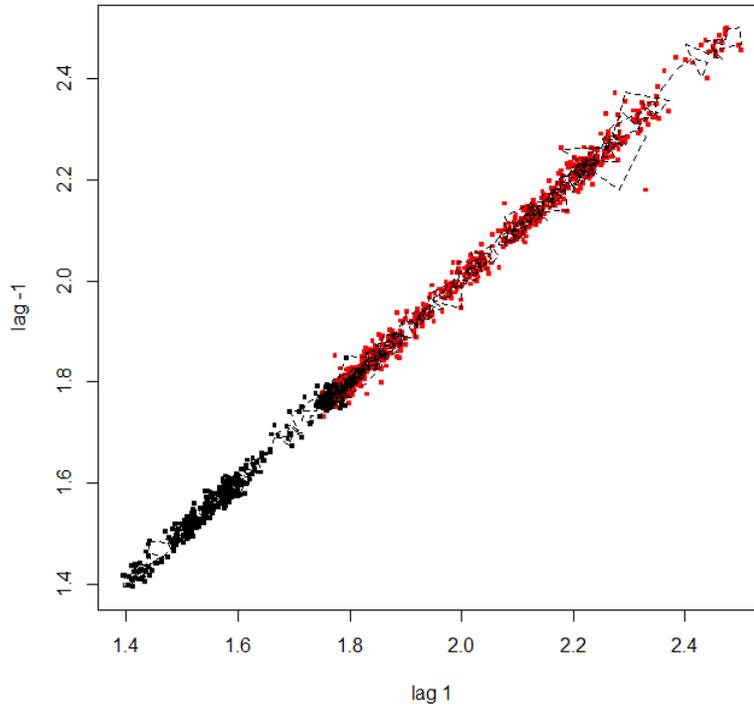
***: Very close to zero, **: Below 0,001, *: Below 0,01, (.) : Below 0,05, () : Very close to 1



Graphic 20. USD/TRY, D1 data and inclusions

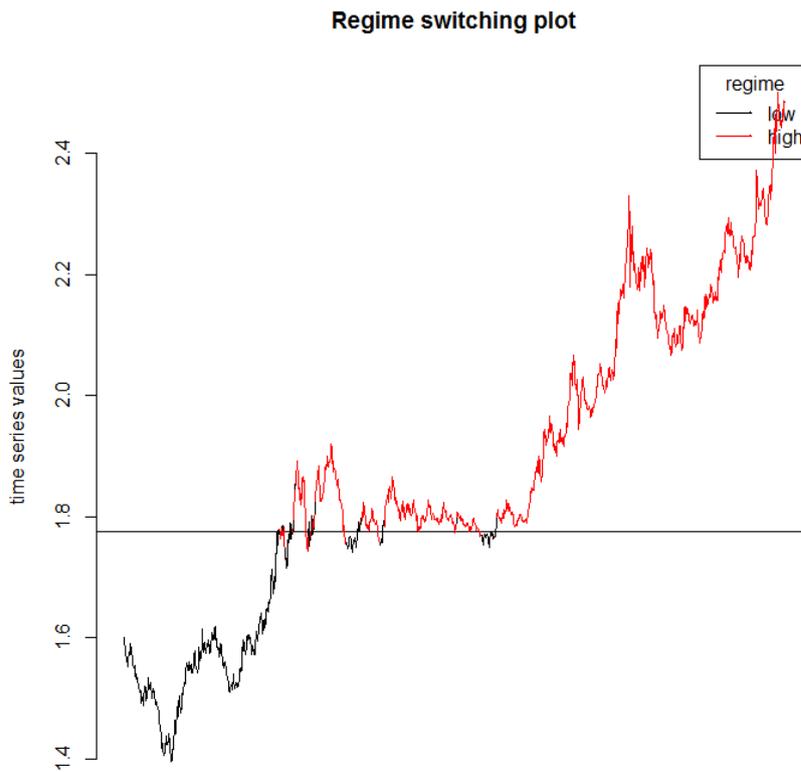


Graphic 21. USD/TRY, D1- Regime-sensitive Scatter Diag

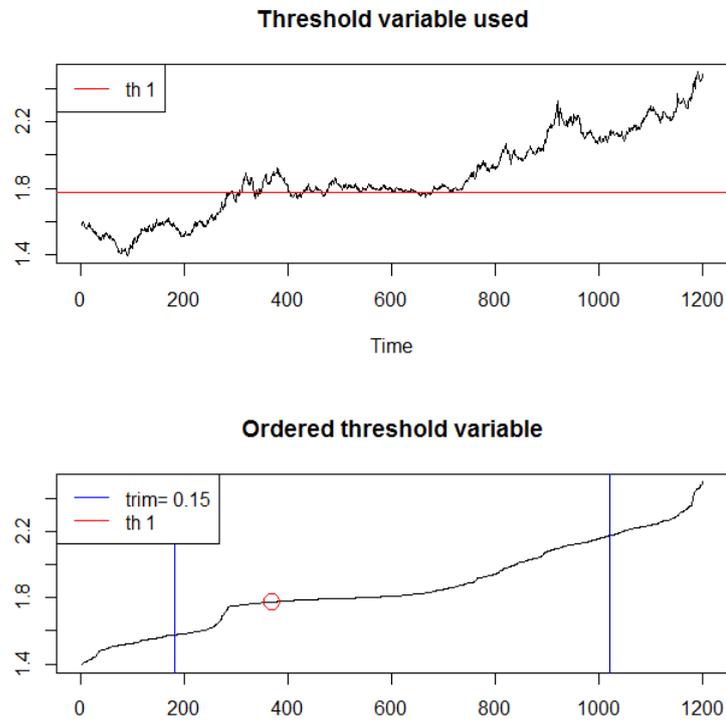


Graphic 22. USD/TRY, D1- Regime-sensitive Scatter Diag

When especially Graphic 21 and Graphic 22 are examined, it attracts attention that polarization in the regimes disappear as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.



Graphic 23. USD/TRY, D1- Regime change graphics

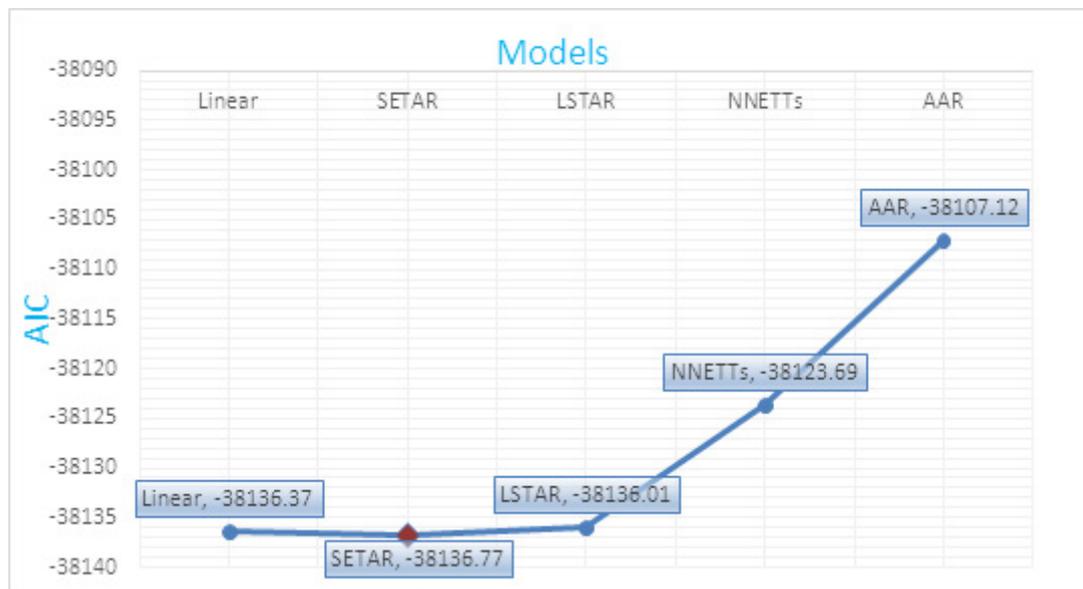


Graphic 24. USD/TRY, D1- Threshold value graphics

The SETAR Model, which is one of the TAR Group modeling, shows a better performance than many other linear and non-linear modeling. In this study, the purpose is to express that the SETAR model performance is superior to the other models by considering the observation values of the parities. In this context, the AIC (Akaike Information Criteria) values of the SETAR Model and the other models were compared, and it was revealed that the SETAR model is more influential in terms of the relevant parities.

Table 9. Some Models and AIC Values

Models	AIC
Linear	-38136.37
SETAR	-38136.77
LSTAR	-38136.01
NNETTs	-38123.69
AAR	-38107.12



Graphic 25. Some Non-Linear Models and AIC Values

Here, in the other modeling in question;

Linear Modeling (Linear): In this modeling, AR(p)-Type modeling was selected; and the delay length $m=2$ was selected.

Logistic Soft-Transition Autoregressive Modeling (LSTAR): It is the type of modeling which is considered within a continuous indicator function.

Artificial Neural Networks Modeling (NNETTs): In a neural network problem with hidden unit and linear output with activation function, the embedded dimension was selected as $m=2$.

Additive Autoregressive Models (AAR): Non-parametric additive model form was handled when $m=2$

6. Conclusions

In this study, self-excited threshold autoregressive models have been used in order to be able to model the volatile changes in currency market. The linearity of the currency parities, which were dealt with before the SETAR modelling approach was applied, have been examined with the help of various approaches (Tsay, Keenan, Likelihood Ratio), and then the SETAR modelling was applied to all parities no matter whether they are linear or not in D1 (daily), W1 (weekly) and MN (monthly) time intervals. In applying the modelling, firstly the model hyper parameters were determined for each parity and each time period, and then in the light of these hyper parameters the SETAR model was estimated.

The SETAR model performance is superior to the other models by considering the observation values of the parities. (Table 9 – Graphic 25)

When the regime-sensitive scatter diagram are examined, it attracts attention that polarization in the regimes disappear

as the delay intervals of the parity values, which are the subject matter of diaphragms, expands.

(The observation values in red are expressed as the High Regime; and the ones in black are expressed as Low Regime).

In this study, a structure integrated with two different linear models was used in the context of single threshold while the non-linear structures of the time series of the parities were examined. Although not used commonly in the macro-economic literature, there are some situations in which the number of the threshold value is more than one (in such a situation, the number of the threshold value being more will also increase the number of the regimes estimated). The non-linear structure in currencies may be investigated with modelling in which there are a number of threshold values.

Even, the chaos estimation of the relevant datasets may also be performed because the non-linear structure in the parities might be handled with chaotic approach (which the other linearity tests lose their values in such a situation, and the linearity tests, which include the basic hypothesis that investigate chaos, are made use of instead).

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