research studies in mathematics education focusing on the Pirie-Kieren model (e.g., [3], [15], [16]) There is also a larger number of studies available concerned with students' general understanding and misunderstanding of concepts with regard to the concept of functions, (e.g., [17], [18], [19], [20]). The researchers also encountered studies concerning the limit concept with regard to functions with two variables (e.g., [11], [21]). Researchers also encountered research concerned with the finding of the domain and the designation of coordinate planes in multivariable functions. For this reason, it is felt that this study will serve to address an issue in the field that is in need of attention.

1.4. Purpose of the Study

The main aim of this study is to investigate the understandings of mathematics preservice teachers with regard to the domain of multivariable functions using the Pirie-Kieren model in a small group of students. The layers which the preservice teachers' responses in were identified. Thus, the participants' understandings were examined through a new perspective. There was an attempt to find an answer to the following research question:

"On which layers of the Pirie-Kieren Model are the understandings of primary school mathematics preservice teachers placed concerning the domain of multivariable functions?

2. Method

This study that examined the understanding of primary school mathematics preservice teachers regarding the domain of multivariable functions was a descriptive study. The data obtained was summarized according to previously-defined dimensions and interpreted. Preservice teachers' responses for each question were placed in Pirie-Kieren layers which were formed previously by the researchers. A coding scheme was created according to Pirie-Kieren model proposed by Pirie and Kieren [6]. All coded student answers were presented numerically in tables and bar charts. Similar analysis method was used in some mathematics education research (e.g., [22], [23], [24]). Students' answers were categorized according to Pirie-Kieren model in the understanding of some mathematical concepts such as infinite numerical series [25], geometric transformations [15] in other research. Besides, example student answers for each question were illustrated according to Pirie-Kieren layers (Image Making, Image Having, Property Noticing) in the present study. Finally, four students' answers were given in the mathematical understanding scheme.

2.1. Study Group

The working group of the study consisted of 40 sophomore preservice teachers studying on the primary mathematics teaching program at a state university. All of the students who participated in the study had completed the Analysis II course. Such learning objectives as finding the domain of multivariable functions and drawing of the related graphs are included in the Primary School Mathematics Teaching Curriculum. Furthermore, topics such as limits in multivariable functions, derivatives, and calculating integrals are also present. The students possess sufficient knowledge, on account of completing this course, to be able to answer questions that arise within the context of this research study.

2.2. Data Collection Tool

In this study, an achievement test consisting of open-ended questions prepared by the researchers was used during the data collection stage. In the question preparation stage, the opinions of two experts in mathematical education were elicited. Four questions were included in the test. On account of the fact that every question was to be examined and answered in detail by the participants, it was felt that the number of questions was adequate. The questions were in particular connected to the finding of domain sets and demonstration of coordinate planes in multivariable functions. Functions with both two and three variables were included within the questions. The preservice teachers, on account of having seen these concepts during the Analysis II course, possessed sufficient knowledge to be able to submit answers concerning the concepts that they faced.

The questions that were used during data collection are as seen below:

1. Find the domain of $f(x,y) = \sqrt{y.\sin x}$ and show this on the coordinate plane.
2. Find the domain of $f(x,y) = \frac{1}{\sqrt{y-\sqrt{x}}}$ and show this on the coordinate plane.
3. Find the domain of $f(x,y) = arctan\frac{x}{y}$ and show this on the coordinate plane.
4. Find the domain of $f(x,y,z) = \ln\,(4 - x^2 - y^2 - z^2)$ and show this on the coordinate plane.

2.3. Analysis of the Data

The obtained data was analyzed according to Pirie-Kieren model. A coding procedure was created depending on the growth of mathematical understandings of preservice teachers and the concept of the domain of multivariable functions. The Pirie-Kieren Model that is used to explain in particular the growth of the individual understandings contains eight action stages. These stages are designated as primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventising. In the data analysis stage, it was determined at which level the responses of the preservice teachers fitted this model. In this stage, two different encoded responses were placed at appropriate stages in the model. When the researchers had placed an answer at different levels, a detailed discussion and debate

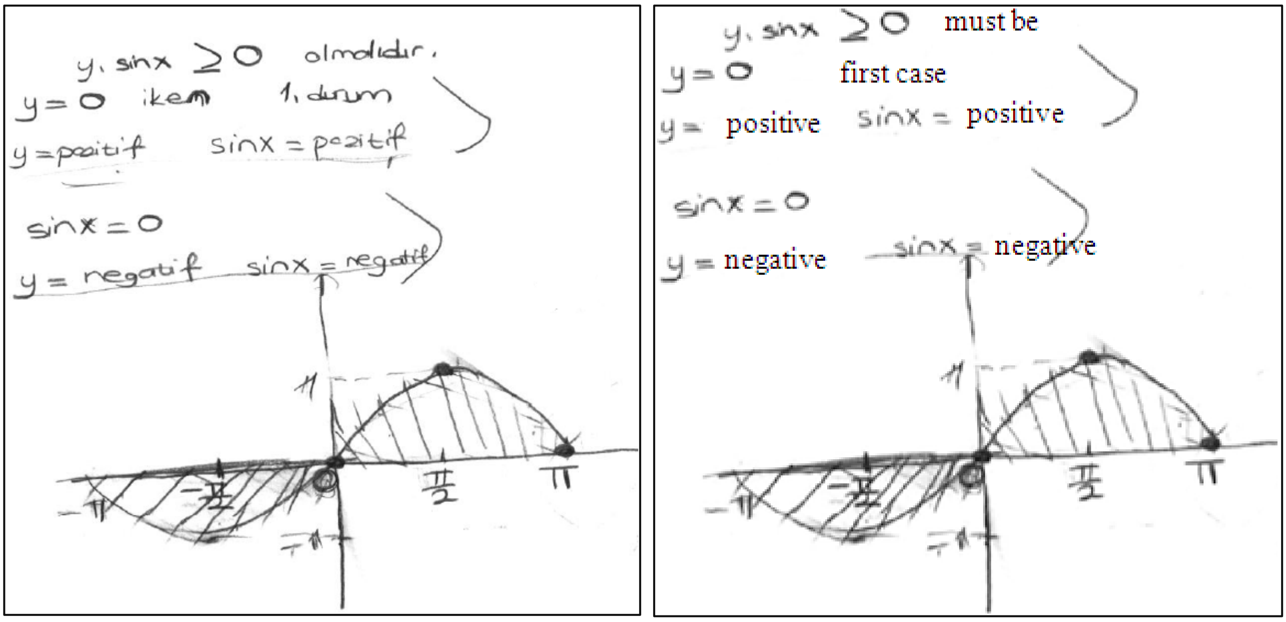

Figure 4. An example student answer included in the “Property Noticing” stage

3.2. Findings with regard to the Second Question

The answers that teacher candidates gave to the question “Find the domain of $f(x,y) = \frac{1}{\sqrt{y-\sqrt{x}}}$ and show this on the coordinate plane” were evaluated according to the Pirie-Kieren model and the findings shown below were obtained.

Table 2. Pirie-Kieren Understanding Levels for the Second Question

	Image making	Image having	Property Noticing
2nd Question	-	8	32

From the answers submitted by 40 students to the first question, 32 were included in the “property noticing” stage, while 8 were included in the “Image Having” stage. The situation regarding the breakdown of the answers is shown in the bar graph below.

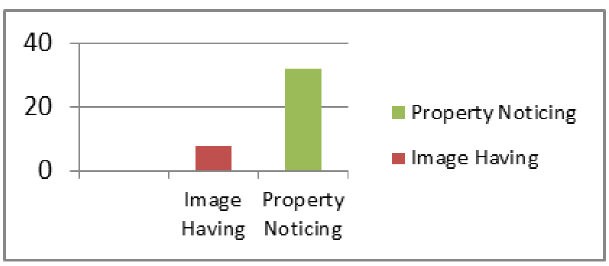

Figure 5. Bar chart for the second question

Example answers for student answers that were included within the “Image Having” and “Property Noticing” stages.

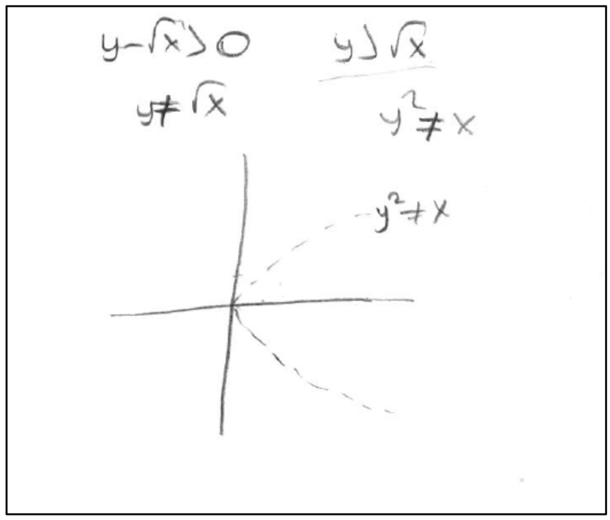

Figure 6. An example student answer included in the “Image Having” stage

3.3. Findings with regard to the Third Question

The answers submitted by preservice teachers for the question “Find the domain of $f(x,y) = arctan\frac{x}{y}$ and show this on the coordinate plane” were evaluated according to the Pirie-Kieren model and the results below were obtained.

Table 3. Pirie-Kieren Understanding Levels for the Third Question

	Image making	Image Having	Property Noticing	Empty
3rd Question	7	18	12	3

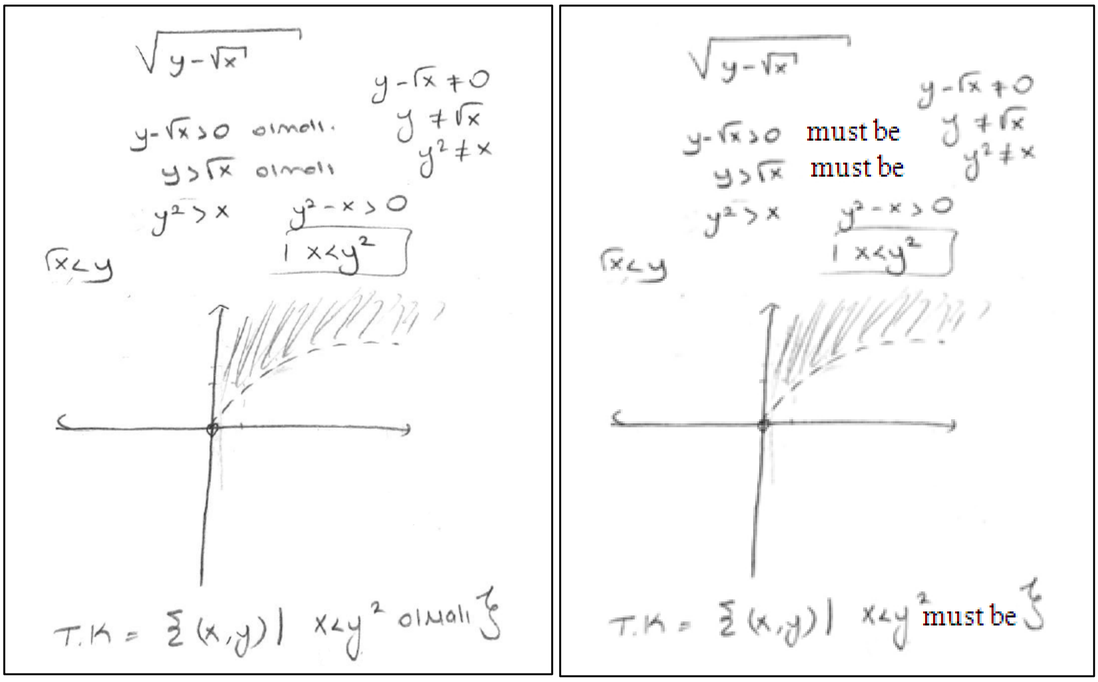

Figure 7. An example student answer included in the "Property Noticing" stage

For the third question, from the 40 answers submitted by students, 18 were included in the "Image Having" stage, 12 were included in the "Property Noticing" stage and 7 in the Image Making Stage. The situation regarding the breakdown of the answers is shown on the bar chart below.

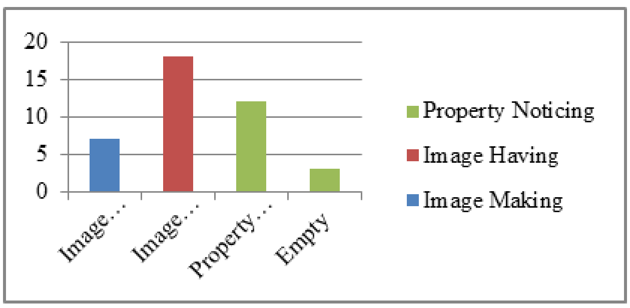

Figure 8. Bar chart for the third question

An example answer submitted by the students to the third question that was included in the "Image Making" stage is shown below:

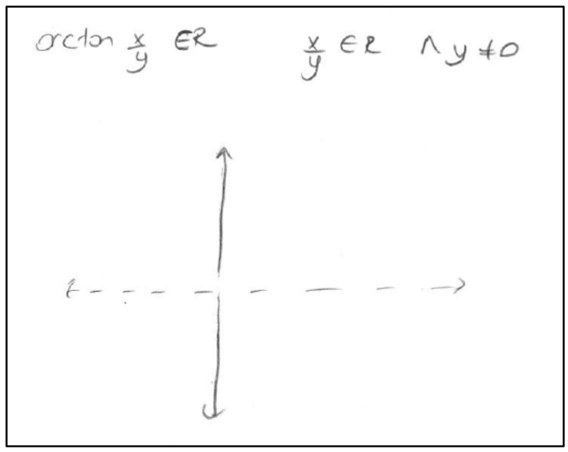

Figure 9. An example student answer included in the "Image Making" stage

Example answers of students whose answers were included in the stages: "Image Making" and "Property Noticing" for the third question:

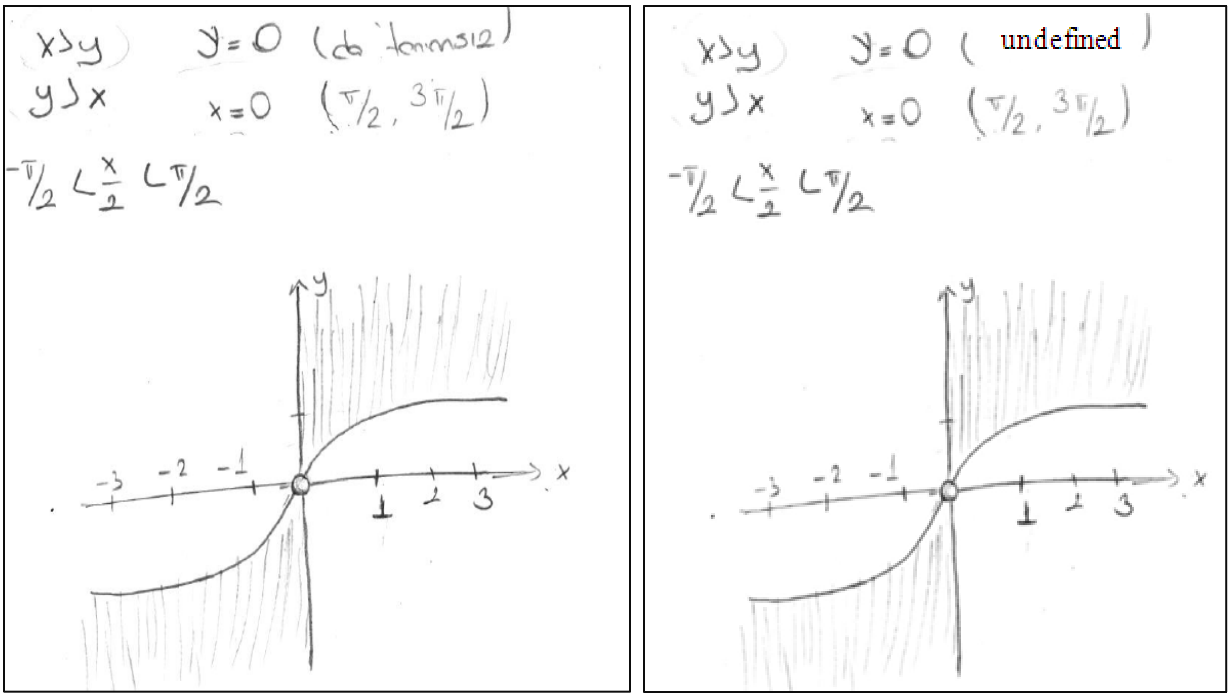

Figure 10. An example student answer included in the "Image Having" stage

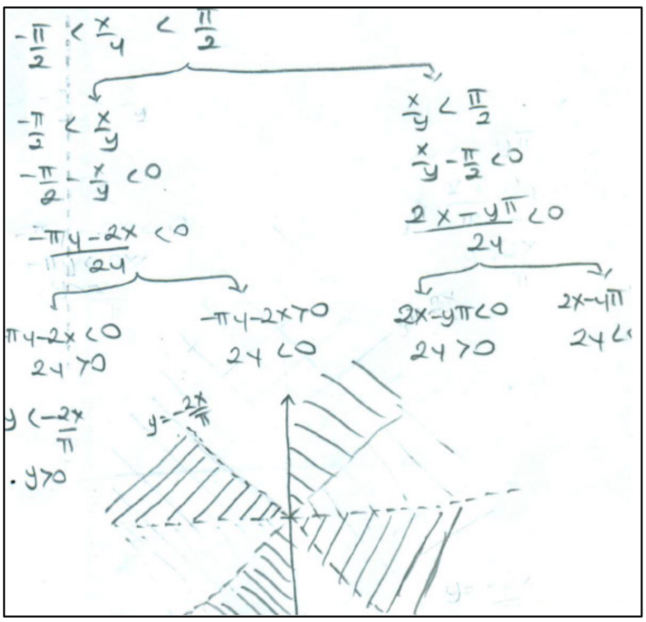

Figure 11. An example student answer included in the "Property Noticing" stage

3.4. Findings with regard to the Fourth Question

The answers submitted by primary school mathematics preservice teachers to the question: "Find the domain of $f(x,y,z) = \ln(4 - x^2 - y^2 - z^2)$ and show this on the coordinate plane" were evaluated according to the Pirie-Kieren model and the findings presented below were obtained:

Table 4. Pirie-Kieren Understanding Levels for the Fourth Question

	Image Making	Image Having	Property Noticing.
4th Question	-	5	35

From the 40 answers submitted by students to the fourth question, 35 were included in the "Property Noticing" stage, while 5 were included in the "Image Having" stage. The situation as regards the breakdown of the answers submitted is shown on the bar chart below.

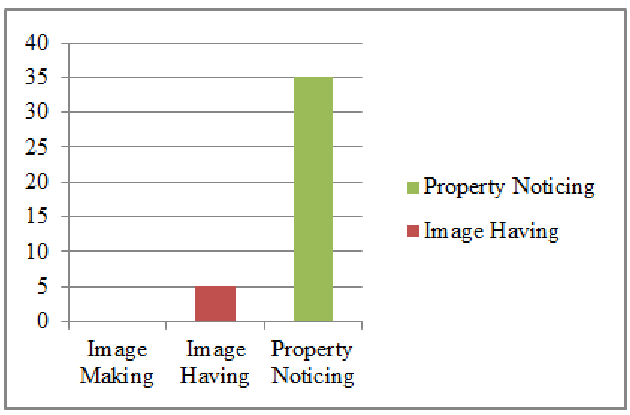

Figure 12. Bar chart for the third question

Examples of student answers to the fourth question that were included within the "Image Having" and "Property Noticing" stages are presented below:

Figure 15. S1's mathematical understanding scheme

Figure 16. S2's mathematical understanding scheme

Figure 17. S3's mathematical understanding scheme

Figure 18. S4's mathematical understanding scheme

4. Discussion

In the present study, the growth of mathematical understanding of primary mathematics preservice teachers regarding the concept of the domain of multivariable functions was described. Depending on the preservice teachers' preconceptions or learning outputs during the Analysis II courses, they gave different answers for questions. Preservice teachers responses moved between three layers of understanding (image making, image having, property noticing) for all questions. In addition, preservice teachers didn't reach the final layer of Pirie-Kieren model. Thus, answers appropriate for each Pirie-Kieren characteristics were not observed. Similar findings that were not obtained all levels of Pirie-Kieren model were found in some studies (e.g., [15], [25]).

The data for every question were evaluated separately. There was a multivariable function in the first question on the achievement test. The expression included both a quadratic square root as well as a trigonometric function. From 40 students, 37 participants, that is 92% were at the "Property Noticing" stage. In other words, they held images that were related both to square root and trigonometric functions. By thinking of both of these concepts at the same time they were able to define a common domain set. For this reason they were able to bring these images together and use them so as to solve the problem [6]. The remaining students held separate images concerning functions but were unable to combine these.

The function in the second question was a rational expression and again constituted a function with two variables. It should be noticed here that the denominator should be different from zero. In the expression of the denominator, there are two interrelated square root functions. In the internal square root only x is present. The x variables should be greater than or equal to zero. At the same time, the expression $\sqrt{y-\sqrt{x}}$ should be greater than zero but not be equal to zero. By drawing the related $y^2 = x$ function, the domain set is defined on the coordinate plane. From 40 students, 32 participants or approximately 80% according to the Pirie-Kieren model were at the "property noticing" stage. It followed that by considering the square root expressions of the denominator, they had made a correct evaluation. They held the image that the interior of second degree square root expressions must be positive; however they considered these concepts separately and in general did not make a correct evaluation.

The answers submitted to the first and second questions correlated with the image having and property noticing stages according to the Pirie-Kieren model. However, on the third question, answers were also encountered that were included on the "image making" stage. Seven of the preservice teachers gave answers that were included in the "image making" stage. It followed that they were at a stage at which they were able to acquire an idea regarding what the question was concerned with the question [7]. The third question contains an inverse trigonometric function and contains a rational expression within a function. Two variables are contained within a rational expression. This was the question from the four with which students experienced the greatest difficulties.

3 preservice teachers left the question blank. In a departure from the first two questions, answers were given that were included in the "image having" stage. From this result, we can deduce that students at this level of understanding may create a mental construct so as to solve the problem without having to carry out specific activities [7]. From 40 students, the number of students that were able to perform this task was approximately 18 from 40, or 45% of the participants. It may also be stated that students experienced difficulties in expressing the domain of an inverse tangent expression. In general, they did not have images with regard to this section or had incorrect knowledge of the topic. However, they were able to see easily that in a rational expression x is greater than 0. Two correct expressions emanating from the starting point have to be determined regarding the definition of the space needed to find x/y. and this then needs to be shown on the coordinate plane. 12 students were only able to conduct an evaluation in this way, and this corresponds to 30% of the total of participants.

While the first three questions included functions with dual variables, the fourth question included three variables. So as to show the domain of the function on the coordinate plane, three axes must be used. The function in the question is a logarithmic function. It follows that the expression within the function needs to be greater than zero $(4 - x^2 - y^2 - z^2) > 0$. The equation $x^2 + y^2 + z^2 = 4$ indicates a sphere with a centre (0,0,0) and radius of it is 2 br. In cases of inequality, the interior region of this sphere constitutes the domain set of the function. The answers were included within the "Image Having" and "Property Noticing" stages of the Pirie-Kieren model. 35 of the preservice teachers were at the "property noticing" stage. In other words, they defined that the expression containing three variables within a logarithmic function expressed a sphere and showed this on the coordinate plane. The remaining five students were either not able to think that this function expressed a sphere or made incorrect evaluations but nevertheless held images that allowed them to define the domain set of a function with three variables.

Because of the limits of this paper, it was presented only students' responses related to the concept of domain of multivariable functions. Embarking from the results of the answers submitted by preservice teachers to the questions in this study, the following may be suggested to researchers and applicants of education.

This study that took as its study group second-class students from the primary mathematics teaching department, who had completed the Analysis II course, may be conducted for other disciplines and lessons. Moreover, an experimental study may also be conducted.

In studies to be carried out involving preservice teachers or students, a comparison may be made between the Pirie-Kieren and APOS theories of education.

A pre-test may be carried out to ascertain the levels of previous knowledge of students and to what extent this knowledge is accurate.

By predicting difficulties and misconceptions that may be faced during teaching, an appropriate teaching program may be designed for incorporation within the curricula of faculties of education.

There were four tasks in this study. If the number of the tasks is increased, it will allow us to demonstrate the different levels of understanding about the domain of multivariable sets for future studies.

The preservice teachers' process of construction of the domain of multivariable sets was described only through their written responses. As a continuation of this study, adding observations of students about the state of the course work can be done with more qualitative data.

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