

What Can Georgescu-Roegen Tell Us Today? On Reproduction, Production, and Sustainability in Peasant Economy Modelling

Ernst-August Nuppenau

Institute of Agricultural Policy and Market Research, Justus-Liebig-University Giessen, Germany

Copyright © 2015 by authors, all rights reserved. Authors agree that this article remains permanently open access under the terms of the Creative Commons Attribution License 4.0 International License

Abstract There is a challenging debate on better ways of linking food production to reproduction of biological resources in agriculture. Currently, food production is mostly considered to be a part of agri-business which uses to profit maximization. This can result in resource depletion and industrial farming working against sustainability. However, peasant economies goals are broader than profit maximization, including reproduction of nature. We use this notion for a revision of objectives. The hypothesis introduced here is as follows: reproduction and production (as goals) are to be connected by joint accounting. The issue is addressed through an approach of energy accounting and through the application of a shadow price analysis. This paper explores how one can apply programming techniques to derive behavioural equations based on energy spending and shadow prices. It is shown how conditional behavioural equations can be readjusted and linked in a system analysis. The adjustment is done in a quasi-market mode, i.e. offered shadow price equivalents are “artificially equated”, which provides a joint optimum. As a result, an optimal population for reproduction fitting the slogan of “living within limits” is achieved, i.e. natural reproduction corresponds to food production.

Keywords Production, Reproduction, Farming Systems, Objectives and Modelling Behaviour

JEL Codes: D51, N50, O13, Q32

1. Introduction

Usually in cases where neoclassical economics is the foundation of economics, almost every ordinary analysis of economic behaviour starts with the ad-hoc assumption of profit and utility maximization (Mankiw [25]). Likewise, “conventional” agricultural economists (Penson et al. [33]) have found that food production and consumption are related

to profit and utility maximization. Though there are maybe scopes to discuss the need to do so and to look at inconsistencies, misconceptions, and flaws, etc., especially with regard to a physical world (Mirowski [27]), few current researchers question this setting. Admittedly there have been long discussions on whether utility maximization vs. profit maximization is relevant (Chayanov [7]). These discussions have apparently shown that rural development aiming at “modernization” requests a separation of farm profit and household utility (de Janvry et al. [9]); for modelling see (Chang et al. [6]). Profit maximization “shall” be the general case and exceptions are accomplished by transaction costs or emerging empirical issues only. Our point is: standard approaches also ignore reproduction in an ecological sense and dynamic effects and issues on sustainability are missing. Modern farm economics largely assumes that natural resources (incl. manpower) can easily be reproduced (by nature). In contrast, in traditionally oriented farm economics, which was built on classical economics of reckoning resource constraints and needs for reproduction, the need for pursuing reproduction and production is, in fact, explicit. Concurrently, food is a source of energy and sustains labour (Breimyer [5]), which is of high interest for us.

We want to return to the origins of objective specification in a world of living within limits, yet at the same time use a modern analysis. The starting hypothesis is: academic wisdom has departed from a deeper analysis of reproduction, eventually for good reason. It looks as if peasant oriented behaviour is not good for growth, but rather capital inflow and embedding farming in a market economy counts (Mundlak [28]). Reproduction seems to disturb the logic of capital in farming and does not fit in farm business concepts. At least (Penson et al. [33]) it is not mentioned anymore in textbooks. Perhaps it is believed that all inputs can be bought on markets and that farming is an industry. The only remedy left is to sustain capital (ibid.). Over decades it has been ideally shown and suggested that peasant and farm-household oriented behaviour, i.e. a combination of consumption and production linked by family labour, is a

special case (Bollman and Bryden [33]) to be ignored. This includes reproduction of labour, etc. Following the logic, it may be right to postulate peasant behaviour as “romantic”, i.e. backward, old, etc., at least as an economic concept, and that it should therefore disappear.

This paper challenges that view looking at modernized peasant behaviour by focusing on reproduction and food provision. Such focus may include old language on peasants, for instance that peasants seek to minimize risk and drudgery (Sahnin [37]). But the aim is to carve a new modelling concept called: *peasantry with reproduction*. It is important to note that terms might be vague since modern farmers still consider themselves peasants and are looking for support or stress independence; yet the concept “*peasantry*” still exists. But one question remains: how can it be described in terms of reproduction, production, nature reliance, sustainability, etc.? Moreover, given importance of agriculture as an energy source for future societies and being in need of energy, it is necessary to broaden the concept of farming beyond production, i.e. aiming at sustainability which includes reproduction (Lopez-Ridaura [27]).

This paper is organized in five chapters. Firstly, the background and underlying concepts of reproduction as aim-driven are discussed. Secondly, a modelling framework of programming and deriving behavioural functions is introduced. Thirdly, we provide a theory on equilibrium and explain it as linking values. Basically production is modified along reproduction. Fourthly, the empirical grounding is discussed and finally possibilities for application are presented. It is a theoretical paper, but shows rigorous empirical methods which can be built on data.

2. Background and State of the Art

There seems to be a deficit in theories on linkages between economics and biological reproduction (Biesecker and Hofmeister [2]). Perhaps, it also seems that modern economics ignores alternative means to explain the needs for reproduction. This paper will model nature and labour interactions with a focus on palatable energy as an alternative understanding for reproduction (Georgescu-Roegen [15]). As it were, it follows critics (as early as Leach [23]): that modern food production (economics) has become energy inefficient and that recent thinking is that food pricing is no longer linked to labour productivity and reproduction, but rather to fossil energy prices (low energy prices seem fundamental for food abundance: Dorward [11]). As this has not been the case in the past, it has to be re-thought (from modern to post-modern). Firstly, referring to Chayanov [7] who emphasized natural returns from labour, we have to focus on peasantry’s objectives. Secondly, the issue of drudgery comes into perspective (Banaji [1]). Drudgery is a mode to describe labouring for food and survival as well as look at localized exchange systems. There was drudgery for reproduction at family and village level.

Thirdly, the work of Georgescu-Roegen [6] offers background knowledge on energy flows between reproduction and production including the subject of peasant economies. In that regard there is still a distinction between peasants and their objectives vs. farmers and their objectives. As Shanin [37] put it: “What is meant by peasant vs. farmer”? For us, the use of the word “peasant” is fairly pragmatic (thus without romantic connotations). We see drudgery and live closely together with nature as the core point. Further note: a “peasant world” was different from modern farming. Farming evolves or revolves (for “revolution” see Merchant [26]) with new technologies and external energy. In contrast, the word “farmer” describes a modern countryman who has access to external (fossil) energy, specializes in production, is strongly integrated in markets, and minimizes recycling. For peasants, vice versa, consumption of their own products is essential for sustaining family labour or exchanges at village level. The unit to be addressed is actually a village exchange system, since it is unrealistic to solely concentrate on the subsistence of single farms (for system analysis: Dalsgaard et al [8]). Nevertheless, the term subsistence comes closer to peasant than modern farming for profits (Banaji [1]), i.e. a situation of reproduction inclusion. Against this background, we seek to model a joint perspective on objectives and reproduction, embed it into energy flows, and seek to limit food production by resource constraints and working for nature (Bonaiuti [4]).

We use a generalized programming approach (Paris and Howitt [30]) for behaviour and the focus of programming will be on labour use decisions and labour must be “reproduced” by palatable energy, obtained from production. This is a typical peasant way of farming with nature (van der Ploeg [39]). Thus, more or less, a closed system is envisaged, which, in modern fashion, includes nature (ecosystem services based on diversified farming: Kremen et al. [21]) and population dynamics. For a further justification: at system level (Dalsgaard et al. [8]) it has to be stated that reproduction, per se, does not seem to be an objective for an individual, but group. At least, this view has been criticised (welfare economics: Just et al. [20]) and we need relevant objectives. We circumvent this problem as labour is always needed. Also, we add a surplus of nature at system level. The specific aim is not only to modify assumptions, but to still retain some logic elements such as equilibrium and price analysis for scarcity from welfare analysis.

3. Concept

Scarcity recognition, its measurement and valuation in reproduction are meant, in general, to help humans solve problems in sustaining their future and, specifically, to support farmers in accounting for reproduction. Especially since the reproductive capacity of nature cannot be completely substituted by humanly produced capital (strong sustainability: Georgescu-Roegen [15]) and since bought inputs (for instance fertilizers for soil fertility) are not full

replaced by chemicals, then how can we measure energy (entropy) scarcity in conjunction with pricing? A new scarcity measure is needed to translate natural or reproductive scarcity into a supplementary accounting (Patterson [31]) for which we should seek similar units as prices. Under scarcity conditions in ecosystems, especially for palatable energy, the reproductive capacity of nature has to be addressed in cases where humans are extracting. The reproductive capacity of nature as provision of assets refers to plants, seeds, animals, etc.; even including human labour retained as manual labour and population. Valuation usually works by linking needs (utility) and installing an exchange system (Dicks [10]). In resource valuation, an immediate stake is to express to what extent the analysis goes beyond simple product valuation. The analysis has to be extended to goals (*peasant* goals, i.e. peasant community goals such as survival). An assumption is, there exist mentality favourable conditions for reproduction (summarized in *peasant behaviour*: van der Ploeg [39]). The argument is that peasants gave priority to clan survival, inter alia. This is a request for extending objectives including working for nature and looking for long-run survival to communities, etc. (i.e. beyond methodological individualism). In general, equilibria of production/consumption in conjunction with reproduction/energy would request joint *quantities* to be equalized and *prices*. We use shadow price analysis.

Particularly an extended version of decision making using shadow prices will be discussed, including drudgery for survival and utility that will deliver (shadow) prices that can adjust. Again, for peasants as community it can be argued that *population growth* and *drudgery* are opposing goals (Ellis [13]). These have to be put into equilibrium. It will be shown that increased population has the price of drudgery. In fact, the paper goes beyond methodological individualism (see Ingham [19]). It ventures into reasons why behaviour of peasants is distinct.

Actually, talking about drudgery immediately has a connotation linked to efforts and energy loss. Efforts could be defined as energy (cost) spent. Spending energy has two components: *time* and *energy per unit of time* (this is like cost). Spending energy shall be a synonym for drudgery in food production serving reproduction of peasants. It can be presumed that energy per unit of time (price) is flexible, as will be modelled later. Both elements (i) time and (ii) time spent per unit must be in equilibrium with provision. So in this case, units of energy (costs) can be assumed as payment giving a demand price. But what is the nature side of supply?

For this paper's purpose, nature (or better peasants' perception of nature) becomes involved as caring for reproduction embedded in an ecosystem. Then, to envision equilibrium between nature and peasants, we believe that nature is a provider of services supporting population. In other words: in balance, palatable energy has to be relinquished for humans by nature. Energy relinquished has a price in form of a valuation equivalent in terms of demand and supply incl. nature. Marginal and average drudgery (in energy equivalents) shall be similar to *price* and average

costs. The average costs can be defined as marginal disutility and cost is spending per unit. Humans are part of a goal of nature, although humans (as a species) are only one element in a broader specification of species reproduction. For this "goal of population achievement" and its reproduction it appears to hold that "current energy spent" vs. long-run "investment" in reproduction should be in balance. Human reproduction is merely considered as part of a system interface to nature (reproduction). As energy gets limited in natural systems it implies that reproduction is endangered in case of an overuse of energy by the human compartment of the agro-eco-system. In contrast, human work for nature (drudgery) is putative and vital.

4. Variables for Formalization

To achieve a formalization of the concept, firstly, variables for the equilibrium have to be specified concerning their features and positions. We use: (i) population, divided in different occupations (families) in a peasant community (core equilibrium variable). The paper introduces reproduction (of a peasant community as size " $N=\sum n$ ") in vector mode " $[n]:= [n_1, \dots, n_n]$ " (i.e. members of a community are classified in different social units/occupations: families, clans, etc.). (ii) Energy " $e \cdot h$ " is spent (mathematically as product of hours multiplied by energy per hour) on labouring in crop production; here, in different activities as related to seeding, birth (offspring), proliferation, etc. At measurement level, energy per hour spent " e " is a *currency* unit for drudgery. Producing more food accelerates energy needs. Then (iii), at technology level, " s " seeds (a vector) are the core of survival. Seeds are embedded in activities of assistance by humans and embedded in a natural system of proliferation. Seeds and offspring, like crops or animals, are extracted as food. Three variables (classes) are categorizing seed needs: they include seeds as (1) input, (2) extraction and (3) seeds for next period. This sustains plants. Input seeds are further promoted by labouring to improve germination probabilities, etc.

Secondly, willpower for drudgery is a central driving force measured as energy spent per hour; and it adjusts. Drudgery has an upper and a lower limit. It is best to start calibration with upper levels of a closed system in which all energy in terms of food consumption is spent for production. Consumption of all energy produced for survival (i.e. without any energy surplus in the system) means, for instance, consumption of starch-based food, like potatoes, cassava, etc. Yet, in that case the underlying concept and measurement is a pure physical input-output-tableau. Of course, this is a situation of misery and it will not work in real world, though it is a reference. Peasants have found equilibria above misery, as will be shown below, i.e. producing tasty food. This can be modelled as artificial surplus like producer and consumer surplus, but physically measured as energy spent. As in neo-classical economics: supply, demand, equilibrium, profit and utility serve as dependent categories.

For that, derivatives to receive behavioural equations are presented. Behavioural equations are constructed as in regular economics, but here to meet the idea of how goods shaped. The goods shall express reproduction.

5. Modelling

5.1. Specification of Surplus and Behavioural Equation

For modelling, since there is no price yet in terms of revenue of reproduction, it is best to step behind, i.e. go outside a straight traditional “economic” formulation of objectives such as revenue minus cost. In contrast, a *semi-economic* approach is chosen in which only costs exist first, but where a target has to be met by minimal cost, secondly. It is assumed that a reference vector of “n” (population size/mix) is declared as a physical objective (target) and costs of generating this vector are then minimized. Costs are physical costs (energy spent). Energy spent is measured in working for “n”. Technology A to achieve “n” is known to peasants. It is used for: $n \geq A h_r$, where “ h_r ” is a vector of hours, as well as energy spent per hour in different occupations. The peasant community optimizes communally (i.e. “as if”). (Linear) programming can be used as a tool to conduct this analysis. The underlying concept is to recover behavioural equations (Paris and Howitt [30]) from limited data using maximum entropy (Heckelei and Wolff [18], Golan et al. [17]: there works with linear programming). From programming, one can recover flexible equations, as has been done for constrained supply of farms in production economics. Next, in programming, one operates with primal and dual solutions. Duals offer shadow prices (Nuppenau [29]). Shadow prices are obtained from mirrored optimization (costs/revenues) with an aim of recovering coefficients that describe behaviour. (Beyond programming, we aim at flexible descriptions of technologies enabling change detection.) A focus is put on optimization of goals, i.e. choice of drudgery. Now “ h_r ”, as energy spent, is energy lost and it makes sense to postulate that it is to be minimized. The energy or its equivalent is palatable biomass and energy is embodied in labour h_r . Further, minimization is indicated as $h_{r,w}$, for reproduction. A primal outline of the programming problem is:

$$\begin{aligned} \text{Min } & e' \cdot h_{r,w} & (1) \\ \text{s.t. } & n_w \geq A_r h_{r,w} \end{aligned}$$

where vectors are: $h_{r,w}$:= working time
 e := energy spent per working time unit
 n_w := population wish,
 A := technology matrix

The result is a vector $h_{r,w}^*$, which is the optimal choice of labouring for fixed reproduction.

Equivalently, the dual maximizes the weights for the constraint, giving the valuation of n_w .

$$\begin{aligned} \text{Max } & \lambda_{e,h}' \cdot n_w & (2) \\ \text{s.t. } & e \leq A_r' \lambda_{e,h} \end{aligned}$$

where: $\lambda_{e,h}$:= shadow price vector

In fact, this representation offers a calculation of the shadow prices to get “revenue” such as:

$$\lambda_{e,h} = A_r'^{-1} e \quad (3)$$

Equation (3) is subject to value detection. Nevertheless, equation (3) does not tell us anything about sizes of “e”; “e” is exogenous, yet to be put in equilibrium. This shall be modelled later endogenously in system equilibrium. For the moment we seek behaviour that must be flexible with a corresponding shadow price evaluation and that adjusts. Revenue minus cost (4) can be constructed jointly from programming and maximum entropy (ibid.). Reproduction becomes specified as surplus in which the shadow price $\lambda_{e,h}$ is a “valuation” of “n”, based on drudgery. Taking function (4) it can be shown how to get a surplus of peasants in terms of reproduction.

$$SP = \lambda_{e,h} n_w - e' \cdot h_{r,w} \quad (4)$$

The anchor for assessment is $\lambda_{e,h}$ as result of drudgery: “ $e' \cdot h_{r,w}$ ”. Since reproduction is a *want* humans have to pay for proliferation; they optimize drudgery. [Excursion for communication: drudgery valuation here is similar to non-market valuation and to observed values in fixed input markets. For example and comparison in input-output market valuation: a feed price determines output (meat) prices. Then on competitive markets, feed prices translate into meat prices (livestock); prices are observable as mark-up. End]. The question remains, why does drudgery assessment not render the valuation of population as an indicator for reproduction? Perhaps it works if we regard population as a “good” (a tradable output as an award of drudgery).

In a next step we construct the counterpart: nature “supply” (see below). However, before *supply of reproduction*, as a request, is modelled, it is necessary to show and work further on outlining flexible behaviour descriptions. Note that “n” is a vector of achieved n_0 . By duality $\lambda_{e,h}$ and by statistical evidence it is feasible to expand the problem of drudgery/reproduction in a quadratic, flexible version. This version is based on optimal h^* and $\lambda_{e,h}^*$, which are results of programming (on technique see Paris and Howitt [30]) and in the Appendix as reference to observations). Also equation (5) bears parallels to an indirect “profit” function”, expressed as the consequence of optimization. Moreover, as in Shepard’s lemma, *supply* is price reliant.

$$\begin{aligned} SP = & \lambda_{e,h}' n - e' \cdot h_{r,w} = .5 \lambda_{e,h}' Q_{11} \lambda_{e,h} + .5 e' Q_{12} e + \\ & + \lambda_{e,h}' Q_{13} e + e' Q_{14} x_1 + \lambda_{e,h}' Q_{15} x_1 \end{aligned} \quad (5)$$

For a generalized approach in (5), the initial programming ((1) and (2) of optimization towards the shadow price) is replicated and Shepard’s lemma gives a *demand* for population. This *demand*, though it cannot be equated yet, is a demand resulting in behavioural terms (5a):

$$n_d = Q_{11}\lambda_{e,h} + Q_{13} e + Q_{15} x_1 \quad (5a)$$

At the same time (5) is optimized to drudgery; i.e. (as shadow prices $\lambda_{e,h}$ prevail) we get:

$$h_{r,w} = Q_{12} e + Q_{13} \lambda_{e,h} + Q_{14} x_1 \quad (5b)$$

In fact the *prices* “e” and “ $\lambda_{e,h}$ ” should become linked (see below) and a behavioural equation appears that is given by a uniformed price, i.e. if $e = A' \lambda_{e,h}$. The aspired population is then:

$$n_w = Q_{13}^{**} \lambda_{e,h} + Q_{14}^{**} x_1 \quad (6)$$

(6) is a flexible wish (*demand*) for reproduction. It is a first behavioural equation for a joint equilibrium. It spells out flexibility in behaviour of a peasant community with respect to the *wish for a certain population* (as vector) of having families for reproduction. This vector “n” will be part of the behavioural concept that is later linked to consumption and production. To supplement the behavioural concept (provided that the pull was delivered) the push is needed.

5.2. The Objective of Nature and Tolerance in the Provision of Human Reproduction

Now for the push aspect of reproduction, i.e. nature’s will or provision of means for reproduction. It is necessary to establish an adequate calculus for surplus. As usual in modernistic welfare economics, a surplus recognition should function as orientation along a goal. A valuation (shadow price analysis) has to be put forward to a new unit (representing nature incl. humans), which has a goal and recognizes energy balances as constraint. The idea is that peasants can infer behaviour. In other words, nature’s behaviour and composition depend on a clever goal orientation of peasants, inferring nature’s behaviour from *supply* construction.

At this point some further remarks on the above outline are needed to understand the context of behaviour. Firstly, nature’s behaviour is considered relevant for a provision of reproduction. Secondly, as this is an economic approach, nature must be considered as response unit in economic terms (Furubotn and Richter [14]: though it is superficial. For a reference: note that the researcher’s wish is to model a “natural” balance oriented depiction of “behaviour”). This is necessary due to the fact that we need an adjustment of nature’s *supply* based on *incentives*.

In our model nature is considered a constrained optimizing unit in energy transformation. The model corresponds to Eichner and Tschirhart [12]. Reproduction is goal oriented. Note: it is not said that nature behaves factually like an economic unit being goal oriented. Rather nature is “represented as if” (i.e. it may have subjective parts; we see reasoning for it as “*tertium datur*”: Latour [22] in epistemology. In simple terms: *supply* shall be envisioned as if nature aims for survival (Riess [35]). A further suggestion is that humans can *assist* nature in survival offering seeds, working for nature, etc. Then the question is what priority setting for species can be found in equilibrium. For those

who believe this is too complicated to understand, please think from another angle: In behavioural learning, one might argue at least logically that nature behaviour has to become represented in peasants’ awareness. Following this logic our approach is comparable to awareness of a principal who is concerned about behaviour of an agent working for him (Forubotn and Richter [14], i.e. assuming peasants have experienced a nature for reproduction drive). Behaviour must match and tolerate humans’ help for one’s own survival. Looking for active survival processes, we focus on reproduction as *supply* in economic terms, similar to service provision. This embodies a modelling along an extended perspective.

For further reading, we refer to Latour [22] assuming that nature can *offer* reproduction in a pre-science style of existence. From a perspective of exchange for reproduction, it is proper to think that peasants should offer labour for a *better* nature, i.e. for one’s own good. Humans can offer labour in form of seeding and other modes. For instance, seeding can be generalized as an activity promoting joint reproduction. For example, there is a meeting topic and a synergy amid seed offering and farming exists, incl. grassland expansion, establishing hedges, etc.

Coming back to calculus, a first necessary step is to establish a “corresponding” communality at peasant community level with shadow prices of labour as a driving force. For nature, a seed vector is taken. Multiplied with shadow prices for scarcity of seeds it shows benefits. After completing this exposition for logically derived peasant behaviour, one can think of revenue minus cost as specification. The revenue is perceived as a “gross surplus” that is the presumed objective of a supplying nature. Next, the natural system will be explored in a mode of cross-checking balances and work on this implicit objective. Using this economic-like approach and making it a nature-related approach, the inclusion of population size has to be understood as a key to linking. At the moment, numbers of humans in energy classes and drudgery, given valuation by “needs”, found a partial segment in nature, indicating the push aspect of nature for reproduction. I.e. in the logic of *supply cost* we seek benefits to “simulate” nature’s response.

$$E = \lambda_{e,h} n_w = \lambda_{e,h} \cdot A' h_{r,o} \quad (7)$$

Equation (7) is partially a benefit equivalent (revenue) of nature, representing the segment of humans as a species; and it is assumed that humans are part of nature which can be separated from the rest. In economic terms, humans are willing to pay for the provision of population sizes to nature at $\lambda_{e,h}$. Equation (7) can be understood as cost as well as benefit depending on “ownership” of reproduction. Equation (7) is calibrated in *quantity* multiplied by *price*. Then, by establishing (8) it is presumed that peasant communities follow nature that provides good natural conditions for reproduction. Notify, the reproduction is founded on optimization and is justified by the reason for getting a behavioural concept. To simulate optimization, a

programmed response (below) along intended behavioural science is obtained. Of course, such a programming is not found in reality. Rather, it serves as a proxy for depicting behavioural adaptability to the system. An essential is to establish payments. Returning to the construction of nature as a response unit for providing population, i.e. as species that underlie a natural process, a surplus shall prevail based on payment. It works with labour *payments* as energy equivalents deducted from food production and requested to sustain nature (here expressed as species vector). Payment is in labour units with similar categories of drudgery as before. Payments are mirrored by intakes as function, i.e. for nature wishes. This is not a real profit, rather a hypothetical one, but it still helps peasants to balance population wishes and fits in nature's "wishes". The benefit (7) can be exemplified as being similar to revenue. Putting the objective function aspect in explicit 'revenue minus cost', where revenue can be defined as species count and cost as humans, we can reckon an equivalent surplus for nature as:

$$SN = R - E \tag{7'}$$

In such a frame, expressed with already given $\lambda_{e,n}$ (for a first round of programming, but now qualified by $\lambda_{e,n}^* = \lambda_{e,h} \cdot A'$), further qualification of drudgery as "price" enables an "objective":

$$SN = \lambda_{e,s} \cdot s_{n,2} - \lambda_{e,n}^* \cdot h_{r,o} \tag{8}$$

In equation (8) as a sequenced type of solving, the problem for valuation is implicit: i.e. $\lambda_{e,s}$ is the new shadow price vector for a vector of seeds that have to be got in nature for reproduction. For qualification: a target supply of s_n^* (seed, baby offspring, etc.) is realized. Payments including a price for s_n^* take a similar position as revenue. For the empirical part, a constraint has to be formulated as condition (9b). At the moment we assume that constraint (9b) is known (or its origin given). I.e. to be built statistically s_n^* is given exogenously. As shown in primal (2), (9) now serves to find behaviour as reconstruction, if one uses programming:

$$\text{Min } \lambda_{e,h}^* \cdot h_{r,o} \tag{9a}$$

$$s_n^* \geq C_1^{**} h_r + C_2^{**} n_s + C_3^{**} x^* \tag{9b}$$

Vice versa, from primal h_r , offered to stir acceptance of nature, one can state the dual as

$$\text{Max } \cdot s_n^* \lambda_{e,s} \tag{10}$$

$$\lambda_{e,n}^* \leq C_1^{**} \lambda_{e,s}$$

Repeating the construction of a quadratic objective function as suggested above in general, and aiming at finding shadow prices specifically (having a data of s_n , $h_{r,o}$ and $\lambda_{e,s}$: using Heckelei and Wolff, 2003) one can finally come to a delineation of a surplus function expressed as quadratic function of shadow prices (given scarcity $\lambda_{e,n}$ and $\lambda_{e,s}$) and it is constrained in n_s :

$$SN = \lambda_{e,s} \cdot s_{n,2} - \lambda_{e,n} \cdot n_s - \lambda_{e,h}^* \cdot h_{r,o} = F(s_n, \lambda_{e,n}, \lambda_{e,s}, x_n) = .5 s_n' Q_{21} s_n + .5 \lambda_{e,s}' Q_{22} \lambda_{e,s} + .5 \lambda_{e,n}' Q_{23} \lambda_{e,n} + \lambda_{e,n}' Q_{24} s_n +$$

$$+ \lambda_{e,s}' Q_{25} s_n + .5 n_s' Q_{26} n_s + .5 \lambda_{e,n}' Q_{27} n_s + .5 \lambda_{e,s}' Q_{28} n_s + n_s' Q_{29} s_n + n_s' Q_{2,10} x_n + \lambda_{e,n}' Q_{16} x_n + \lambda_{e,s}' Q_{17} x \tag{11}$$

From the optimization of (11) one can derive a behavioural equation (12) as reproduction *supply*. Also, human labour needed for nature is derived as the first derivative and given as:

$$h_{r,o} = Q_{21}^* \lambda_{e,s} + Q_{22}^* s_n + Q_{23}^* \lambda_{e,n} + Q_{28}^* n_s + Q_{23}^* \lambda_{e,h}^* + Q_{15}^* x_n^* \tag{12}$$

The distinction between shadow prices is that $\lambda_{e,n}$ and $\lambda_{e,s}$ are both endogenous to reproduction. $\lambda_{e,h}^*$ is exogenous to reproduction but endogenous to the system adjustment, i.e. with production (as will be shown later). The parallel of reproduction supply (push equation 12) to a previous function of human request for reproduction (pull, equation 5b) is manifested in the need to equate internally derived shadow prices $\lambda_{e,s}$ and $\lambda_{e,n}$. (Note that the *survival* aim is set by nature). In terms of labour needs equation (12) enables inference by peasants. Consequently, taking population as an aim, the solution for drudgery in equilibrium means to adjust offer and wish: $h_{r,w} = h_{r,o}$. The balance delivers a population size that is given at that equilibrium:

$$n^* = A h_r^*$$

Human reproduction, i.e. population size, now depends on labour devoted to a hopefully positive interaction with nature. It means an established recognition of nature by humans, built on reproduction, is feasible and can be described as balance. The balance depends on the information of nature's need for labour and production capacities. For making decisions in the human sphere one has to connect (in parallel) the reproduction with production as specified next.

6. Conditions Anticipated from Nature and Energy Balance

Before we enter into linking reproduction to production, the condition mentioned above for reproduction has to be further explained. We suggest: by a specific type of relationship (9b) reproduction shall be assured. This gives behaviour as a constraint specified for nature:

$$s_n^* \geq C_1^{**} h_r + C_2^{**} n_s + C_3^{**} x^* \tag{9b}$$

Here and now, for further appreciation of the background (from a peasant point of view), it is postulated: "peasants must work for a functioning human-nature-interaction by injecting additional seeds s_h beyond $s_{n,2}$ in the system". This position will enable reproduction beyond nature without humans. Recognizing natural seeds, $s_{n,2}$ especially, means that humans care for eco-system reproduction in case of peasants. A simple version is offered in (13). Peasants expect biomass from seeds, since nature works on biomass as an indicator. The outcome is organic matter bent, "b", split between humans and nature. Organic matter balances are taken as:

$$b_h + b_n \leq B_1 [s_{n,1} + s_h] + B_2 s_{n,2} + B_3 x_2 \quad (13)$$

where: s_h := seeds by humans

s_n := seeds by nature

b_h := organic matter by humans

b_n := organic matter by nature

We distinguish between humanly appropriated biomass, such as seeds (like grains) and natural seeds. For further explanation, we look at two levels: first, the technical level and second, the content-level. Technically, biomass for human usage can come from nature that works by seeds to reproduce biomass. To keep it simple, only a static version is deployed due to the fact that dynamic issues are more complex. Biomass is a vector resulting from different species. Peasants may just look at their seeds s_h (biomass), but s_n also matters. Sustainability is not only crops in agriculture. Secondly, natural seeds play a role as they can serve as an interface. Hereafter, some parts of biomass are taken apart, b_h for humans as food, the rest is for nature:

$$[b_{h,t}] \leq A_{11} b_{h,t-1} + A_1 b_{n,t-1} + B_{11h} [s_{n,1}] + B_{12h} [s_h] + B_{2h} s_{n,2} + B_{3h} x_2 \quad (14)$$

Alternatively b_n is free or planned. The related issue is an extraction of biomass that not damages nature, but works with compensation, and may perhaps be explored in a further discussion on interaction. However, peasants as they are modelled here, are perceived as careful managers of farm species living in food webs (nature) and depend on interactions with the natural environment $[s_{n,1}]$. An immediate question is whether they see regeneration of palatable biomass by nature as indirect effect or action (labouring). Another question related to interferences is how impacts influence the functioning of nature or on the reproduction of biomass.

In our model (15) biomass b_n is used to support humans, i.e. it is extracted, but should be biomass for nature. Other biomass b_n is for nature and it corresponds to seeds as means of nature's reproduction. As dependent on seeds (15') peasants can realize how far to go with nature extraction. Actually this issue could be worked out with a dynamic presentation (beyond).

$$[b_t] - \phi [b_t - b_{t-1}] = B_{11} [s_{n,1,t}] + B_{12} [s_{h,t}] + B_2 s_{n,2,t} + B_3 x_{2,t} \quad (14)$$

Here, for reasons of operational simplicity, we nevertheless have to skip the change part and assume a steady state. For restoration it works differently.

$$[b_h] = B_{11} [s_{n,1}] + B_{12} [s_h] + B_2 s_{n,2} + B_3 x_2 - [b_n] \quad (15')$$

Further, to simplify, $[b_n]$ is a vector of humanly used biomass from species that are palatable. Also, $[b_h]$ is included in a food system. Food can be extracted from total biomass with the result that only some natural biomass is left. The vector corresponds to population of species. To summarize, regarding the knowledge on natural population, equation (15) is needed for reproduction. The need is considered a threshold in nature. On the other hand, from the perspective

of peasants, natural biomass is a concession. And, from a system perspective above, the condition is a balance compared to natural balances or reference situations, i.e. a situation of no-farming.

$$[b_h] - [b_h^r] = -B_{11} [[s_{n,1}] - [s_{n,1}^r]] + B_{12} [s_h] + B_3 [x_2 - x_2^r] \quad (15)$$

Equation (16) corresponds to a concept of *willingness to concede* a certain number of potential populations by humans as "requested" by nature to humans to provide service. This population for a certain biomass is settled by nature (as a vector for producing/consuming units).

$$C_2 [[s_{n,1}] - [s_{n,1}^r]] + C_3 [n_s - n_{norm}] = C_1 [b_h - b_h^r] \quad (16)$$

Again, in equation (16) it is postulated that only some species' biomass is used by humans and some goes into seed. (Skipping norm, (16) converts to a measure of natural population.)

$$C_2 [[s_{n,1}] - [s_{n,1}^r]] + C_3 [n_s] = C_1 [b_h - b_h^r] \quad (16')$$

Also for summation one gets population size N as the sum of nutrients supporting humans:

$$v'[n_s] = N \quad (16'')$$

Peasants' survival is based on seeds from nature and biomass extraction in balance. It means a nature seed (species) vector is recognized and nature *accepts* a population of humans. Further, biomass can be extracted for the purpose of feeding humans. Still, the system is likewise limited by biomass that is not taken by humans. Moreover, human biomass depends on seeding for crops s_h , that is labour dependent. The consequence is that seed injection, given in (17), is inserted in (15). In other words, seeds produced by humans require energy expenses.

$$s_h = D_1 h_{hr} \quad (17)$$

where: h_r := realized labour for reproduction

Inserting conditions (17) and (16) into (15) gives an outline (balance) of what can be obtained, i.e. from *patient* nature in case of a simple version of biomass substitution possibilities:

$$C_2 [s_{n,1}] + C_3 [n_s] \leq B_{11} [s_{n,1}] + B_{12} D_1 h_{hr} - B_{11} [s_{n,1}^r] + C_2 [s_{n,1}^r] + B_3 dx_2 \quad (18)$$

where: n_s := population supplied by nature as nutrients sum up

Equation (18) is the basis for equation (9b). And rearranged (18) is a constraint obtained as:

$$s_{n,2} \geq C_1^{**} h_r + C_2^{**} n_s + C_3^{**} x^* \quad (18')$$

Equation (18) outlines the necessary frame where humans (peasants) can reach reproduction from nature by *voluntarily supporting humans*. If condition (18') is met it helps to balance off nature with the human side of reproduction. To illustrate that, equation (19) must be translated into constrained optimization. Gaining behavioural equations in programming, peasants face:

$$n_s \leq C_1^* h_r + C_2^* b_n + C_3^* s_{n,2} + C_4^* x_2 \quad (19)$$

Equation (19) is a condensed version of equation (15), i.e. by calculated joint matrices. Then, if inverted, a greater equal condition appears for programming that still contains n and h.

$$s_{n,2} \geq C_1^{**} h_r + C_2^{**} n_s + C_3^{**} x^* \quad (19')$$

To further reduce the number of variables, peasants may additionally check the balance of species existence and link it straight to labouring, similar to the above notion. Labour can and may be delivered in interaction with nature and reproduction. Chiefly, things should improve. Equation (19) spells out delivery of actions for reproduction. It is energy balanced provision.

$$n_s = B [h_r + h_p] \quad (19'')$$

This gives:

$$s_n \geq C_1^{***} h_r + C_2^{***} h_h + C_3^{***} x_2^* \quad (20)$$

Finally, condition (20) supports seed population s_n and shows labour requests for reproduction. For a further understanding of the equilibrium: equation (20) is based on a supposition that nature tolerates humans, though humans extract palatable biomass. Compensation is achieved via seed provision at human labour expenses, drudgery. Conversely, other means could be used. The reason for or meaning of taking seed proliferation as a tool is to compromise that it *satisfies* nature beyond cropped seeds. Also, it may include planting of wild species, etc.

7. Production and Consumption

7.1. Reference to Market and Welfare Economics and Integration

So far, the paper has dealt with a reproduction system that is built on labouring and includes nature in terms of a partial equilibrium analysis, here as supplying units of human population reproduction. Reproduction must now be linked to food consumption and the production of energy (labour to get support and consumption). The aim is to achieve a joint equilibrium built on energy. Also, since no one can really (re)-invent a completely new economy theory, standard approaches should be modified in a way that they suit the approach outlined so far. We use the concept of welfare economics (Just et al. [20]) and modify it. We modify that and start with objective function. Welfare functions and optimal pricing are usually dependent on utility as an objective. As is revealed in preference theory, consumer and producer surplus are areas below demand and above supply functions (Ibid). This formal method shall deliver an equivalent description of production/consumption based on objectives. A crude way to integrate reproduction in production is to take into account resource constraints, inter alia on labour. In the above outline, reproduction was based on energy spent on reproduction on basis of drudgery multiplied by hours spent. A similar outline is possible for

production: minimizing drudgery. Then, reproduction and production compete on energy produced and spent as labouring for both. And food is consumed or sold. Sales are important since we do not model subsistence, only. Finally, consumption preferences and energy requirements must match.

Actually, on preference, one could start with utility maximization (de Janvry, et al. [9]). But it is important to embed consumption in energy equivalents of food from production, not only as preference. The issue matters in programming i.e. needed food to survive. This raises questions of what “utility” is and how it shapes production. We substitute utility by a taste index and treat consuming food as taste and energy needs. For a taste index, the peasants will minimize energy expenses or look at exchanging commodities on the basis of market prices.

7.2. Adjusted Production Economics

Taking a semi-peasant style of production, reproduction, consumption and market exchange of foods, i.e. a partly commercial perspective, a version of decoupling production and consumption decisions is appropriate (de Janvry et al. [9]). Yet, it is primarily sales as revenues (cash or income for labour and then food: Chayanov [7]) that counts and peasants can buy food at markets, though there might be a gap between sale and purchase prices. An integration of reproduction in this regard is done in two steps. First, a peasant has no other (labour) costs than relaxing production to reproduction seeking minimal loses. Second, a multipart world of transaction costs and market involvement may complete this. For now, objective (21) is formulated in a way that revenue maximizing and energy loss are paired by a Lagrange style:

$$PP = p_p [q_{p,p} - q_{c,p}] - \lambda_{e,p} [e' \cdot h_p + e' \cdot h_r - n_{e,c}' \cdot c_c] \quad (21)$$

where $q_{p,p} :=$ production

$q_{p,c} :=$ own consumption

$\lambda_{e,p} :=$ shadow price

To make it clearer, production is outlined by optimization which is driven by existing prices:

$$PP = p_p \cdot q_{p,n} - \lambda_{e,p} [e' \cdot h_p - c_{nc}] \quad (21')$$

where $c_{n,c} :=$ fixed given energy requirement for reproduction, of farm, etc., by population

$q_{p,n} :=$ fixed given energy surplus by population

Distinctly, the envisaged peasant community seeks to maximize revenues from surplus over consumption. The own consumption will be clarified in the next sections as addition to purchases. A technology prevails that is labour-oriented and considers humans and natural seeds:

$$q_{p,n} \geq T_{p,1} h_p + T_{p,2} s_n + T_{p,3} s_n$$

and $q_{p,n} - T_{p,2} s_n - T_{p,3} s_n \geq T_{p,1} h_p \quad (22a)$

Production is also constrained by factors as land, building, etc.; in the programming we incl.:

$$x_p \geq T_{p,4} h_p \quad (22b)$$

Programming translates *revenue vs. effort* (21b') in a quadratic revenue function (23). In fact, (23) is an indirect revenue function. It contains a quadratic result that can be retrieved from statistical methods such as *Maximum Entropy* if applied. In principle producer surplus is:

$$\begin{aligned} RP = F(p_c, n, c_c - e' \cdot h_r, s_n, x_p) = & .5 p_p' Q_{32} p_p + \\ & +.5[n c_c - e' \cdot h_r]' Q_{33} [n c_c - e' \cdot h_r] + p_p' Q_{34} [n c_c - e' \cdot h_r] + \\ & + p_p' Q_{35} x_p + [n c_c - e' \cdot h_r]' Q_{35} x_p + p_p' Q_{36} s_n + \\ & + [n c_c - e' \cdot h_r]' Q_{37} s_n + s_n' Q_{38} x_p \end{aligned} \quad (23)$$

Note natural conditions of seed proliferation enter production conditions as *ecosystem services ES*. Revenue or surplus are used as synonyms and provide supply if derivatives are taken:

$$[q_{p,p} - q_{c,p}] = Q_{32} p_c + Q_{34} [n c_c - e' \cdot h_r] + Q_{36} s_n + Q_{34} x_p \quad (24a)$$

Importantly, shadow prices for the constraint include energy and this is explicitly given as:

$$\lambda_{e,p} = Q_{33} [n c_c - e' \cdot h_r] + Q_{34} p_c' + Q_{37} s_n + Q_{35} x_p \quad (24b)$$

Peasant consumption creates residual supply: $[q_{p,p} - q_{c,p}]$. Vice versa, this is sold for revenue for which one can buy food. Own consumption can be equated and we work with demand from outside (and it is assumed that a fixed price for consumption of non-peasants exists). Additionally, at production level, function (24a) can be considered as *demand* for labour and energy in production, yet based on food consumption (as will be shown below in the conclusion).

It is important to note that at this level production delivers energy but also requires energy embodied in labour. Actually, we model a partially closed economy. For that reason, concisely, one must take energy balances and look at net energy. Talking about net effects brings up a debate on what "production minus consumption" means. In the given framework of peasants, real consumption $q_{c,p}$ can be defined as a net of energy in consumption over energy requirements to reproduce manual labour. Net is the taste and not reproduction aspect. Energy in labour is only part of it. In fact, since the focus is not on absolute needs for reproducing labour, it is rather taste that matters. An energy surplus (waste) in consumption can be realized. This surplus above pure reproduction of a workforce may for instance include energy losses, vitamin checks, need for minerals, etc. The energy losses could be caused due to more tasty foods, i.e. beyond energy reproduction. Thus, one must distinguish *true consumption* from reproductive consumption and *production services of eating*. For any food item a balance has to be established between *spent* and *obtained* energy (Pimental and Pimental [34]) and exogenous energy is included in x_p . This means that peasants, consuming food, use *taste* for decision and do not only work to get energy. Fairly, *interest* is in net-energy, i.e. *non-balance*.

7.3. Peasant Consumption of Own Crops and Purchased Food

For food consumption, a difference has to be made between foods of origin, i.e. from the community, vs. purchased food. Own food reduces cash needs and then, in an internal equilibrium, consumption surplus can be derived based on minimal expenditures. Surplus is *utility* minus expenditure (utility is given as taste index: Just et al. 2008). And a distinction is to be made between p_c (bought) and p_p (sold). We firstly specify expenditures and surplus by:

$$CP = p_c \cdot q_{c,c} - \lambda_{c,p} [y - p_c q_{c,c} + p_p [q_{c,p} - q_{s,p}]] \quad (25)$$

where p_c and $p_p := c$ consumer (purchase) price; p producer (sales) price;

This statement for the consumption side of peasants contains two elements, minimization of expenditures and maintenance of cash flow as a constraint. The minimization of expenditures describes purchases of food from markets and the maintenance of a cash flow (as a constraint) is dual to food needs. Cash, i.e. sales of food, increases the budget, but an external income can also augment cash. Here, the issue of subsistence vs. commercial farming is depicted.

For further explanation: consumption is represented by two variables, own food and purchased food for which a preference exists. How can we link them? Preferences are detected by an index. The construction of the index works along three aspects. First, the index (26) is introduced with a minimum food requirement that is defined by population size. Second, the index (26) shows a kind of scoring for satisfaction. Scoring can be accomplished by getting an arranging from interviews. (Note that each food item is linked to sustain labour by energy; but there is a possibility of deviation from own food, i.e. produced on a farm, due to taste). Thirdly, by choice of coefficients in a matrix I_p (in equation 26: stratifying needs) one can simulate preferences of bought over local food. Since index "i" can meet characteristics of choice sets, provided information on the strength of community preferences exists. "i" can be normalized, for example, as an assessment of being a peasant $i = 0$ or farmer 1. That means as from 0 to 1 "i" is exogenous and constructed by weights as preferences. Indeed we see a big difference in satisfying minimum needs vs. taste orientation between *peasants* and *farmers*. A preference against local food (porridge) may be off-set by instead buying food (bread). Yet buying food is costly and it has to be balanced with peasants' own needs. Index (26) enables a deviation from a "norm":

$$i_{c,c} \geq i' [I_c q_{c,c} - I_p [q_{c,p} - c_n' n_c]] \quad (26)$$

where I_c and $p :=$ matrices for ordering

Norm (26), food requisite, is energy defined. Equation (26) also implies that one can balance food of good and bad taste (qualitatively), thus reducing the degree of freedom. For instance, meeting exactly food needs by local produce can be 0 %. and buying the entire food is 100 % (i.e. purchasing external food has been given a preference increasing wellbeing if it is 100%).

Finally, equation (26) serves to minimize costs as a constraint. As a measure retrieved from a consumer preference it indicates behavioural directions to be met. Matching programming for the primal is a problem of simultaneous planning of $q_{c,c}$, $q_{c,p}$ and $\lambda_{c,p}$. As the primal is

$$\begin{aligned} \text{Min } & p_c \cdot q_{c,c} - \lambda_{c,p}[y - p_c q_{c,c} - p_p [q_{c,p} - q_{s,p}]] \\ \text{s.t. } & i_{c,c}^* \geq x_c [I_c q_{c,c} - I_p [q_{c,p} - c_n' n_e]] \end{aligned} \quad (27)$$

We get eating as choice. Received intake, own and bought food and information on shadow price can be supplemented with dual (28). For shadow prices of indices 3 constraint duals are:

$$\begin{aligned} \text{Max } & \lambda_{c,i,1} i_{c,c,1}^* + \lambda_{c,i} i_{c,c,2}^* + \lambda_{c,i} y \\ & p_c \cdot q_{c,c} \leq I_c' \lambda_{c,i} \\ & [q_{c,p} - c_n' n_e] \leq I_p' \lambda_{p,i} \\ & y \leq [p_c q_{c,c} + p_p [q_{c,p} - q_{s,p}]] \lambda_{c,p} \end{aligned} \quad (28)$$

Primal and dual programming offer numerical representation for choices and values such as:

$$\begin{aligned} \text{CS} = & \lambda_{c,i,1} i_{c,c,1}^* + \lambda_{c,i} i_{c,c,2}^* - p_c \cdot q_{c,c} + \\ & + \lambda_{c,p} [y - p_c q_{c,c} - p_p [q_{c,p} - q_{s,p}]] \lambda_{c,i} \end{aligned} \quad (29)$$

Equation (29) depicts both, choices and revealed preferences. In theory, a quadratic description (30) matches the surplus calculus of equation (25). Also, the minimization of expenditure by self-production provides an expenditure function (being an indirect choice); on index “i”:

$$\begin{aligned} \text{CS} = F(p_c, y + p_p q_{s,p}, x_c) = \\ = .5 i_c' Q_{41} i_c + .5 p_c' Q_{42} p_c + .5 [y + p_p q_{s,p}]' Q_{43} [y + p_p q_{s,p}] + \\ + p_c' Q_{44} [y + p_p q_{s,p}] + i_c' Q_{45} p_c + i_c' Q_{46} [y + p_p q_{s,p}] + p_c' Q_{47} x_p + \\ + [y + p_p q_{s,p}]' Q_{48} x_c + i_c' Q_{49} x_p \end{aligned} \quad (30)$$

Behind this finding of peasants’ surplus (30) is a calculus without further constraints. Yet, after having included all constraints, also a quadratic function with nutrition constraints is feasible. It may serve to complete the approach. Then we can derive purchase (30a): own consumption (30b), consecutively, and determined by purchase price, income

and production:

$$q_{c,c} = Q_{42} p_c + Q_{44} [y + p_p q_{s,p}] + Q_{45} i_c' + Q_{47} x_c \quad (30a)$$

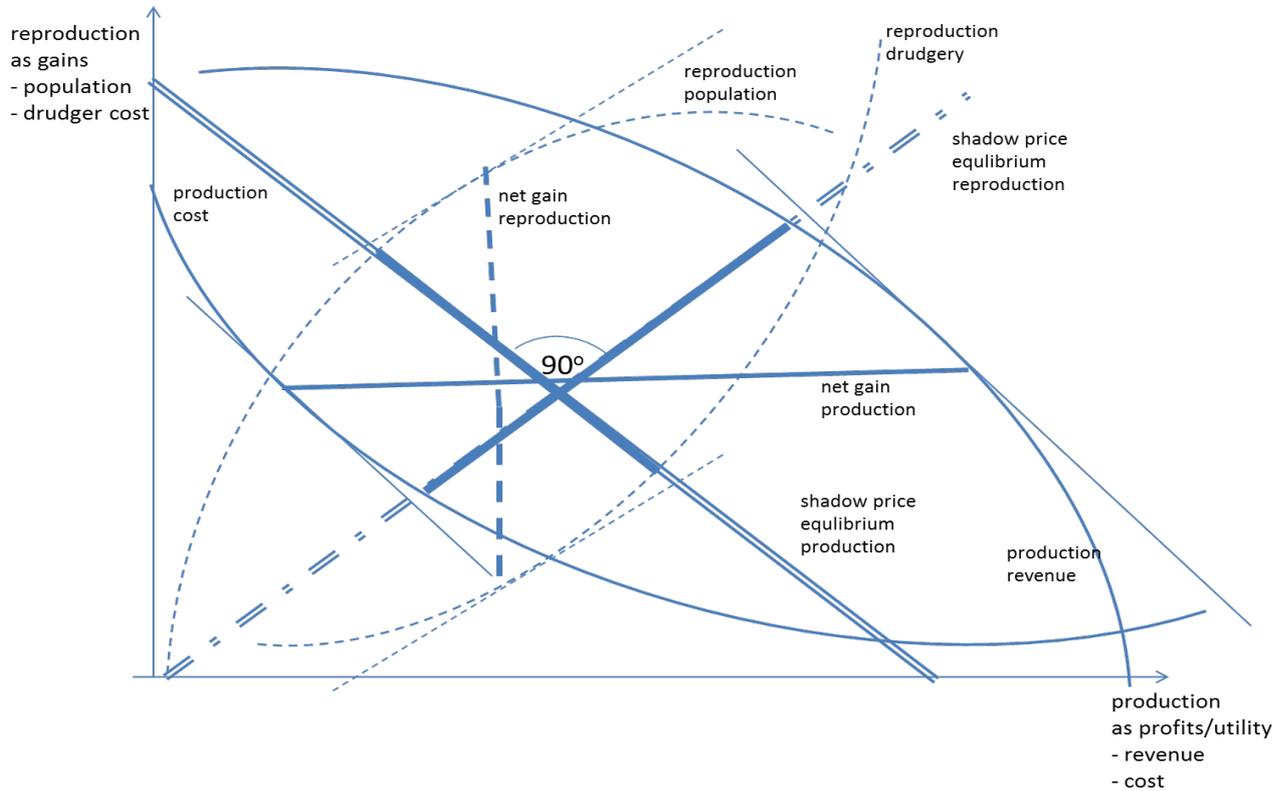
$$q_{c,p} = Q_{44} p_c + Q_{43} [y + p_p q_{s,p}] + Q_{46} i_c' + Q_{48} x_c \quad (30b)$$

Both equations are derivatives of equation (30). And shadow prices for constraints are specified as endogenous. We add retention of food, whereby sales and expenditures equate and surplus is generated by sales. Then the final intake is obtained. As a reminder, shadow prices for production and consumption are dependent on population as well as labour in reproduction and are therefore contingent. (Still, it looks similar to a usual solution). Conditions (30) give a case as if households may buy from production offering unified welfare, and that is deliberate.

8. Equilibrium between Reproduction and Production

8.1. Idea, Concept and Depiction

So far, partial equilibria have been modelled for reproduction, production and consumption. Now the question is: how can one perceive a joint equilibrium between reproduction and production which is based on system optimization and which contains a *price* vector which enables adjusting of the physical variables? We see shadow prices adjusting as the core and modelled shadow prices as related to energy for both, production and reproduction. It is important to rethink that the concept of equilibrium serves to optimize a system. As prices are a medium to optimize welfare on markets, a central planner is substituted by “self-regulation”. Do we have such a market with peasants? The answer on how this could be realized is: with shadow prices. A second question is how institutional setups can promote equilibrium? Since markets typically request a medium of exchange (usually money) a cultural setup in *economies* must be found. The pertinent question is: what is the currency between both aspects?



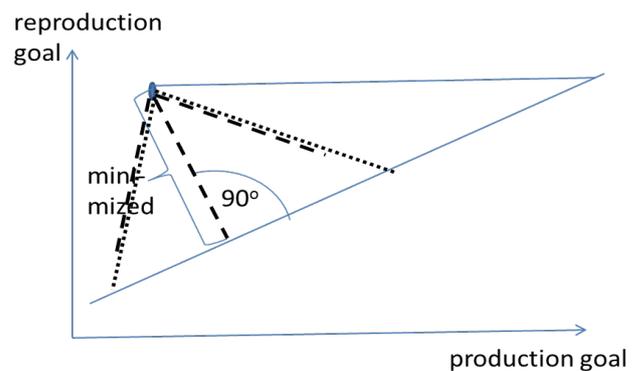
Source: author's own design

Diagram 1. Equilibrium between Reproduction and Production

For the first question, we might think, energy flows can be depicted in a production/ reproduction plane (Diagram 1). The argument is as follows: reproduction obtains energy from production as part of surplus and vice versa reproduction delivers labour for production as population. This argument is built from technical point of view, but enables the depiction of lines for marginal costs and benefits on both sides (see Diagram 1). It helps us to better understand the topic on joint optimization. The axes show production and reproduction as orthogonal and as physically referenced systems. Resources such as labour, i.e. humanly acquired energy, can either be used in production or reproduction. For instance, having more children requires additional food. This has to be accomplished by releasing food for reproduction. The overall welfare from production is on the x-axis and the measure for *welfare of reproduction* is given on the y-axis. Since this paper works with a concept of measuring utility in revealed preferences of drudger, a comparison of both objectives is feasible; both surplus measures are in energy units spent. The location of costs and utility in production depends on interactions with reproduction and length is jointly maximized. For peasant economies there is no independence of production from reproduction and survival, e.g. necessary biomass, soil fertility, energy, etc. Diagram (1) serves to draw joint slopes, shadow prices, and shows how they equate.

Then, in Diagram (2), we can argue that the optimal distance is obtained if a 90 degree angle is chosen. The

vertical axis represents energy expenditures measured as distance between reproduction (achieved minimum) and labour, while the distance between the slopes of production and consumption is utility. Here, the energy spent function offers a dual for minimal surplus for nature. The distances have to be minimized, which requires the production optimum,



Source: author's own design

Diagram 2. Minimization of the Secondary Objective Function

i.e. slope of the shadow price to deviate from the partial equilibrium. Diagram (2) solely sketches the argument (for a spatial mathematical proof, see Tu [38]). The two distances, again, become related, i.e. become mutually optimized if variations follow orthogonal conditions. In other words, prices are dual to each other. In fact, it occurs if inverted

values are:

$$\begin{aligned} \text{Optimal Price (for Production)} &= \\ &= 1 / \text{Optimal Shadow Price (for Reproduction)} \quad (31) \end{aligned}$$

For an intuitive proof of optimality: the distance of a deviation as shown in Diagram (2) by means of the dashed line is minimal, if it is orthogonal to the distance of the corresponding primary objective. Distances can be used to express joint costs, in this case mutually, and we add costs. The two slopes are to be put in optimal position (90°) and then added as joint optimization shifting positions of production and reproduction from a partial to a joint maximum.

Optimization implies that derivatives on corresponding utility functions and energy functions equate inversely. Positions on one side move to the other side and are mirrored. This implies that slopes become orthogonal; hence prices correspond by inverse slopes. In the geometry of triangles, the sum of angles must be 180 degrees, which is π . Then, if a slope is α , the counter angle is $\pi - \alpha$. For an approximate expression of $\lambda_{e,p}$ this is $\lambda_{e,p} = 1/\lambda_{e,r} \approx 1/\lambda_{e,r,0} [1 - \lambda_{e,r}]$.

Now, since we have got the idea of equilibrium, by adjusting shadow prices, an inclusion of prerequisites for reproduction in production is feasible. Note, shadow prices are incentives behind behaviour and at the same time they offer partial optimality. Vice versa, production sustains survival, i.e. offspring for future. In that setup, 2 shadow prices $\lambda_{e,r}$ and $\lambda_{e,p}$ emerge as trigger. And they are both valuations of thresholds in constraining behaviour, brought in as endogenous constraints in optimization. *This is a quasi-market solution.* Allocating labour towards reproduction is given in energy calculus for reproduction; the same with labouring for consumption. (Similar as to land allocation, market prices are means of an apportion system: i.e. who gets how much land.) An issue might be that scaling differs? But in production labour costs are in energy units. Since the two conceptual frameworks are based on the same units, they only differ in economic and natural science accounts. In system accounting they show similarities. Theoretical shadow prices match. Nevertheless, in practices rules concerning a mutual recognition of shadow prices must be found. The issue is which currency?

Again, theoretically one can work with duality and expand it. Duality is a concept already referred to above. In programming, there are primal solutions that are physical ones and duals offer to bind them to the alike accounting. Dual optimization offers a corresponding fixing of prices for constraints, i.e. one gets shadow prices as per unit values for the same optimization.

In this regard production and reproduction can be considered a goal oriented maximization of surplus in which the energy aspect is jointly minimized. I.e. reproduction as a maximization of human offspring (population goal) is a physical goal, given that costs (energy) and utility loss are minimized. Minimization vice versa has a duality in nature where it means minimization of impacts. Having shadow

prices, a shadow price is typically a marginal value of a constraint where marginal physical units can equate and shadow prices direct maximization.

As shown in the two diagrams, this applied to fix partially as well as jointly production and reproduction. For the purpose of equating one can use slopes as the interaction of shadow prices to optimize the whole. Any parallel representation is obtained in the space of energy and costs, hereby maximizing the sum of the two surpluses. Then, there remains the second question of practical implications. As a hint: ratio $\lambda_{e,p} = 1/\lambda_{e,r}$ already uses prices as energy spent per labour $\lambda_{e,p} := e_p/h_p$. The same applies to the shadow price of reproduction $\lambda_{e,r} := e_r/h$ (energy per hour). From the definitions (if the ratio $\lambda_{e,p} = 1/\lambda_{e,r}$ is inserted), we obtain the following results:

$$\begin{aligned} e_p / h_p &= - 1 / e_r / h_r \quad (31) \\ e_p / h_p &= - h_r / e_r \\ \ln e_p - \ln h_p &= (- 1) [\ln h_r - \ln e_r] \\ \text{and } h_p &= - h_r \text{ for fixed labour} \\ e_p &= - 1 / e_r \quad (32) \end{aligned}$$

Thence ratio (32) is a sort of exchange rate in energy. Exchange rates link “currencies”. For such study reproduction and production can be expressed in similar (energy) currencies. Labour per unit of work is provided by reproduction, while in production, labour and energy is given in food. Yet, this needs to be further elaborated, to show the underlying consequences.

8.2. Equilibrium, Selection of Equations, System-Closure and Variables for Solution

From the above outline of behavioural equations, i.e. in compartments of reproduction and production, we receive a set of equations and variables that are to be brought into equilibrium. At the same time, thoughts are to be provided as to which variables are endogenous and exogenous. At the core are shadow prices for drudgery and labour ($\lambda_{e,p}$ and $\lambda_{e,r}$) being endogenous and nearly linear. Yet, they are inverse related to each other; if approximated, we get:

$$\lambda_{e,p} = 1/\lambda_{e,r} = 1/\lambda_{e,r,0} [1 - \lambda_{e,r}] \quad (33)$$

Subsequently, the labour-energy constraint counts in the system (either as $h_r + h_p = C_r n_e$ or $1' h_r + 1' h_p = c 1' n_e$). This reflects a closure between population and spending energy for reproduction vs. production spending based on endogenous population. Furthermore, at the level of production and consumption, balances $q_{p,p} - q_{p,p}^s = q_{c,p}$ and $q_{c,p} + q_{c,p}^d = q_{c,c}$ must hold. This requires an external definition of non-community food demand that is a source of cash and herein depends on external prices which are determining exchange. For exogenous food demands, the demands from outside the system rely on market prices. Extra closures are on cash balance and accounting. Another problem is that factor costs are only represented by shadow prices. Finally,

the above list of equations has to be supplemented by this extension. Yet, we use accounting with shadow prices. In the accounting of production, revenues must be equal to costs plus income, but here no surplus profit exists. Instead, we use cash for non-food.

$$p_p q_p + y_i - y_o - p_p q_c = \lambda_{e,p} h_t \quad (34)$$

To supplement all accountings, production technology $q_p = A h_p$ can firstly be inserted and, secondly, a reference is made to the already introduced interaction of biomass and population. Thirdly, net returns and off-farm labour income add to an extra equation with an assumption that net off-farm income contributes to food purchases, which depends on labouring/drudgery. Then, as income can be supplemented by additional activities of off-farm labouring, we get: $y_i - y_o = \zeta_o h_o$; also total labour is $h_t = h_r + h_p + h_o$. Finally, total labour corresponds to consumption $h_t = \zeta_c [q_{p,p} + q_{c,c}]$, which offers a measurement for $y_i - y_o = \zeta_o [\zeta_c [q_{p,p} + q_{c,c}] - h_r - h_p]$. Then it is:

$$\begin{aligned} p_p q_{pp} + \zeta_o [\zeta_c [q_{p,p} + q_{c,c}] - h_r - h_p] - p_p q_c = \\ = \lambda_{e,p} [h_r + h_p - \zeta_c [q_{p,p} + q_{c,c}]] \end{aligned} \quad (35)$$

In a nutshell, a relationship $q_c = Z q_p$ corresponds to biomass b_h and b_h as q_p can be expressed in biomass. The same applies to q_c . The appropriate intermediary, being the biomass, gives:

$$q_c = Z_1 b \text{ and } q_p = Z_2 b \text{ whence } q_c = Z_2^{-1} Z_1 q_p$$

Equipped with this information (incl. markets charge transaction costs $p_p = p_p + c_c$) production and consumption are regulated by the peasant community along prices and transactions with the outside. Furthermore, we can refer to closures at cash and labour markets that represent the usual external economy matters. In that regard, a quantification of links between prices and reproduction is feasible by taking internal mark-ups. No distribution issue prevails so far. If income is neglected or statistically ignored, the needed net income prevails as follows:

$$\begin{aligned} p_p q_p - [p_c + c_c] Z_2^{-1} Z_1 q_p - q_p A^{-1} q_p \lambda_{e,p} - y_i - y_o \quad (36) \\ p_p = [1 - c_c] Z_2^{-1} Z_1^{-1} A^{-1} q_p \lambda_{e,p} - Z^* p_p \end{aligned}$$

At this stage, evidence has been reduced to flexible *shadow pricing* in production for λ_e . And we can commence to set up final conditions for equilibrium (37). The equilibrium is embedded in the system-closure mentioned above. Coefficients are derived from partial supply, demand analysis, etc. and cater for realities such as market integration of peasants. Ecological conditions are expressed by natural seed demand. Elsewise this can become endogenous. Yet, the number of variables and vectors has been reduced. 4 equations appear system-wise, which can work out a new tuning for a joint equilibrium (37). See, shadow prices λ_e are endogenous:

$$\begin{aligned} n_e &= q_{10} + Q_{11} \lambda_e + Q_{13} e + Q_{15} x_1 \\ n_e &= q_{20} + Q_{21} \lambda_e + Q_{22} s_n + Q_{23} h_p + Q_{15} x_n^* \quad (37) \\ q_{p,n} &= Q_{32} Z [\lambda_{e,0} / [1 - \lambda_e] - y_p] + \end{aligned}$$

$$+ Q_{34} [c_c n_e - e' A h_r] + Q_{36} s_n + Q_{34} x_p$$

$$q_{p,n} = [Q_{42} - Q_{44}] [\lambda_{e,0} / [1 - \lambda_e] + c - y_p] +$$

$$+ [Q_{44} - Q_{43}] [y + C^* Q_{21}] \lambda_{e,0} / [1 - \lambda_e] + [Q_{45} - Q_{46}] i_c + Q_{47} x_c$$

In this equilibrium, we see 5 variables to adjust; they are: n_e , h_p , $q_{p,n}$, h_r , λ_e . Two are further internally determined because $[h_p + h_r + h_o] = \Xi n_e$ and $q_{p,n}$ stand for $q_{p,n} = [q_{p,p} - q_{c,p}]$. The other variables are exogenous (x_p , x_n , x_1 , x_c , incl. market prices p , etc.). Equations like outside sale and demand are reduced form driven. The system can be solved by matrix inversion (under the notion of surplus). It serves as depicting a peasant society living within limits given by nature.

9. Discussion

Apparently, an important question is how such interactive and balanced optimization of reproduction and production can be transferred to operational resource using concepts, based on valuation and accounting for reproduction. Importantly, the consumption side is given as index i_c and this also results in a surplus of peasants. This means the concept is still preference oriented, but has an objective meaning. In this regard, the economic view that consumption is driven by preferences has been given a more objective grounding because we introduced a need for reproduction. The prescribed peasantry can live well beyond pure drudgery, wherein reproduction is the scope that promotes sustainability instead of profit maximization. A question is: what is the difference? Nature reproduction is included and the approach has an interface with nature. A problem, not yet be discussed, is income distribution and how to specify claims for food in a community. In neo-classical production theory, the marginal product of labour is an accepted distribution criterion which channels income (food) between different actors. Applying this institution to our modus of labour, individual food appropriation is perhaps justified, if food is given to families and Σn is total populace getting reproduction rights as a whole.

As far as other, eventually needed controlling mechanisms of communities are concerned, more work is needed to evaluate schemes of visual recognition of shadow prices. A cover of rights or credit as entitlement is also needed because it may give scope for individual improvements. Since shadow prices (multiplied by drudgery, i.e. labour in terms of energy spent) are comparable to income in a market economy, they are perhaps a mode to become material. How they can become a public variable is unclear. But, they can be retrieved and calculated, yes; but only rights make them tradable. A final question is: if there is a general surplus, who claims it? Surpluses can be controlled by rights and shadow prices, but how? A point for discussion is who sets rights, access to food, resources, etc. and enforces drudgery.

10. Summary

This paper started with a list of issues of sustainability of food provision as related to reproduction, production, consumption and peasant behaviour. Some historical references were made towards peasantry and behaviour in farming systems that were more sustainable than today, although they include drudgery (associated with caring for nature). The paper referred to a need of modelling interactions between nature, reproduction, production as well as consumption, using valuation schemes based on extended equilibriums of behavioural equations.

Then, particularly, a novel approach was presented in which shadow price analysis was used to link peasant behaviour of production to reproduction creating interfaces. As a way to recognize natural processes (sustainability) caring for seeds was introduced. Using a quadratic objective function as primal and dual programming to derive behaviour, indirect objective functions were stated. From programming and Maximum Entropy, i.e. marginal analyses of optimizing behaviour of sections, flexible functions were, as usual, achieved. Also, in Chapter 8.1 we showed how behavioural functions can be adjusted. Adjustment can be done virtually as in a quasi-market mode of adjusting prices/quantity. A chapter (8.2) on inverse pricing showed the linkage. This mode offered equated shadow prices and labour allocation for reproduction and production. Prices were free to be equivalent to dual characterization of optimality.

As a result, four equations (as sets in a system) offered an optimal population size for reproduction which fitted with carrying capacities of natural systems. Since the approach was still based on a type of welfare analysis, it was unveiled how consumption and production interact with reproduction for sustaining populations and surplus. This concept put reproduction and production in a perspective of survival and welfare. To a certain extent, the envisaged peasant community can be characterized as aiming for sustainable farming and ES systems maintenance.

Appendix

To generalize from a single observation, a maximum entropy approach can be used (Golan, et al. 1996). A first step is to generalize through a quadratic value function which is consecutively subject to the definition of matrices. As has been discussed (Howitt and Paris, 2000) and suggested for fulfilling certain criteria (Heckelei and Wolff, 2003) at a minimum, a quadratic approximation can be obtained:

$$V(r, e) = 0.5 r' Q_1 r + r' Q_2 e + 0.5 e' Q_3 e \quad (A1)$$

which in this case is the result of a linear programming of minimizing costs. For this representation the derivatives can be derived and determine the constraints for the maximum entropy

$$A \underline{e} = Q_1 \underline{r} + Q_2 \underline{e} \quad (A2a)$$

$$A' \underline{r} + \lambda + \underline{c} - \underline{p} = Q_2 \underline{r} + Q_3 \underline{e} \quad (A2b)$$

The maximum entropy itself is a function of probabilities that are maximized for stratified coefficient ranges. For instance, for the matrix Q use the representation matrices L and D

$$Q = L D L' \quad (A3)$$

where D is a diagonal matrix and L is a lower and L' is an upper triangular matrix (Cholezky factorisation). Basically, maximum entropy (Golan et al. 1996) works with a concept and a specification of probabilities "π" that are used to find the most likely distribution function for the coefficients of L and D which are supported by a matrix Z of an a priori given spectrum of a support matrix. The number of observations can be singular.

$$L = \sum_s Z_L(j,j',s) \Pi_L(j,j',s) \quad (A4)$$

$$D = \sum_s Z_D(j,j',s) \Pi_D(j,j',s) \quad (A5)$$

A specification of the entropy function

$$H = - \sum_{i,j,s} \Pi_{L,i}(j,j',s) \log(\Pi_{L,i}(j,j',s)) - \sum_{i,j,s} \Pi_{D,i}(j,j',s) \log(\Pi_{D,i}(j,j',s)) \quad (A6)$$

$$\begin{aligned} \text{s.t.} \quad A \underline{e} &= (Z_{L,1} \Pi_{L,1})' (Z_{D,1} \Pi_{D,1}) (Z_{L,1} \Pi_{L,1}) \underline{r} + \\ &+ (Z_{L,2} \Pi_{L,2})' (Z_{D,2} \Pi_{D,2}) (Z_{L,2} \Pi_{L,2}) \underline{e} \\ A' \underline{r} + \lambda + \underline{c} - \underline{p} &= (Z_{L,2} \Pi_{L,2})' (Z_{D,2} \Pi_{D,2}) (Z_{L,2} \Pi_{L,2}) \underline{r} + \\ &+ (Z_{L,2} \Pi_{L,3})' (Z_{D,3} \Pi_{D,3}) (Z_{L,3} \Pi_{L,3}) \underline{e} \end{aligned}$$

gives the needed information on the coefficients.

REFERENCES

- [1] Banaji, J. (1975). The Peasantry in the Feudal Mode of Production: Towards an Economic Model. *The Journal of Peasant Studies*.3(6), 299- 320.
- [2] Biesecker, A., Hofmeister, S. (2010). Focus: (Re)productivity: Sustainable relations both between society and nature and between the genders. *Ecological Economics*, 69 (8), 703-1711.
- [3] Bollman R.D., Bryden, J. M. (1997). *Rural employment An international Perspective*. Wallingford.
- [4] Bonaiuti, M. (2011). *From Bioeconomics to Degrowth*. Georgescu-Roegens "New Economics" in eight essays. Milton Park.
- [5] Breimyer, H.F. (1974). *Agricultural Economics in a Less Expansive Economy*. *American Journal of Agricultural Economics*. 812-815.
- [6] Chang, Y.-M., Huang, B.-W., Chen, Y.-U. (2012). Labor supply, income, and welfare of the farm household. *Labour Economics*, 19, 427-437.
- [7] Chayanov, A.V. (1966). *The Theory of Peasant Economy*. Illinois.
- [8] Dalsgaard, J.P.T., Lightfoot, C., Christensen, V. (1995). Towards quantification of ecological sustainability in farming systems analysis. *Ecological Engineering*, 4(3), 181-189.

- [9] De Janvry, A., Fafchamp, M., Sadoulet, E. (1991). Peasant household behaviour with missing markets: some paradoxes explained. *The Economic Journal* 101, 1400-1417.
- [10] Dicks, M.R. (1998). Linking Sector and Resource Allocation Models for Agricultural and Environmental Policy. pp. In Robertson, T., English, B.C. Alexander, R.R., *Evaluating Natural Resource Use in Agriculture*. Ames, pp.319-332.
- [11] Dorward, A. (2013). Agricultural labour productivity, food prices and sustainable development impacts and indicators. *Food Policy*, 39, 40-50.
- [12] Eichner, T., Tschirhart, J. (2007). Efficient ecosystem services and naturalness in an ecological/economic model. *Environmental and Resource Economics*, 37, 733-755.
- [13] Ellis, F. (1993). *Peasant Economics. Farm Households and Agrarian Development*. Wye Studies, Cambridge.
- [14] Furubotn, E. G., Richter, R. (2005), *Institutions and Economic theory. The Contribution of the New Institutional Economics*. Ann Arbor.
- [15] Georgescu-Roegen, N. (1960). *Economic theory and Agrarian Economics*. *Oxford Economic Papers*, 12(1), pp. 1-40.
- [16] Georgescu-Roegen, N. (1993). The Entropy Law and Economic Problem. In: Daly H., Townsend, K.N.(ed.). *Valuing the Earth. Economics, Ecology and Ethics*. Cambridge (Mass.) p.75-87.
- [17] Golan, A., Judge, G.g., Miller, D. (1996). *Maximum Entropy Econometrics: Robust Estimation of Limited Data*. Wiley.
- [18] Heckeleei T., Wolff H. (2003). "Estimation of Constrained Optimisation Models for Agricultural Supply Analysis Based on Generalised Maximum Entropy". *European Review of Agricultural Economics*, 30(1), 27-50.
- [19] Ingham, B. (1999). Human behaviour in development economics, *The European Journal of the History of Economic Thought*, 6(4), 606-623.
- [20] Just, R.E., Hueth, D.L., Schmitz, A. (2008). *Applied Welfare Economics and Public Policy*. Prentice-Hall.
- [21] Kremen, C. Iles, A., Bacon, C. (2012). Diversified Farming Systems: An Agroecological, Systems-based Alternative to Modern Industrial Agriculture. *Ecology and Society* 17(4): 44- 50.
- [22] Latour, B. (2013). *An Inquiry in the Modes of Existence*, Paris.
- [23] Leach, G. (1975). *Energy and Food Production*. *Food Policy*, 1((1), 62-73.
- [24] Lopez-Ridaura, S., Van Keulen, H., van Ittersum, M.K., Leffelaar, P.A., (2005). *Multiscale Methodological Framework to Derive Criteria and Indicators for Sustainability Evaluation of Peasant Natural Resource Management Systems*. *Environment, Development and Sustainability*, 7(1), 51-69.
- [25] Mankiw, N.G. (2014). *Principles of Microeconomic Theory*. 7th. Edition, Stanford.
- [26] Merchant, C. (2010). *Ecological Revolution, Nature, Gender and Science in New England*. Chapel Hill.
- [27] Mirowski, P. (1992). *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics (Historical Perspectives on Modern Economics)*. Cambridge.
- [28] Mundlak, Y., (2000). *Agriculture and Economic Growth. Theory and Measurement*. Cambridge.
- [29] Nuppenau, E.-A. (2014). *Integrated Modelling of Payment for Ecosystem Services: Using Willingness to Pay and Accept, for Nature Provision and Addressing Public Management in Cultural Landscape*. *Operational Research* 14, pp. 151-175.
- [30] Paris, Q. Howitt, R.E. (2000). The Multi-Output and Multi-Input Symmetric Positive Equilibrium Problem. In: Heckeleei, T., H.P. Witzke, W. Henrichsmeyer (eds.). *Agricultural Sector Modeling and Policy Information Systems*. Kiel, Vauk Verlag, *Proceedings 65th EAAE Seminar at Bonn University*, 88-100.
- [31] Patterson, M. (2002). Ecological production based pricing of biosphere processes. *SPECIAL ISSUE: The Dynamics and Value of Ecosystem Services: Integrating Economic and Ecological Perspectives*. *Ecological Economics* 41, pp. 457-478.
- [32] Pender, J., Place, F., Ehui, S., 1999. *Strategies for sustainable agricultural development in the East African Highlands*. Environment and Production Technology Division Working Paper #41. International Food Policy and Research Institute(IFPRI), Washington, DC, p. 86.
- [33] Penson, J.B., Capps, O., Rosson, C.P. (1995). *Introduction to Agricultural Economics*. London.
- [34] Pimental, D. and Pimental, M. (2008). *Food, Energy and Society*. Third Edition. Boca Ration.
- [35] Riess, J. (2009). *Not by Design: Retiring Darwin's Watchmaker*. Berkeley.
- [36] Ruben, R., van Ruijven, A. (2001). Technical coefficients for bio-economic farm household models: a meta-modelling approach with applications for Southern Mali. *Analysis*. *Ecological Economics*, 36, pp. 427-441.
- [37] Sahnin, T. (1987). *Peasant and Peasant Societies*. New York.
- [38] Tu, P.V.N. (1984). *Introductory Optimization Dynamics: optimal control with economics and management science applications*. Heidelberg.
- [39] van der Ploeg, J.D.(2013). *Peasants and the Art of Framing. A Chayanovian Manifest*. *Agrarian Change and Peasant Studies Series*. Black Point.