Universal Journal of Management 3(8): 337-345, 2015 http://www.hrpub.org
DOI: 10.13189/ujm.2015.030805

Solving Fuzzy MCDM by Subtracting Benefit Criteria from Cost Criteria

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Abstract This paper suggests a fuzzy MCDM (multiple criteria decision-making) approach, where ratings of alternatives versus criteria and weights of criteria are assessed in fuzzy numbers or linguistic values represented by fuzzy numbers. Criteria are classified into cost and benefit ones. In the proposed method, the ratings assigned by decision makers to each alternative versus each criterion and the weights assigned by decision makers to each criterion are averaged. The averaged cost and benefit ratings are further normalized into comparable scales respectively. The membership function of subtracting the summation of weighted normalized benefit ratings from that of weighted normalized cost ratings for each alternative can be developed by interval arithmetic of fuzzy numbers. The fuzzy number ranking method of centroid is then applied to determine the ordering of the alternatives. A numerical example of robot selection demonstrates feasibility of the proposed method.

Keywords Fuzzy MCDM, Cost, Benefit, Interval Arithmetic, Centroid

1. Introduction

Fuzzy multiple criteria decision-making (MCDM) is a powerful tool for evaluation and selection of alternatives versus different criteria, where ratings of alternatives under different criteria and the importance weights of criteria are usually assessed in fuzzy numbers or linguistic values (Zadeh, 1975) represented by fuzzy numbers. Numerous fuzzy MCDM methods have been investigated. A review of many of these methods can be found in Carlsson and Fuller (1996), Ribeiro (1996), Chu and Varma (2012), and Moghimi and Anvari (2014). Some other recent methods can be found in (Afkham, et al., 2012; Akdag, et al., 2014; Büyüközkan, et al., 2011; Chung et al., 2015; Ghorbani, et al., 2013; Govindan, et al., 2013). However, clear development for the membership function of the aggregation of the fuzzy weighted ratings of each alternative cannot be found in the above works. This limitation deters their applicability to real world problems.

To resolve the above problem, a new fuzzy MCDM approach for alternative selection is suggested. In this work, criteria are categorized into cost and benefit ones. Cost criteria have the property: the smaller, the better. Conversely, benefit criteria have the property: the larger, the better. Via the proposed method, the ratings given by decision-makers to each alternative under each criterion and the weights given by decision-makers to each of the criteria are averaged. Since criteria may have incommensurable units (Chen and Hwang, 1992), all averaged cost and benefit ratings are further normalized into comparable scales respectively before weighted. Obviously, if the summation of the weighted normalized cost ratings (SWNCR) is smaller and/or the summation of the weighted normalized benefit ratings (SWNBR) is larger, the higher priority the alternative will have. The larger value the SWNBR offsets the SWNCR, the higher priority the alternative will have. Thus, the concept of subtracting SWNBR from SWNCR is used to evaluate the alternative performance (or suitability). If final fuzzy evaluation value is smaller, priority of the alternative is higher. Using interval arithmetic of fuzzy numbers can develop the membership function of subtracting SWNBR from SWNCR for each alternative. Because the final fuzzy evaluation values are still fuzzy numbers, then a ranking method is needed.

Many fuzzy number ranking methods have been proposed since fuzzy set theory was introduced by Zadeh in 1965. A comparison of many ranking methods can be seen in Wang and Kerre (2001) and Wang and Lee (2008). Some recent works can be found in (Asady, 2010; Ezzati, et al., 2012; Hari Ganesh and Jayakumar, 2014; Jafarian and Rezvani, 2013; Rao and Shankar, 2013; Sharma, 2015; Thorani, et al., 2012). Despite the merits, some methods are computational complex and others are difficult to present connection by formula between the ranking procedure and the final fuzzy evaluation values of alternatives under fuzzy MCDM model. To resolve the above limitations, this work applies the centroid method (Yager, 1981) to rank alternatives under fuzzy MCDM. The suggested method provides an extension to the fuzzy MCDM techniques available. A numerical example of robot selection demonstrates feasibility of the proposed method.

2. Fuzzy Numbers

Definition 1. A real fuzzy number *A* is described as any fuzzy subset of the real line *R* with membership function f_A which possesses the following properties (Dubois and Prade, 1978):

1. f_A is a continuous mapping from *R* to the closed interval [0,1];
2. $f_A(x)$=0, for all $x \in (-\infty, a]$;
3. f_A is strictly increasing on $[a,b]$;
4. $f_A(x)$=1, for all $x \in [b,c]$;
5. f_A is strictly decreasing on $[c,d]$;
6. $f_A(x)$=0, for all $x \in [d,\infty)$,

where *a*, *b*, *c* and *d* are real numbers. We may let $a = -\infty$, or *a*=*b*, or *b*=*c*, or *c*=*d*, or *d*= $+\infty$. Unless elsewhere specified, it is assumed that *A* is convex, normal and bounded, *i.e.* $-\infty < a$, $d < \infty$.

The membership function f_A of the fuzzy number *A* can also be expressed as:

$$f_A(x) = \begin{cases} f_A^{\mathrm{L}}(x), & a \le x \le b, \\ 1, & b \le x \le c, \\ f_A^{\mathrm{R}}(x), & c \le x \le d, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $f_A^{\mathrm{L}}(x)$ and $f_A^{\mathrm{R}}(x)$ are the left and right membership functions of fuzzy number *A* respectively.

The fuzzy number *A* is a triangular fuzzy numbers if its membership function f_A is given by (Laarhoven and Pedrycz, 1983):

$$f_A(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b, \\ (x-c)/(b-c), & b \le x \le c, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

where *a*, *b* and *c* are real numbers.

Definition 2. The α-cut of fuzzy number *A* can be defined as (Kaufmann and Gupta, 1991)

$$A^\alpha = \{x | f_A(x) \ge \alpha,\}, \text{ where } x \in R,\ \alpha \in [0,1].$$

A^α is a non-empty bounded closed interval contained in *R* and it can be denoted by $A^\alpha = [A_l^\alpha, A_u^\alpha]$, where A_l^α and A_u^α are the lower and upper bounds of the closed interval respectively. For example, if triangular fuzzy number *A*=(*a*, *b*, *c*), then the α-cut of *A* can be expressed as:

$$A^\alpha = [A_l^\alpha, A_u^\alpha] = [(b-a)\alpha + a, (b-c)\alpha + c] \tag{3}$$

Given fuzzy numbers *A* and *B*, $A, B \in R^+$, the α-cuts of *A* and *B* are $A^\alpha = [A_l^\alpha, A_u^\alpha]$ and $B^\alpha = [B_l^\alpha, B_u^\alpha]$, respectively. By the interval arithmetic, some main operations of *A* and *B* can be expressed as follows (Kaufmann and Gupta, 1991):

$$(A \oplus B)^\alpha = [A_l^\alpha + B_l^\alpha, A_u^\alpha + B_u^\alpha], \tag{4}$$

$$(A \ominus B)^\alpha = [A_l^\alpha - B_u^\alpha, A_u^\alpha - B_l^\alpha] \tag{5}$$

$$(A \otimes B)^\alpha = [A_l^\alpha \cdot B_l^\alpha, A_u^\alpha \cdot B_u^\alpha], \tag{6}$$

$$(A \oslash B)^\alpha = \left[\frac{A_l^\alpha}{B_u^\alpha}, \frac{A_u^\alpha}{B_l^\alpha}\right] \tag{7}$$

$$(A \otimes r)^\alpha = [A_l^\alpha \cdot r, A_u^\alpha \cdot r], \quad r \in R^+. \tag{8}$$

3. A New Fuzzy MCDM Approach

Assume that a committee of *k* decision-makers (*i.e.*, $D_1, D_2, \ldots, D_k$) is responsible for evaluating *m* alternatives (*i.e.*, $A_1, A_2, \ldots, A_m$) under *n* selection criteria (*i.e.*, $C_1, C_2, \ldots, C_n$), where the ratings of alternatives versus each of the criteria as well as the weights of all criteria are assessed in fuzzy numbers or linguistic values (Zadeh, 1975) represented by triangular fuzzy numbers. Both cost criteria (*C*), *j*=1~*h*, and benefit criteria (*B*), *j*=*h*+1~*n*, are considered.

3.1. Average ratings and Perform Normalization

Let $x_{ijt} = (o_{ijt}, p_{ijt}, q_{ijt})$, $x_{ijt} \in R^+$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $t = 1,2,\ldots,k$, be the rating assigned to alternative A_i by decision-maker D_t for criterion C_j. The averaged rating, $x_{ij} = (o_{ij}, p_{ij}, q_{ij})$, of alternative A_i under criterion C_j assessed by the committee of *k* decision-makers can be evaluated as:

$$x_{ij} = (1/k) \otimes (x_{ij1} \oplus x_{ij2} \oplus \cdots \oplus x_{ijk}) \tag{9}$$

where $o_{ij} = \sum_{t=1}^{k} o_{ijt}/k$, $p_{ij} = \sum_{t=1}^{k} p_{ijt}/k$, $q_{ij} = \sum_{t=1}^{k} q_{ijt}/k$.

All averaged criteria, both *C* and *B*, are further normalized by the Chen method (Chen, 2001) into comparable scales respectively. This method preserves the property in which the ranges of normalized triangular fuzzy numbers belong to [0,1]. The normalization of the averaged ratings, *i.e.* x_{ij}'s, is as follows:

$$\bar{x}_{ij} = \left(\frac{o_{ij}}{q_j^*}, \frac{p_{ij}}{q_j^*}, \frac{q_{ij}}{q_j^*} \right),\ q_j^* = \max_i q_{ij},\ j \in B \text{ or } j \in C \tag{10}$$

where $\bar{x}_{ij}$ is the normalized value of x_{ij}. For convenience, let $\bar{x}_{ij} = (r_{ij}, s_{ij}, t_{ij})$.

3.2. Average Weights

Let $w_{jt} = (a_{jt}, b_{jt}, c_{jt})$, $w_{jt} \in R^+$, $j = 1,2,...,n$, $t = 1,2,...,k$, be the weight assigned by decision-maker D_t to criterion C_j. The averaged weight, $w_j = (a_j, b_j, c_j)$, of criterion C_j assessed by the committee of k decision-makers can be evaluated as:

$$w_j = (1/k) \otimes (w_{j1} \oplus w_{j2} \oplus \cdots \oplus w_{jk}) \tag{11}$$

where $a_j = \sum_{t=1}^{k} a_{jt} / k$, $b_j = \sum_{t=1}^{k} b_{jt} / k$, $c_j = \sum_{t=1}^{k} c_{jt} / k$.

3.3. Develop Membership Function

The membership function of subtracting the SWNBR from the SWNCR for each alternative, *i.e.* $G_i = \sum_{j=1}^{h} \bar{x}_{ij} \otimes w_j - \sum_{j=h+1}^{n} \bar{x}_{ij} \otimes w_j$, can be developed by Eqs. (3)~(8) as follows:

$$\left(\bar{x}_{ij} \otimes w_j\right)^{\alpha} = \Big[(s_{ij} - r_{ij})(b_j - a_j)\alpha^2 + [r_{ij}(b_j - a_j) + a_j(s_{ij} - r_{ij})]\alpha + r_{ij}a_j,$$
$$(s_{ij} - t_{ij})(b_j - c_j)\alpha^2 + [t_{ij}(b_j - c_j) + c_j(s_{ij} - t_{ij})]\alpha + t_{ij}c_j\Big] \tag{12}$$

Thus, we have

$$G_i^{\alpha} = \left(\sum_{j=1}^{h} \bar{x}_{ij} \otimes w_j - \sum_{j=h+1}^{n} \bar{x}_{ij} \otimes w_j \right)^{\alpha}$$
$$= \Bigg[\sum_{j=1}^{h} (s_{ij} - r_{ij})(b_j - a_j)\alpha^2 + \sum_{j=1}^{h} [r_{ij}(b_j - a_j) + a_j(s_{ij} - r_{ij})]\alpha + \sum_{j=1}^{h} r_{ij}a_j,$$
$$\sum_{j=1}^{h} (s_{ij} - t_{ij})(b_j - c_j)\alpha^2 + \sum_{j=1}^{h} [t_{ij}(b_j - c_j) + c_j(s_{ij} - t_{ij})]\alpha + \sum_{j=1}^{h} t_{ij}c_j \Bigg] -$$
$$\Bigg[\sum_{j=h+1}^{n} (s_{ij} - r_{ij})(b_j - a_j)\alpha^2 + \sum_{j=h+1}^{n} [r_{ij}(b_j - a_j) + a_j(s_{ij} - r_{ij})]\alpha + \sum_{j=h+1}^{n} r_{ij}a_j,$$
$$\sum_{j=h+1}^{n} (s_{ij} - t_{ij})(b_j - c_j)\alpha^2 + \sum_{j=h+1}^{n} [t_{ij}(b_j - c_j) + c_j(s_{ij} - t_{ij})]\alpha + \sum_{j=h+1}^{n} t_{ij}c_j \Bigg] \tag{13}$$

Let $E_{i1} = \sum_{j=1}^{h} (s_{ij} - r_{ij})(b_j - a_j)$, $F_{i1} = \sum_{j=1}^{h} [r_{ij}(b_j - a_j) + a_j(s_{ij} - r_{ij})]$,

$E_{i2} = \sum_{j=1}^{h} (s_{ij} - t_{ij})(b_j - c_j)$, $F_{i2} = \sum_{j=1}^{h} [t_{ij}(b_j - c_j) + c_j(s_{ij} - t_{ij})]$,

$I_{i1} = \sum_{j=h+1}^{n} (s_{ij} - r_{ij})(b_j - a_j)$, $J_{i1} = \sum_{j=h+1}^{n} [r_{ij}(b_j - a_j) + a_j(s_{ij} - r_{ij})]$,

$$I_{i2} = \sum_{j=h+1}^{n}(s_{ij} - t_{ij})(b_j - c_j),\ J_{i2} = \sum_{j=h+1}^{n}[t_{ij}(b_j - c_j) + c_j(s_{ij} - t_{ij})],$$

$$V_{i1} = \sum_{j=1}^{h} r_{ij}a_j,\ Y_{i1} = \sum_{j=1}^{h} s_{ij}b_j,\ Z_{i1} = \sum_{j=1}^{h} t_{ij}c_j,\ V_{i2} = \sum_{j=h+1}^{n} r_{ij}a_j,$$

$$Y_{i2} = \sum_{j=h+1}^{n} s_{ij}b_j,\ Z_{i2} = \sum_{j=h+1}^{n} t_{ij}c_j.$$

We now have two simplified equations to solve, namely:

$$(E_{i1} - I_{i2})\alpha^2 + (F_{i1} - J_{i2})\alpha + (V_{i1} - Z_{i2}) - x = 0 \tag{14}$$

$$(E_{i2} - I_{i1})\alpha^2 + (F_{i2} - J_{i1})\alpha + (Z_{i1} - V_{i2}) - x = 0 \tag{15}$$

Only roots in [0,1] will be retained in (14) and (15). The left membership function, *i.e.* $f_{G_i}^{\mathrm{L}}(x)$, and the right membership function, *i.e.* $f_{G_i}^{\mathrm{R}}(x)$, of G_i can then be produced as:

$$f_{G_i}^{\mathrm{L}}(x) = \left\{(J_{i2} - F_{i1}) + \left[(F_{i1} - J_{i2})^2 + 4(E_{i1} - I_{i2})(x - V_{i1} + Z_{i2})\right]^{1/2}\right\} \Big/ 2(E_{i1} - I_{i2}),$$

$$V_{i1} - Z_{i2} \le x \le Y_{i1} - Y_{i2}, \tag{16}$$

$$f_{G_i}^{\mathrm{R}}(x) = \left\{(J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(x - Z_{i1} + V_{i2})\right]^{1/2}\right\} \Big/ 2(E_{i2} - I_{i1}),$$

$$Y_{i1} - Y_{i2} \le x \le Z_{i1} - V_{i2}. \tag{17}$$

For convenience, G_i, $i = 1,2,\ldots,m$, can be expressed as:

$$G_i = (V_{i1} - Z_{i2}, Y_{i1} - Y_{i2}, Z_{i1} - V_{i2}; J_{i2} - F_{i1}, E_{i1} - I_{i2}; J_{i1} - F_{i2}, E_{i2} - I_{i1}) \tag{18}$$

Proposition 1. For Eq. (16), $f_{G_i}^{\mathrm{L}}(x) = 0$ if $x = V_{i1} - Z_{i2}$.

Proof.

$$f_{G_i}^{\mathrm{L}}(x)$$

$$= \left\{(J_{i2} - F_{i1}) + \left[(F_{i1} - J_{i2})^2 + 4(E_{i1} - I_{i2})(x - V_{i1} + Z_{i2})\right]^{1/2}\right\} \Big/ 2(E_{i1} - I_{i2})$$

$$= \left\{(J_{i2} - F_{i1}) + \left[(F_{i1} - J_{i2})^2 + 4(E_{i1} - I_{i2})(V_{i1} - Z_{i2} - V_{i1} + Z_{i2})\right]^{1/2}\right\} \Big/ 2(E_{i1} - I_{i2})$$

$$= \left\{(J_{i2} - F_{i1}) + \left[(F_{i1} - J_{i2})^2\right]^{1/2}\right\} \Big/ 2(E_{i1} - I_{i2}) = 0.$$

Proposition 2. In Eq. (16), $E_{i1} - I_{i2} - J_{i2} + F_{i1} = Y_{i1} - Y_{i2} - V_{i1} + Z_{i2}$.

Proof.

$$E_{i1} - I_{i2} - J_{i2} + F_{i1}$$

$$= \sum_{j=1}^{h}(s_{ij}-r_{ij})(b_j-a_j) - \sum_{j=h+1}^{n}(s_{ij}-t_{ij})(b_j-c_j)$$

$$-\sum_{j=h+1}^{n}[t_{ij}(b_j-c_j)+c_j(s_{ij}-t_{ij})] + \sum_{j=1}^{h}[r_{ij}(b_j-a_j)+a_j(s_{ij}-r_{ij})]$$

$$= \sum_{j=1}^{h}s_{ij}b_j - \sum_{j=1}^{h}r_{ij}b_j - \sum_{j=1}^{h}s_{ij}a_j + \sum_{j=1}^{h}r_{ij}a_j - \sum_{j=h+1}^{n}s_{ij}b_j + \sum_{j=h+1}^{n}t_{ij}b_j + \sum_{j=h+1}^{n}s_{ij}c_j - \sum_{j=h+1}^{n}t_{ij}c_j$$

$$-\sum_{j=h+1}^{n}t_{ij}b_j + \sum_{j=h+1}^{n}t_{ij}c_j - \sum_{j=h+1}^{n}s_{ij}c_j + \sum_{j=h+1}^{n}t_{ij}c_j + \sum_{j=1}^{h}r_{ij}b_j - \sum_{j=1}^{h}r_{ij}a_j + \sum_{j=1}^{h}s_{ij}a_j - \sum_{j=1}^{h}r_{ij}a_j$$

$$= \sum_{j=1}^{h}s_{ij}b_j - \sum_{j=h+1}^{n}s_{ij}b_j - \sum_{j=1}^{h}r_{ij}a_j + \sum_{j=h+1}^{n}t_{ij}c_j = Y_{i1}-Y_{i2}-V_{i1}+Z_{i2}. \quad \square$$

Corollary 2.1. For Eq. (16), $f_{G_i}^{\mathrm{L}}(x)=1$ if $x=Y_{i1}-Y_{i2}$.

Proof.

$$f_{G_i}^{\mathrm{L}}(x)$$

$$=\left\{(J_{i2}-F_{i1})+\left[(F_{i1}-J_{i2})^2+4(E_{i1}-I_{i2})(x-V_{i1}+Z_{i2})\right]^{1/2}\right\}\Big/2(E_{i1}-I_{i2})$$

$$=\left\{(J_{i2}-F_{i1})+\left[(F_{i1}-J_{i2})^2+4(E_{i1}-I_{i2})(Y_{i1}-Y_{i2}-V_{i1}+Z_{i2})\right]^{1/2}\right\}\Big/2(E_{i1}-I_{i2})$$

Proposition 2 obtains:

$$f_{G_i}^{\mathrm{L}}(x)$$

$$=\left\{(J_{i2}-F_{i1})+\left[(F_{i1}-J_{i2})^2+4(E_{i1}-I_{i2})(E_{i1}-I_{i2}-J_{i2}+F_{i1})\right]^{1/2}\right\}\Big/2(E_{i1}-I_{i2})$$

$$=\left\{(J_{i2}-F_{i1})+\left[(F_{i1}-J_{i2})^2+4(E_{i1}-I_{i2})^2+4(E_{i1}-I_{i2})(F_{i1}-J_{i2})\right]^{1/2}\right\}\Big/2(E_{i1}-I_{i2})$$

$$=\left\{(J_{i2}-F_{i1})+\left[\left(2(E_{i1}-I_{i2})+(F_{i1}-J_{i2})\right)^2\right]^{1/2}\right\}\Big/2(E_{i1}-I_{i2})=1. \quad \square$$

Proposition 3. In Eq. (17), $F_{i2}-J_{i1}+E_{i2}-I_{i1}=Y_{i1}-Y_{i2}-Z_{i1}+V_{i2}$.

Proof.

$$F_{i2}-J_{i1}+E_{i2}-I_{i1}$$

$$=\sum_{j=1}^{h}[t_{ij}(b_j-c_j)+c_j(s_{ij}-t_{ij})] - \sum_{j=h+1}^{n}[r_{ij}(b_j-a_j)+a_j(s_{ij}-r_{ij})]$$

$$+\sum_{j=1}^{h}(s_{ij}-t_{ij})(b_j-c_j) - \sum_{j=h+1}^{n}(s_{ij}-r_{ij})(b_j-a_j)$$

$$= \sum_{j=1}^{h} t_{ij} b_j - \sum_{j=1}^{h} t_{ij} c_j + \sum_{j=1}^{h} s_{ij} c_j - \sum_{j=1}^{h} t_{ij} c_j - \sum_{j=h+1}^{n} r_{ij} b_j + \sum_{j=h+1}^{n} r_{ij} a_j - \sum_{j=h+1}^{n} s_{ij} a_j + \sum_{j=h+1}^{n} r_{ij} a_j$$

$$+ \sum_{j=1}^{h} s_{ij} b_j - \sum_{j=1}^{h} t_{ij} b_j - \sum_{j=1}^{h} s_{ij} c_j + \sum_{j=1}^{h} t_{ij} c_j - \sum_{j=h+1}^{n} s_{ij} b_j + \sum_{j=h+1}^{n} r_{ij} b_j + \sum_{j=h+1}^{n} s_{ij} a_j - \sum_{j=h+1}^{n} r_{ij} a_j$$

$$= \sum_{j=1}^{h} s_{ij} b_j - \sum_{j=h+1}^{n} s_{ij} b_j - \sum_{j=1}^{h} t_{ij} c_j + \sum_{j=h+1}^{n} r_{ij} a_j = Y_{i1} - Y_{i2} - Z_{i1} + V_{i2}. \quad \square$$

Corollary 3.1. For Eq. (17), $f_{G_i}^{\mathrm{R}}(x) = 1$ if $x = Y_{i1} - Y_{i2}$.

Proof.

$$f_{G_i}^{\mathrm{R}}(x)$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(x - Z_{i1} + V_{i2}) \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(Y_{i1} - Y_{i2} - Z_{i1} + V_{i2}) \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

Proposition 3 produces:

$$f_{G_i}^{\mathrm{R}}(x)$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(F_{i2} - J_{i1} + E_{i2} - I_{i1}) \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(F_{i2} - J_{i1}) + 4(E_{i2} - I_{i1})^2 \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[\left(2(E_{i2} - I_{i1}) + (F_{i2} - J_{i1}) \right)^2 \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[\left(2(I_{i1} - E_{i2}) + (J_{i1} - F_{i2}) \right)^2 \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1}) = 1. \quad \square$$

Proposition 4. For Eq. (17), $f_{G_i}^{\mathrm{R}}(x) = 0$ if $x = Z_{i1} - V_{i2}$.

Proof.

$$f_{G_i}^{\mathrm{R}}(x)$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(x - Z_{i1} + V_{i2}) \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(F_{i2} - J_{i1})^2 + 4(E_{i2} - I_{i1})(Z_{i1} - V_{i2} - Z_{i1} + V_{i2}) \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1})$$

$$= \left\{ (J_{i1} - F_{i2}) - \left[(-F_{i2} + J_{i1})^2 \right]^{1/2} \right\} \Big/ 2(E_{i2} - I_{i1}) = 0.$$

3.4. Determine the Ordering of Alternatives

Herein, an intuitive ranking method, the centroid index (Yager, 1981) is applied to help rank all the final fuzzy numbers.

Suppose fuzzy numbers A_i, $A_i = [a_i, b_i, c_i, d_i]$, as in Definition 1, with membership function $f_{A_i}(x)$. The centroid of A, $C(A_i)$, on the horizontal axis can be defined as:

$$C(A_i) = \frac{\int_{a_i}^{b_i} (x f_{A_i}^{\mathrm{L}})\,\mathrm{d}x + \int_{b_i}^{c_i} x\,\mathrm{d}x + \int_{c_i}^{d_i} (x f_{A_i}^{\mathrm{R}})\,\mathrm{d}x}{\int_{a_i}^{b_i} (f_{A_i}^{\mathrm{L}})\,\mathrm{d}x + \int_{b_i}^{c_i} \mathrm{d}x + \int_{c_i}^{d_i} (f_{A_i}^{\mathrm{R}})\,\mathrm{d}x} \quad (19)$$

According to the proposed concept, the smaller the $C(A_i)$, the higher priority the alternative A_i will have. Thus, for any two final fuzzy numbers G_i and G_j, if $C(G_i) < C(G_j)$, then $G_i > G_j$. If $C(G_i) = C(G_j)$, then $G_i = G_j$. Finally, if $C(G_i) > C(G_j)$, then $G_i < G_j$.

4. Numerical Example

Assume that a manufacturing company needs a robot to perform a material-handling task (Liang and Wang, 1993). After preliminary screening, three robots R_1, R_2 and R_3 are chosen for further evaluation. A committee of three decision-makers D_1, D_2 and D_3 is formed to conduct the evaluation and selection of the three robots. Two cost criteria, purchase cost (C_1) and positioning accuracy (C_2), and two benefit criteria, programming flexibility (C_3) and man-machine interface (C_4), are considered.

Further assume that the decision-makers use the linguistic rating set *S*={VP, P, F, G, VG}, where VP=Very Poor=(0,0,3), P=Poor=(0,3,5), F=Fair=(2,5,8), G=Good=(5,7,10), and VG=Very Good=(7,10,10) to evaluate the suitability of each robot under each of benefit criteria. And the decision-makers employ a linguistic weighting set *W*={VL, L, M, H, VH}, where VL=Very Low=(0,0.1,0.3),L=Low=(0.1,0.3,0.5),M=Medium=(0.3,0.5,0.7), H=High=(0.5,0.7,0.9), and VH=Very High=(0.7,0.9,1), to assess the importance of all the criteria. Moreover, assume that the cost (million NT$) and positioning accuracy ($\pm$ in.) assessed by decision-makers for the three robots are presented in Table 1 and the suitability ratings of three robots versus benefit criteria are also shown in Table 1. The importance weights of the criteria are displayed in Table 2.

Table 1. Ratings of robots, averaged ratings and normalized ratings

C.	R.	Decision-makers			Averaged ratings x_{ij}	Normalized ratings $\bar{x}_{ij}$
		D_1	D_2	D_3		
	R_1		(3.1,3.5,3.7)		(3.1,3.5,3.7)	(0.7561,0.8537,0.9024)
C_1	R_2		(3.3,3.5,3.8)		(3.3,3.5,3.8)	(0.8049,0.8537,0.9268)
	R_3		(3.4,3.7,4.1)		(3.4,3.7,4.1)	(0.8293,0.9024,1.0000)
	R_1		(0.10,0.13,0.14)		(0.10,0.13,0.14)	(0.5882,0.7647,0.8235)
C_2	R_2		(0.09,0.13,0.17)		(0.09,0.13,0.17)	(0.5294,0.7647,1.0000)
	R_3		(0.09,0.12,0.14)		(0.09,0.12,0.14)	(0.5294,0.7059,0.8235)
	R_1	VG	VG	G	(6.3333, 9.0000, 10.0000)	(0.6333,0.9000,1.0000)
C_3	R_2	G	G	G	(5.0000, 7.0000, 10.0000)	(0.5000,0.7000,1.0000)
	R_3	G	VG	VG	(6.3333, 9.0000, 10.0000)	(0.6333,0.9000,1.0000)
	R_1	F	G	G	(4.0000, 6.3333, 9.3333)	(0.4286,0.6786,1.0000)
C_4	R_2	G	F	G	(4.0000, 6.3333, 9.3333)	(0.4286,0.6786,1.0000)
	R_3	F	G	F	(3.0000, 5.6667, 8.6667)	(0.3214,0.6071,0.9286)

C.: Criteria, R.: Robots.

By Eqs. (9) and (10), the averaged rating and normalized averaged rating of each robot R_i under each criterion from the decision-making committee can be obtained as also shown in Table 1. By Eq. (11), the averaged weights of the criteria from the decision-making committee can be obtained as also shown in Table 2.

Table 2. The weights of criteria and averaged weights

C.	Decision-makers			Averaged weights w_j
	D_1	D_2	D_3	
C_1	H	H	VH	(0.5667, 0.7667, 0.9333)
C_2	VH	VH	H	(0.6333, 0.8333, 0.9667)
C_3	H	M	H	(0.4333, 0.6333, 0.8333)
C_4	M	L	L	(0.1667, 0.3667, 0.5667)

C.: Criteria.

By Eqs. (12)-(18), the membership function for G_i, $i = 1 \sim 3$, can be obtained as follows:

G_1=(-0.5900,0.4729,1.2925; -1.1014,-0.0295; 0.7322,-0.0874),

G_2=(-0.6086,0.5996,1.5436; -1.2757,-0.0675; 0.8976,-0.0464),

G_3=(-0.5543,0.4875,1.4014; -1.0762,-0.0344; 0.8354,-0.0785).

Finally, Eq. (19) produces $C(G_1)$=0.3988, $C(G_2)$ =0.5210 and $C(G_3)$=0.4543. Obviously, the ranking order of the three robots is $R_1 > R_3 > R_2$. Thus, the best selection is robot R_1.

5. Conclusions

This work suggests a new fuzzy MCDM algorithm, which is established based on subtracting the summation of weighted normalized benefit ratings from that of weighted normalized cost ratings for each alternative. Using interval arithmetic of fuzzy numbers can then clearly develop the membership function of the final aggregation of fuzzy numbers. The centroid method of fuzzy number ranking has been applied to help determine ordering of all alternatives.

The suggested method provides an extension to the fuzzy MCDM techniques available. A numerical example has demonstrated feasibility and computational process of the proposed method.

Acknowledgements

This work was supported in part by Ministry of Science and Technology, Taiwan, ROC, under Grant MOST 103-2410-H-218-008-MY2.

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