

# The Arithmetic in Galois's Equations

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**Abstract** In this article I demonstrate the method for solve Galois's equations, is applying simply arithmetic in his coefficients.

**Keywords** Arithmetic, Determinant Coefficients, Factoring, Galois's Inverse Problem

**AMS 11** Number Theory

## 1 Introduction

Throughout the latter century Galois's polynomials have taken great relevancy in mathematics and by it, the great number of articles that to they reference. By associating Galois theory with the problems of the Diophantine equations (elliptical curves) and in the inverse problem (Hilbert). Nevertheless none of them reflected an process of beginning to the solution, and if has got improved with the Newton's-Rapson Algorithm. Therefore the article that I present will be of interest and relevancy for the mathematicians.

The equations of Galois are polynomials in expansion[8].  
 $\forall (a, b, c, \dots, k, L, x) \in \{\mathbb{N}\}$

$$ax^n + bx^{n-1} + \dots + kx^{n-(n-1)} = 0 \quad (1.1)$$

$$ax^n + bx^{n-1} + \dots + kx^{n-(n-1)} + L = 0 \quad (1.2)$$

The groups of Galois are the equations that have same number of addends and the same variable.

$$(ma)x^n + (mb)x^{n-1} + \dots + (mk)x^{n-(n-1)} = 0; \quad (1.3)$$

$$(m > 1) \in \{\mathbb{Z}\}.$$

## 2 Main Results

All the equations have the form following, always will have a minimum of two negative numbers; for example:

$$ax^n + bx^{(n-1)} + cx^{(n-2)} + \dots + (-jx^2) = -kx^{n-(n-1)}.$$

**Therefore if we apply arithmetic we have**

Is to say:

$$ax^n + bx^{(n-1)} + cx^{(n-2)} + \dots + \dots < (jx^2) \text{ with which}$$

$$ax^n + bx^{(n-1)} + cx^{(n-2)} + \dots + \dots = (j - m')x^2$$

is to say:  $(j - m')x^2 = \sum_a^i ax^n$  consequently.

$m'.x^2 = k.x$  **Is equality generalized for all the Galois's equations.[6],[7],[10]**

In the "(1.2)", we have the same thing is to say that:

$$L \langle kx \rangle \sum ax^n$$

$$\text{with the which: } (k - m_1)x = \sum_a^j ax^n \quad y \quad m_1x = L$$

We differentiate the values of the coefficients in two form: random coefficient and determinant coefficients. **(the determinant coefficients are designated for the existence of Galois's equations;** his numerical value is major that of the random. Therefore to solve Galois's equations we will do the following: we take two of the coefficients of major value; example (e, d), factoring both values in such a way that, the common value of them is the value that corresponds to him to (x) it is to say.

$$(e \cap d) = (a^n; b; b \times a) = x$$

Now well, if the equation belong to the group of Galois's "(1.3)" we will have the numbers (m.x). And therefore we divide all this coefficients by (m), for his checking.

If the presented equations allows to deduce the value give (x) we apply the process and, we substitute (x) for his value, if we verify that is correct, we will have an equation falsified.

That better way of corroborating and debating the process, that with the following examples

### Example 1.

$$5.x^6 + 7.x^5 + 2.x^4 + 4.x^3 + x^2 + 373.x + 134 = 0$$

If we factor: 373 y 134

$$373 = (3.126 + 1) 2.3^3; \quad 134 = (3.22 + 1) 2;$$

**common number 2**  $\rightarrow (x = 2)$

$$5.2^6 + 7.2^5 + 2.2^4 + 4.2^3 + 2^2 + (-373.2) = -134$$

**Example 2.**

$$11.x^6 + 7.x^5 + 9.x^4 + 8.x^3 + 6.x^2 + 4218.x + 1935 = 0$$

If we factor: 4218 y 1935

$$4218 = (3.234 + 1) 2.3; 1935 = (3.71 + 3) 3^2;$$

**common number 3**  $\rightarrow (x = 3)$

$$11.3^6 + 7.3^5 + 9.3^4 + 8.3^3 + 6.3^2 + (-4218.3) = -1935$$

**Example 3.**

$$12.x^5 + 7.x^4 + 20.x^3 + 1032.x^2 + 280.x + 32 = 0$$

If we factor: 1032 y 280

$$1032 = (14.3 + 1) 3.2^3; 280 = (17.2 + 1) 2^3;$$

**common number is the 8, nevertheless the number 32 = 2<sup>3</sup>.4**

**Therefore the value of (x) is (x = 8) or (x = 4); this fact appears because the variable is a power (x = 2<sup>n</sup>); (it transforms the equation into a polynomial with exponents even number).**

$$12.4^5 + 7.4^4 + 20.4^3 + (-1032.4^2) = (-280.4) - 32$$

$$12.2^{10} + 7.2^8 + 20.2^6 + (-1032.2^4) = (-280.2^2) - 32$$

**Example 4.**

$$24.x^4 + 10.x^3 + 3.x^2 + 3341.x + 435 = 0$$

If we factor: 3341 y 435

$$3341 = 167.2^2.5 + 1; 435 = 5.87;$$

**common number 5**  $\rightarrow (x = 5)$

$$24.5^4 + 10.5^3 + 3.5^2 + (-3341.5) = -435$$

**Example 5.**

$$13.x^7 + 5.x^6 + 4.x^5 + 9.x^4 + 56376.x^3 + 1731.x^2 + 1403.x + 872 = 0$$

$$56376 = 2^3.3^5.29; 872 = (3.36 + 1) 2^3;$$

**common number 2<sup>3</sup>**  $\rightarrow (x = 8)$

$$13.8^7 + 5.8^6 + 4.8^5 + 9.8^4 + (-56376.8^3) = -1731.8^2 - 1403.8 - 872$$

**Example 6.**

$$48.x^4 + 20.x^3 + 6.x^2 + 6682.x + 870 = 0$$

If we factor: 6682 y 870

$$6682 = 167.2^3.5; 870 = 2.5.3.29; \text{common number } 2 \text{ y } 5$$

**This one will be an equation of group [2],[3],[11] because all coefficients are even numbers  $\frac{6682}{2} = 3341 \text{ y } \frac{48}{2} = 24$**

**Therefore (m = 2) y (x = 5) corresponds to the Group of the example 4**

$$48.5^4 + 20.5^3 + 6.5^2 + (-6682.5) = -870$$

### 3 Discussion

**Galois's inverse problem.[5],[9]**

Any finite group that converge on (0), admits multiply all his coefficients for a number (m), and to have a polynomial of the same degree that that of the finite group.

$$P(x) = 0$$

$$m[P(x)] = 0$$

$P(x) \subset m[P(x)]$ ; this is reflected in the **example 6.**

We observe also that the inverse problem, allows to give the solution of the polynomial, because.

$$P(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + ex^{n-4} + fx^{n-5} + gx^{n-6} + h = 0$$

With (n = 7) and the finite group is an elliptical curve such as  $ex^{n-4} + fx^{n-5} + gx^{n-6} + h$ ; with (e = 1) [ see 12], [4] if we substitute in the polynomial, the elliptical curve we have that.

$$P(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + y^2 = 0$$

Wihebery the polynomial is solved because we know value of the variable with equation Enfer[12]. The same we will have if in the polynomial, there are one quadratic function.

### 4 Conclusion

The value of the variable in Galois's polynomials the we know, with the factoring of two of the coefficients of major value of such form that.

$$e \cap d = (a^n; b; b \times a) = x$$

The developed process in this paper contradict Abel theorem because we resolve Galois's equation at apply laws of of arithmtic over the coefficients.

### REFERENCES

- [1] Michael Rosen, Abel's theoremon la lemniscate, *Amerias Mathematical Monthly*,ISSN 0002-9890. (6), 387-395 (1981).
- [2] John G. Thompson, Glois groups, *London Math. Soc.*, vol 2, 455-462 (1989).
- [3] I. Efracr and D. Haran, on Galois group ove Pythagorean and semi-real closed fields, *Israel J. Math* 85, n 1-3. 57-78 (1994)
- [4] Andrew Wiles, modular elliptical curve and fermat's last theorem, *Annals of Mathematic* 141 (3) 443-551 (1995)
- [5] Malle Gunder and B. Heinrich Matzat, inverse Galois theory, Berlin, Heidelberg, *Apringer- verlap* pp460. (1999).

- [6] H. Koch, Galois theory of p-extensions, *Springer Monographs in Mathematics*, 2001
- [7] J. P. Escofier, Galois theory, *Springer- verlag*, New York (2001)
- [8] J.P. Tignol, Galois's theory of Algebraic equations, *Word Scientific Publishing* Cp. Pte, pp348 (2001)
- [9] P. Deuflhard, Newton methods for nonlineas problems, affine invariance and adaptive algorithms, *Springer Series in Computational Mathematics* vol 35 (2004).
- [10] Bewersdorff. Jrg, Galois theory for Beginners, a historial perspective (1ª edicion) providence, *American Mathematical Society* pp180. 2006.
- [11] I. Efrat and J. Minac, on the descending central sequence of absolute Galois groups, *Amer. J. Math* 133. n 6, 1503-1532 (2011).
- [12] Enfer Diez, equation to solve the elliptical curves, *Uni. Journal of Computational Mathematic*, vol 1 June (2013).