Computer Science and Information Technology 3(6): 227-232, 2015 http://www.hrpub.org
DOI: 10.13189/csit.2015.030601

A Grey Stochastic Multi-criteria Decision-making Method Based on Hausdorff Distance

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Abstract Aiming at stochastic multi-criteria decision problems brought by criterion value with extended grey number, a grey stochastic multi-criteria decision-making method based on Hausdorff distance is proposed. Firstly, definitions, calculation rules and distance formulas of expanded grey number random variables and expectations are given. Then based on grey decision matrix and natural state probability, expectation decision matrix of the extended grey number is obtained. Combined with the weight vector calculation of each criterion, positive and negative ideal solution distance for each program is then obtained. Ultimately, the relative nearness is determined and the programs are sorted depending on the size of value. Through case study, it verifies the feasibility and effectiveness of the proposed method.

Keywords Extended Grey Number, Grey Stochastic, Hausdorff Distance, Multi-criteria Decision-making, Interval Grey Number

1. Introduction

Multi-criteria decision-making, an important part of modern scientific decision making, focuses on solution of limited program decision making problem under the circumstances of multiple criteria. Its essence is to use the existing decision making information, to sort and prioritize a limited number of alternative options through a certain way. Its theory and methods have been widely applied to social life, engineering design, system engineering, management science and other fields. In real life, due to the complexity of the external environment, the ambiguity of the objective thing itself and limitations of human knowledge, there exist a lot of uncertainties in decision making process, so decision-making information in the actual decision making usually have blurred, random or grey uncertainties.

Grey stochastic multi-criteria decision-making is with two characteristics of grayness and randomness. Its related research proceeds slowly, with relatively little research result. At present, the study of such decision making problems has caused an active interest in the experts and scholars at home and abroad, such as: Nese Yalcin *et al.* [1] proposed a new financial performance evaluation approach to rank the companies of each sector in the Turkish manufacturing industry. For this purpose, a hierarchical financial performance evaluation model is structured based on the AFP and VFP main-criteria and their sub-criteria. Renato A. Krohling [2] proposed a hybrid approach combining prospect theory and fuzzy numbers to handle risk and uncertainty in MCDM problems. Zhou *et al.* [3] defines possibility degree and distance formula of extended grey number, studies uncertain multi-criteria decision-making with program criteria value as extended grey number and proposes a multi-criteria decision-making method with uncertain probability based on Hurwicz. Fatih Emre Boran [4] proposed the integration of intuitionistic fuzzy preference relation aiming to obtain weights of criteria and intuitionistic fuzzy TOPSIS method aiming to rank alternatives for dealing with imprecise information on selecting the most desirable facility location. Wang *et al.* [5] studies grey stochastic multi-criteria decision-making problem with probability as interval number and criteria value as interval grey number and proposes a decision-making method based on maximum degree of membership. S. Meysam Mousavi *et al.* [6] developed a new Fuzzy Grey Multi-Criteria Group Decision Making model to solve evaluation and selection problems under uncertainty in real-life situations. Wang *et al.* [7] defines expectation possibility degree of grey random variables, and studies the stochastic multi-criteria decision-making problem with uncertain weight and with criteria value as interval grey number. Ren *et al.* [8] aiming at multi-criteria decision-making problem with incomplete criteria weight information and with criteria value as normally distributed random variable, proposes stochastic multi-criteria decision-making method based on interval arithmetic. The above methods have provided some good research ideas for solution of multi-criteria decision-making problem. It can also be found that there lacks research on stochastic multi-criteria decision-making problem with criteria value as extended grey number that considers the natural state. To this end, this study proposes a corresponding decision-making method to meet the needs of such decisions.

2. Basic Theories

2.1. Extended Grey Number

Grey number refers to a number with only probable range but not the exact value [9], which can effectively measure the grayness of things. In practical applications, the number of grey number is limited to a certain interval or a general set of numbers, usually referred to as "$\otimes$".

Definition 1: Suppose $\otimes$ is a grey number, D is a set that covers $\otimes$, then:

1) If D is an interval, then $\otimes$ is referred to as interval grey number, denoted by $\forall\otimes \Rightarrow d^* \in [a,b]$, or $\otimes = [a,b]$;

2) If D is a discrete set, then $\otimes$ is referred to as discrete grey number, denoted by, $\forall\otimes \Rightarrow d^* \in D$, $D = \{d_1, d_2, \cdots, d_n\}$ or $\otimes = \{d_1, d_2, \cdots, d_n\}$.

Wherein, the size of the interval grey number can be compared with the possibility degree of interval grey number. To better describe the grayness of decision making information, extended grey number with a combination of discrete grey number and continuous grey number can be used [10].

Definition 2: If D is a set of a series of interval grey number, and then $\otimes$ is referred to as extended grey number, denoted by $\otimes = \bigcup_{i=1}^{n}[a_i, b_i]$. Wherein, $[a_i, b_i] \cap [a_j, b_j] = \varnothing (i \neq j)$, $a_i \leq b_i$ (i = 1, 2, ..., n). The set of all extended grey number is denoted as $R(\otimes)$.

2.2. Extended Grey Number Distance and Expectation

Grey number distance describes the disjoint degree between the two grey numbers, and plays an important role in the description of the distance between the criteria evaluation value and the ideal value. In view of the current studies [11] on definition of interval grey number distance, and considering that the theory does not fit the extended grey number, this study presents definition of extended grey number distance.

Definition 3: If $\otimes_1 = \bigcup_{i=1}^{n}[a_i, b_i], \otimes_2 = \bigcup_{j=1}^{m}[c_j, d_j], \in R(\otimes)$ $a_i \leq b_i$ (i = 1, 2, ..., n), $c_j \leq d_j$ (j = 1, 2, ..., m), then Hausdorff distance between extended grey number $\otimes_1$ and $\otimes_2$ is:

$$D(\otimes_1, \otimes_2) = \max\{h(\otimes_1, \otimes_2), h(\otimes_2, \otimes_1)\} \quad (1)$$

Wherein, $h(\otimes_1, \otimes_2) = \max_{i=1}^{n} \min_{j=1}^{m} \| \otimes x_i - \otimes y_j \|$ is the Hausdorff distance between $\otimes_1$ and $\otimes_2$, $\otimes x_i = [a_i, b_i]$, $\otimes y_j = [c_j, d_j]$, (i = 1, 2, ..., n; j = 1, 2, ..., m), $\|\cdot\|$ represents any of the norm, such as L_p.

When $\|\cdot\|$ is L_p,

$$\| \otimes x_i - \otimes y_j \| = \sqrt[p]{|a_i - c_j|^p + |b_i - d_j|^p}$$

Thus obtained:

$$D(\otimes_1, \otimes_2) = \max\{\max_{i=1}^{n} \min_{j=1}^{m} \sqrt[p]{|a_i - c_j|^p + |b_i - d_j|^p}, \max_{j=1}^{m} \min_{i=1}^{n} \sqrt[p]{|c_j - a_i|^p + |d_j - b_i|^p}\} \quad (2)$$

Wherein, p = 1, 2, …, l; l tends to $+\infty$.

Definition 4: The extended grey number random variable is a set of random variables by a limited number of different extended numbers $\otimes$, denoted by $\xi(\otimes)$. Its probability distribution is shown in Table 1, which can also be expressed with the probability distribution function $f(\xi(\otimes))$.

Table 1. Probability distribution of extended grey number random variable $\xi(\otimes)$

$\xi(\otimes)$	$\otimes_1$	$\otimes_2$	$\cdots$	$\otimes_i$	$\cdots$	$\otimes_n$
p	p_1	p_2	$\cdots$	p_i	$\cdots$	p_n

In table 1, $\otimes_i$ represents the value of extended grey number random variable $\xi(\otimes)$ at the i state, $\otimes_i \in \bigcup_{i=1}^{n}[\underline{x}_i, \overline{x}_i]$, $\underline{x}_i \leq \overline{x}_i$, $1 \leq i \leq n$; p_i is the probability at the i state, and $\sum_{i=1}^{n} p_i = 1$ is met, n is the number of possible values of extended grey random variables [12]. The probability distribution function $f(\xi(\otimes))$ is $f(\xi(\otimes) = \otimes_i) = p_i$.

Definition 5: Suppose $\xi(\otimes)$ is an extended grey random variable, then $\sum_{i=1}^{n} p_i \times \otimes_i$ is referred to as expectation value of extended grey random variables, denoted by:

$$E(\xi(\otimes)) = \sum_{i=1}^{n} p_i \times \otimes_i$$

3. Grey Stochastic Multi-criteria Decision-making Method Based on Hausdorff Distance

3.1. Description of Decision Problems

With regard to stochastic multiple criteria decision problems with extended grey number as criterion value, if $A = \{A_1, A_2, \ldots, A_m\}$ is the scheme sets, $B = \{B_1, B_2, \ldots, B_n\}$ is the mutual independent criterion sets, weight vector of the criterion is $w = \{w_1, w_2, \ldots, w_n\}$, and $\sum_{j=1}^{n} w_j = 1$, $w_j \geq 0$ (j = 1, 2, …, n). Due to the no determinacy of decision-making environment, the scheme will show s numbers of natural states under various criterions, and the state sets are $\theta = \{\theta_1, \theta_2, \cdots, \theta_s\}$, Suppose the probability for the state in number t ($t \leq s$) is shown is P_t. The value of scheme A_i under the criterion in number j is the random variable of extended grey number u_{ij}, of which the value under the state in number t is the extended grey number $\otimes u_{ij}^t$, denoted by $\otimes u_{ij}^t = \bigcup_{k=1}^{l} [a_{ijk}^t, b_{ijk}^t]$, therefore the decision matrix $R^t = \{u_{ij}^t\}_{m \times n}$ can be obtained [13].

3.2. Steps of Algorithm

The decision-making method discussed in this study states that, under the condition of all criterion weights are known, in order to determine the optimal solution or sequence of the scheme set, the specific steps are as follow:

Step 1 Normative approach of decision matrix. In order to eliminate the factors affecting the decision-making results due to the dimension differences from criterion to criterion, a normative approach can be carried out for the decision matrix R^t [14]. For the multi-criteria decision-making problems, the common criterion types include benefit-oriented type and cost-oriented type. For the benefit-oriented type, the bigger value is always preferred, while for the cost-oriented once, the smaller value is the preference.

The value of benefit-oriented criterion is:

$$\otimes r_{ij}^t = \frac{\otimes u_{ij}^t}{b_{ijk}^{t\ (\max)}} = \bigcup_{k=1}^{l} [\frac{a_{ijk}^t}{b_{ijk}^{t\ (\max)}}, \frac{b_{ijk}^t}{b_{ijk}^{t\ (\max)}}] \quad (3)$$

The value cost-oriented criterion is:

$$\otimes r_{ij}^t = \frac{a_{ijk}^{t\ (\min)}}{\otimes u_{ij}^t} = \bigcup_{k=1}^{l} [\frac{a_{ijk}^{t\ (\min)}}{b_{ijk}^t}, \frac{a_{ijk}^{t\ (\min)}}{a_{ijk}^t}] \quad (4)$$

Wherein, $b_{ijk}^{t\ (\max)} = \max_{1 \leq k \leq l; 1 \leq i \leq m} b_{ijk}^t$, $a_{ijk}^{t\ (\min)} = \min_{1 \leq k \leq l; 1 \leq i \leq m} a_{ijk}^t$. During corresponding the criterions, s numbers of standardized decision matrixes in natural state are $G^t = \{\otimes r_{ij}^t\}_{m \times n}$.

Step 2 Determining expected values.

Based on the grey decision matrix G^t and the probability of t in natural states P_t, the expected values of each scheme under various states should be calculated by definition 5, thus getting the expected value of decision matrix $G = \{\otimes r_{ij}\}_{m \times n}$.

The calculation formula for expected values is:

$$E(\xi(\otimes r_{ij})) = \sum_{t=1}^{s} p_t \times \otimes r_{ij}^t \quad (5)$$

Step 3 Determining the positive ideal solution and the negative ideal solution.

The positive ideal solution A^+ is:

$$\begin{cases} A^+ = (A_1^+, A_2^+, \cdots, A_n^+) \\ A_j^+ = [\max_{i=1,2,\cdots,m} \underline{r}_{ij1}, \max_{i=1,2,\cdots,m} \overline{r}_{ijh}] \end{cases} \quad (6)$$

The negative ideal solution A^- is:

$$\begin{cases} A^- = (A_1^-, A_2^-, \cdots, A_n^-) \\ A_j^- = [\min_{i=1,2,\cdots,m} \underline{r}_{ij1}, \min_{i=1,2,\cdots,m} \overline{r}_{ijh}] \end{cases} \quad (7)$$

Step 4 Calculating the distances between each scheme and positive/negative ideal solution.

Distance between A_i and A^+ is:

$$d_i^+(A_i, A^+) = \sum_{j=1}^{n} w_j D_1(\otimes r_{ij}, A_j^+) \quad (8)$$

Distance between A_i and A^- is:

$$d_i^-(A_i, A^-) = \sum_{j=1}^{n} w_j D_1(\otimes r_{ij}, A_j^-) \quad (9)$$

Wherein, $D_1(\otimes r_{ij}, A_j^+)$ is the distance between $\otimes r_{ij}$ and A_j^+, $D_1(\otimes r_{ij}, A_j^-)$ is the distance between $\otimes r_{ij}$ and A_j^-.

Step 5 Calculating the relative closeness coefficient K_i, and sequencing the schemes.

$$K_i = \frac{d_i^+(A_i, A^+)}{d_i^+(A_i, A^+) + d_i^-(A_i, A^-)} \quad (10)$$

4. Illustrative Examples

Suppose a certain bank is giving investment to 3 enterprises (A_1, A_2, A_3) from C city, the criterions for evaluating the investment values are respectively: Annual output value B_1 (Units: ten million Yuan), social benefit B_2 (Units: ten million Yuan), pollution level B_3, weight vectors of the criterions w = (0.25, 0.33, 0.42). Evaluations were given respectively to the enterprises under various criterions, and during investment, each criterion is corresponded with 3 kinds of natural states i.e. good, medium, poor, of which the corresponded probability $p = (0.3, 0.4, 0.3)$. Under each state, evaluation information was given in the form of random variables of extended grey number, of which the decision-making data are shown in Table 2 to Table 4. Try to determine the enterprise with best investment effectiveness [15].

Step 1 In up-mentioned criterions, annual output value, social benefit belong to the benefit-oriented criterion, while the pollution level is included in cost-oriented criterion. Conducting normative approach to matrix R^t (t = 1, 2, 3) based on formula (3) and formula (4), the standardized decision matrix can be obtained as shown in Table 5 to Table 7.

Table 2. Decision matrix under good state R^1

	B_1	B_2	B_3
A_1	$[3.0,3.4]\cup[3.5,3.6]$	$[3.1,3.3]\cup[3.7,4.0]$	$[0.25,0.40]\cup[0.60,0.75]$
A_2	$[3.1,3.2]\cup\{3.4\}$	$[3.5,3.7]\cup[3.7,3.9]$	$[0.40,0.55]\cup[0.60,0.80]$
A_3	$[2.5,2.7]\cup[2.8,2.9]$	$[2.7,3.0]\cup[3.3,3.5]$	$[0.30,0.50]\cup[0.50,0.65]$

Table 3. Decision matrix under medium state R^2

	B_1	B_2	B_3
A_1	$[2.8,3.0]\cup[3.0,3.2]$	$[3.3,3.8]\cup[3.9,4.4]$	$[0.25,0.40]\cup[0.40,0.60]$
A_2	$[2.1,2.3]\cup\{2.6\}$	$[3.8,4.1]\cup[4.4,4.5]$	$[0.25,0.40]\cup[0.60,0.75]$
A_3	$[3.0,3.2]\cup[3.4,3.7]$	$[2.6,3.1]\cup[3.2,3.7]$	$[0.40,0.60]\cup[0.60,0.75]$

Table 4. Decision matrix under poor state R^3

	B_1	B_2	B_3
A_1	$[2.8,3.0]\cup[3.0,3.3]$	$[3.3,3.5]\cup[3.6,4.0]$	$[0.30,0.40]\cup[0.50,0.70]$
A_2	$[2.6,2.9]\cup[3.0,3.2]$	$[2.6,2.8]\cup[3.3,4.0]$	$[0.40,0.50]\cup[0.50,0.65]$
A_3	$[2.5,2.8]\cup[3.1,3.3]$	$[2.6,2.9]\cup[3.0,3.4]$	$[0.25,0.40]\cup[0.50,0.70]$

Table 5. Standardized decision matrix under good state G^1

	B_1	B_2	B_3
A_1	$[0.833,0.944]\cup[0.972,1]$	$[0.775,0.825]\cup[0.925,1]$	$[0.333,0.417]\cup[0.625,1]$
A_2	$[0.861,0.889]\cup\{0.944\}$	$[0.875,0.925]\cup[0.925,0.975]$	$[0.313,0.417]\cup[0.455,0.625]$
A_3	$[0.694,0.750]\cup[0.778,0.806]$	$[0.675,0.750]\cup[0.825,0.875]$	$[0.385,0.500]\cup[0.500,0.833]$

Table 6. Standardized decision matrix under medium state G^2

	B_1	B_2	B_3
A_1	$[0.757,0.811]\cup[0.811,0.865]$	$[0.733,0.844]\cup[0.867,0.978]$	$[0.417,0.625]\cup[0.625,1]$
A_2	$[0.568,0.622]\cup\{0.703\}$	$[0.844,0.911]\cup[0.978,1]$	$[0.333,0.417]\cup[0.625,1]$
A_3	$[0.811,0.865]\cup[0.919,1]$	$[0.578,0.689]\cup[0.711,0.822]$	$[0.333,0.417]\cup[0.417,0.625]$

Table 7. Standardized decision matrix under poor state G^3

	B_1	B_2	B_3
A_1	$[0.848,0.909]\cup[0.909,1]$	$[0.825,0.875]\cup[0.900,1]$	$[0.357,0.500]\cup[0.625,0.833]$
A_2	$[0.788,0.879]\cup[0.909,0.970]$	$[0.650,0.700]\cup[0.825,1]$	$[0.385,0.500]\cup[0.500,0.625]$
A_3	$[0.758,0.848]\cup[0.939,1]$	$[0.650,0.725]\cup[0.750,0.850]$	$[0.357,0.500]\cup[0.625,1]$

Table 8. Expected value decision matrix G

	B_1	B_2	B_3
A_1	[0.7762,0.9054]	[0.7607,0.9578]	[0.3338,0.9500]
A_2	[0.6662,0.8153]	[0.7662,1.0000]	[0.3288,0.7876]
A_3	[0.7868,1.0000]	[0.5995,0.8305]	[0.3206,0.7000]

Step 2 Based on nature state probability $p = (0.3, 0.4, 0.3)$ and the operational rule of extended grey number, expected values can be calculated using formula (5), therefore the expected value decision matrix can be obtained $G = \{\otimes r_{ij}\}_{3\times 3}$, of which the results are shown in Table 8.

Step 3 Calculating the positive ideal solution and negative ideal solution using formula (6) and formula (7).

A^+ = ([0.7868, 1.0000], [0.7662, 1.0000], [0.3206, 0.7000])

A^- = ([0.6662, 0.8153], [0.5995, 0.8305], [0.3338, 0.9500])

Step 4 Calculating the distances between each scheme and positive/negative ideal solution using formula (8) and formula (9), given that the weight vectors of criterions are $w = (0.25, 0.33, 0.42)$.

Distance between A_i and A^+ is:

$$d_1^+(A_1, A^+) = 0.1430,\ d_2^+(A_2, A^+) = 0.0921,$$
$$d_3^+(A_3, A^+) = 0.0785$$

Distance between A_i and A^- is:

$$d_1^-(A_1, A^-) = 0.1033,\ d_2^-(A_2, A^-) = 0.1467,$$
$$d_3^-(A_3, A^-) = 0.1603$$

Step 5 Calculating the relative closeness coefficient using formula (10).

$$K_1 = 0.5805, K_2 = 0.3857, K_3 = 0.3286$$

Therefore it can get $K_3 < K_2 < K_1$, and the sequence result for various criterions is: $A_3 \succ A_2 \succ A_1$. As a result, the enterprise A_3 is the best invested enterprise. The result is roughly the same with the conclusion in reference [15], which proved that method was available, In addition, according to the process of calculation analysis, it can be seen that compared with the method in the reference, the method proposed in this study can meet more practical application requirements and possess better operability.

5. Conclusions

In this study, through analyzing the stochastic multi-criteria decision-making problems, of which the criterion value is extended grey number, it proposed the grey stochastic multi-criteria decision-making method based on Hausdorff distance. Meanwhile, an illustrative example was adopted to identify the practicability and rationality of the method. With advantages of clear concept, simple calculation procedure and easiness in understanding, the decision-making method proposed in the study possessed excellent application range and high practical decision-making value, which can be widely used in related decision-making issues such as supply chain management, project evaluation and investment decision-making.

Acknowledgements

This study was supported by the Scientific Research Fund of Hunan Provincial Education Department (No. 14C0184), the Hunan Province Philosophy and Social Science Foundation (No. 14YBA065), and the construct program of the key discipline in Hunan province.

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