

Grey-correlation Multi-attribute Decision-making Method Based on Intuitionistic Trapezoidal Fuzzy Numbers

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Abstract This study proposes a grey-correlation multi-attribute decision-making method based on intuitionistic trapezoidal fuzzy numbers to solve the problem that the attribute weight depends on the various statuses and the attribute values offer multi-attribute decision making in the form of intuitionistic trapezoidal fuzzy numbers. Firstly, this paper gives the definitions of intuitionistic trapezoidal fuzzy numbers, and the distance formula. Then, the grey-correlation coefficient about the intuitionistic trapezoidal fuzzy numbers is obtained through grey-correlation analysis. The correlation degree between different options is obtained through calculation based on the correlation coefficient. With that, the options are ranked based on the values to identify the optimal option. Finally, the result of analysis of examples demonstrates the feasibility and effectiveness of the proposed method.

Keywords Intuitionistic Trapezoidal Fuzzy Numbers, Grey-correlation Analysis, Multi-attribute Decision Making, Hamming Distance, Grey-correlation Coefficient

1. Introduction

Since the fuzzy set theory was proposed by Zadeh [1] and applied to the multi-attribute decision making, related researches have seen rapid development. In 1986, K.T. Atanassov [2] proposed the concept of intuitionistic fuzzy sets based on the fuzzy set theory. This concept was mainly characterized by consideration of degree of membership, degree of non-membership and hesitancy degree at the same time, which could better reflect the information on three statuses of the participants, such as the certainty, denying degree and hesitancy degree. It can be more expressive than traditional fuzzy sets when it comes to processing of uncertain information. In recent years, the intuitionistic fuzzy set theory has been drawing great

attention in the academic circle and the engineering technical field. Its extended forms mainly include the interval-valued intuitionistic fuzzy sets (IVIFSs), triangular intuitionistic fuzzy numbers (TIFN) and intuitionistic trapezoidal fuzzy numbers (ITFN).

The multi-attribute decision making method based on intuitionistic trapezoidal fuzzy numbers is one of the hot research topics of today. Further probing into such issues provides more objective description of a number of decision making problems. References [3-4] propose intuitionistic fuzzy sets, and further probe into the interval-valued intuitionistic fuzzy sets. Reference [5] proposes an ideal solution method to solve the problem with interval-valued intuitionistic fuzzy MADM with the attribute weight not completely known or completely unknown. Reference [6-7] defines the desired value and expected scores of intuitionistic trapezoidal fuzzy numbers, and the orderly weighted integration operator, and establishes a multi-attribute group decision making model based on ITFN. References [8-9] probes into the multi-criteria decision making problems with the criteria value being ITFN and information not completely certain and proposes related decision making methods. Reference [10] proposes an improved ITFN arithmetic operation, and defines various intuitionistic trapezoidal fuzzy aggregation operators, and applies them to group decision making. In addition, it also proposed related multi-criteria group decision making methods. Reference [11] uses the weighted possibilistic mean of the non-membership function and membership function to obtain the new ranking method of ITFN, and proposes the ITFN matrix game problem, to which the solutions are also given. Based on the above analysis, this study describes the specific steps to solve the problems with intuitionistic trapezoidal fuzzy number multi-attribute decision making based on the basic idea of grey-correlation analysis method, and proposes a decision making method based on grey-correlation analysis, which is applied to analysis of examples. The results of analysis of examples indicate that this decision making

method enriches and develops theory and methods in the multi-attribute decision making problems, and provides a new path to solve the problems with intuitionistic trapezoidal fuzzy multi-attribute decision making that the attribute weights are in different decision-making statuses.

2. Preliminaries

2.1. Intuitionistic Trapezoidal Fuzzy Numbers

Definition 1: Assume the intuitionistic trapezoidal fuzzy numbers on the real number set are $\tilde{a} = \langle ([a, b, c, d]; \mu_{\tilde{a}}), ([a', b, c, d']; \nu_{\tilde{a}}) \rangle$, and then its membership degree function can be expressed as follows [12]:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a}{b-a} \mu_{\tilde{a}} & a \leq x < b \\ \mu_{\tilde{a}} & b \leq x \leq c \\ \frac{d-x}{d-c} \mu_{\tilde{a}} & c < x \leq d \\ 0 & otherwise \end{cases} \quad (1)$$

The non-membership degree function should be:

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{b-x+(x-a')\nu_{\tilde{a}}}{b-a'} & a' \leq x < b \\ \nu_{\tilde{a}} & b \leq x \leq c \\ \frac{x-c+(d'-x)\nu_{\tilde{a}}}{d'-c} & c < x \leq d' \\ 0 & otherwise \end{cases} \quad (2)$$

Fuzzy set [1] is treated as a generalization of the classical set theory where the membership functions are only treated by their belongings or not. In fuzzy set theory, this relaxation is overlapped with the condition that it allows the partial membership between 0 and 1 also. Where, $0 \leq \mu_{\tilde{a}} \leq 1$, $0 \leq \nu_{\tilde{a}} \leq 1$, $\mu_{\tilde{a}} + \nu_{\tilde{a}} \leq 1$. When $b = c$, the intuitionistic trapezoidal fuzzy numbers are degraded into intuitionistic triangular fuzzy numbers; when $a = a'$ and $d = d'$, the above intuitionistic trapezoidal fuzzy numbers can be written as $\tilde{a} = ([a, b, c, d]; \mu_{\tilde{a}}, \nu_{\tilde{a}})$. All numbers in this paper refer to such kind of fuzzy numbers. $\mu_{\tilde{a}}$ and $\nu_{\tilde{a}}$ are respectively the maximum membership degree and the minimum non-membership degree. $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$ is the hesitancy function of \tilde{a} . A smaller value indicates a more ascertained fuzzy number.

Definition 2: Assume $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1})$ and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}_2}, \nu_{\tilde{a}_2})$ are two intuitionistic trapezoidal fuzzy numbers, then [13]:

$$1) \quad \tilde{a}_1 + \tilde{a}_2 = ([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \mu_{\tilde{a}_1} \wedge \mu_{\tilde{a}_2}, \nu_{\tilde{a}_1} \vee \nu_{\tilde{a}_2}) \quad , \quad \text{where,}$$

symbols \wedge and \vee respectively indicate the operation of taking the smaller value or the greater value through comparison between two.

$$2) \quad \tilde{a}_1 \tilde{a}_2 = ([a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2]; \mu_{\tilde{a}_1} \wedge \mu_{\tilde{a}_2}, \nu_{\tilde{a}_1} \vee \nu_{\tilde{a}_2});$$

$$3) \quad \text{If } \lambda \geq 0, \quad \lambda \tilde{a}_1 = ([\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1});$$

$$\text{If } \lambda < 0, \quad \lambda \tilde{a}_1 = ([\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1});$$

$$4) \quad \tilde{a}_1^\lambda = ([a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1}), \quad \lambda \geq 0$$

Definition 3: Assume $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1})$

and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}_2}, \nu_{\tilde{a}_2})$ are two intuitionistic trapezoidal fuzzy numbers. The Hamming distance and Euclidean distance of fuzzy numbers \tilde{a}_1 and \tilde{a}_2 are respectively as follows based on definition of Hausdorff distance [14]:

$$d_h(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4} (|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|) + \max\{|\mu_{\tilde{a}_1} - \mu_{\tilde{a}_2}|, |\nu_{\tilde{a}_1} - \nu_{\tilde{a}_2}|\} \quad (3)$$

$$d_e(\tilde{a}_1, \tilde{a}_2) = \frac{1}{2} ((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2 + \max\{|\mu_{\tilde{a}_1} - \mu_{\tilde{a}_2}|^2, |\nu_{\tilde{a}_1} - \nu_{\tilde{a}_2}|^2\})^{\frac{1}{2}} \quad (4)$$

If $\mu_1 = \mu_2 = 1$, $\nu_1 = \nu_2 = 0$, then the intuitionistic trapezoidal fuzzy numbers \tilde{a}_1 and \tilde{a}_2 are degraded into trapezoidal fuzzy numbers. Therefore, Formulas (3) and (4) are both degraded into the Hamming distance and Euclidean distance of trapezoidal fuzzy numbers.

2.2. Grey-correlation Analysis Method

In 1980s, grey system theory [15] was a system engineering discipline established based on Professor Deng Julong of Huazhong University of Science and Technology based on the mathematical theory. In particular, grey-correlation analysis method, as one of the important methods in the grey system theory, has been widely used by a number of experts and scholars. The basic idea of this method is to utilize the geometrical relationship between different scheme orders and the similarity between different curve geometries to identify the correlation degree between different schemes. If the change tendencies of two sub-schemes (factors) are consistent, then their correlation degree is high; otherwise their correlation degree is low.

3. Decision Making Methods

3.1. Problem Description

For a certain fuzzy multi-attribute decision making problem, assume the scheme set is $A = \{A_1, A_2, \dots, A_m\}$, the attribute set is $C = \{C_1, C_2, \dots, C_n\}$, the weight sector of attributes is $w = \{w_1, w_2, \dots, w_n\}$ and $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$. The attribute value of Scheme A_i under Attribute C_j can be expressed using the intuitionistic trapezoidal fuzzy numbers as follows: $\tilde{a}_{ij} = ([h_{1j}(a_i), h_{2j}(a_i), h_{3j}(a_i), h_{4j}(a_i)]; \mu_j(a_i), \nu_j(a_i))$. In particular, $\mu_j(a_i)$ and $\nu_j(a_i)$ respectively indicate the satisfaction degree or dissatisfaction degree \tilde{a}_{ij} about the evaluation values of Scheme A_i under Attribute C_j , and $0 \leq \mu_j(a_i) \leq 1, 0 \leq \nu_j(a_i) \leq 1$ and $\mu_j(a_i) + \nu_j(a_i) \leq 1$.

3.2. Steps of Algorithm

The discussion in this study is based on the grey-correlation analysis method based on ITFN multi-attribute decision making problems. The specific steps are as follows:

Step 1 Offer the intuitionistic trapezoidal fuzzy number evaluation value of each scheme under each attribute, and build its decision making matrix $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$.

Step 2 Standardized processing of the decision making matrix: To remove the impact of different dimensions on the result of decision making between attributes, the calculating formula of standardized fuzzy decision making matrix can be used to convert the intuitionistic trapezoidal fuzzy number decision making matrix \tilde{A} into a standardized decision making matrix $\tilde{R} = (\tilde{r}_{ij})_{m \times n}$, where $\tilde{r}_{ij} = ([r_{1j}(a_i), r_{2j}(a_i), r_{3j}(a_i), r_{4j}(a_i)]; \mu_j(a_i), \nu_j(a_i))$.

When it comes to multi-attribute decision making, the common attribute types include the benefit type and the cost type. A greater benefit type attribute value indicates a better condition; and a smaller cost type attribute value also indicates a better condition. The method to build a standardized intuitionistic trapezoidal fuzzy number decision making matrix is as follows:

For benefit type attributes:

$$r_{ij}(a_i) = \frac{h_j(a_i) - \min_j(h_j(a_i))}{\max_j(h_j(a_i)) - \min_j(h_j(a_i))} \quad (5)$$

For cost type attributes:

$$r_{ij}(a_i) = \frac{\max_j(h_{4j}(a_i)) - h_j(a_i)}{\max_j(h_{4j}(a_i)) - \min_j(h_{1j}(a_i))} \quad (6)$$

Where, $i = 1, 2, \dots, m; j = 1, 2, \dots, n; l = 1, 2, \dots, 4$.

Step 3 Determine the reference number series and define the reference number series as follows:

$$R_0 = (r_{01}, r_{02}, \dots, r_{0n}) \quad (7)$$

Where, $r_{0j} = ([1, 1, 1, 1]; 1, 0)$

Step 4 Calculate the Hamming distance σ_{ij} between two related elements of standardized attribute value number series and reference number series based on Definition 3.

$$\sigma_{ij} = \frac{1}{4} (|r_{1j}(a_i) - 1| + |r_{2j}(a_i) - 1| + |r_{3j}(a_i) - 1| + |r_{4j}(a_i) - 1|) + \max\{|\mu_j(a_i) - 1|, |\nu_j(a_i) - 0|\} \quad (8)$$

Step 5 Calculate the maximum value σ_{\max} and minimum value σ_{\min} of Hamming distance.

$$\sigma_{\max} = \max_{i,j} \sigma_{ij}, \quad \sigma_{\min} = \min_{i,j} \sigma_{ij} \quad (9)$$

Step 6 Calculate the grey correlation coefficient ε_{ij} between the reference number series and each optional scheme attribute value number series. Obtain the grey-correlation coefficient matrix $(\varepsilon_{ij})_{m \times n}$.

$$\varepsilon_{ij} = \frac{\sigma_{\min} + \lambda \sigma_{\max}}{\sigma_{ij} + \lambda \sigma_{\max}} \quad (10)$$

Where, λ is the resolution factor; a smaller value of λ indicates a greater resolution capacity. The value of λ is generally 0.5.

Step 7 Calculate the correlation degree r_i between the reference number series and each optional scheme attribute value number series under different decision making statuses.

$$r_i = \sum_{j=1}^m \varepsilon_{ij} \cdot w_j \quad (11)$$

In the Formula, considering the variable-weight multi-attribute decision making theory of the correlated intuitionistic trapezoidal fuzzy numbers, the weight vector w_j will have different values under different decision making statuses, which fully reflects the flexibility of the decision making process. Therefore, the correlated variable weight multi-attribute decision making method is used to obtain their correlation degree through solving the single object optimization model to calculate the weight w_j of each attribute under different decision making statuses.

Step 8 Rank the optional schemes based on the correlation degree values under different decision making statuses. A greater correlation degree value indicates a better scheme. This way, the optimal scheme can be identified.

4. Analysis of Examples

A company selects 3 enterprises in the same industry $\{A_1, A_2, A_3\}$ to improve its market competitiveness by partnering with the optimal enterprise. The company hired experts to evaluate those 3 enterprises in terms of 3 index attributes, namely, production capacity C_1 , R&D capacity C_2 and capital turnover capacity C_3 , with all attributes being of the benefit type [16]. The intuitionistic trapezoidal

fuzzy numbers decision making matrix of the schemes provided by the experts is shown in Table 1. Please find out the optimal partner enterprise. During determination of the optimal partner enterprise, the decision making method proposed in this study can be used as follows:

Table 1. Intuitionistic Trapezoidal Fuzzy Number Decision-making Matrix

Scheme	C_1	C_2	C_3
A_1	([3,5,6,8];0.5,0.4)	([2,3,4,5];0.8,0.2)	([2,4,5,7];0.7,0.1)
A_2	([1,2,3,4];0.8,0.0)	([3,4,6,8];0.5,0.4)	([3,4,6,7];0.7,0.2)
A_3	([2,3,4,6];0.7,0.2)	([1,3,5,8];0.6,0.2)	([1,2,4,6];0.7,0.2)

1) Based on **Step 2**, standardize Table 1. The result is shown in Table 2.

Table 2. Standardized Decision-making Matrix

Scheme	C_1	C_2	C_3
A_1	([0.286,0.571,0.714,1.000];0.5,0.4)	([0.143,0.286,0.429,0.571];0.8,0.2)	([0.167,0.500,0.667,1.000];0.7,0.1)
A_2	([0.000,0.143,0.286,0.429];0.8,0.0)	([0.286,0.429,0.714,1.000];0.5,0.4)	([0.333,0.500,0.833,1.000];0.7,0.2)
A_3	([0.143,0.286,0.429,0.714];0.7,0.2)	([0.000,0.286,0.571,1.000];0.6,0.2)	([0.000,0.167,0.500,0.833];0.7,0.2)

2) Calculate the Hamming distance between the optional schemes and reference number series based on **Step 4**. The results are shown in Table 3.

Table 3. Distances between Each Scheme and Reference Number Series

Attribute	C_1	C_2	C_3	$\min_i \sigma_{ij}$	$\max_i \sigma_{ij}$
σ_{1j}	0.857	0.843	0.717	0.717	0.857
σ_{2j}	0.986	0.893	0.633	0.633	0.986
σ_{3j}	0.907	0.936	0.925	0.907	0.936
σ_{\min}				0.633	
σ_{\max}					0.986

3) Calculate the grey correlation coefficient according to **Step 6** to obtain the grey correlation coefficient matrix as shown in Table 4.

Table 4. Grey-correlation Coefficient Matrix

Attribute	C_1	C_2	C_3
r_{1j}	0.834	0.843	0.931
r_{3j}	0.762	0.813	1.000
r_{2j}	0.804	0.788	0.794

4) Take a proper coefficient of balance using the solution to the single-object optimal model based on data in analysis of examples in Reference [16], with the fact that the 3 objective functions are in a fair competition, so as to obtain the weight of each attribute under different decision making statuses as shown in Table 5.

Table 5. Weight of Each Attribute under Different Decision Making Statuses

	C_1	C_2	C_3
α_1	0.346	0.209	0.233
α_2	0.359	0.472	0.376
α_3	0.173	0.420	0.336

5) Calculate the correlation degree of each scheme under different decision making statuses based on **Step 7**. The result is shown in Table 6.

Table 6. Correlation Degree under Different Decision Making Statuses and the Ranking Results

	r_1	r_2	r_3	Ranking of correlation degree
α_1	0.6818	0.6664	0.6282	$r_1 \succ r_2 \succ r_3$
α_2	1.0475	1.0330	0.9595	$r_1 \succ r_2 \succ r_3$
α_3	0.8113	0.8091	0.7371	$r_1 \succ r_2 \succ r_3$

With that, the ranking result of the schemes is as follows $A_1 \succ A_2 \succ A_3$. Therefore, the optimal partner enterprise should be A_1 . Based on the specific steps and process, the method proposed in this study takes the attribute weight in different decision-making statuses into consideration. In practical application, for many complicated decision making problems, decision-making information in different statuses is really exist, it is consistent with the reality. References [7-8] does not take the change or different of decision-making information in different decision-making statuses into consideration, does not comply with the fact. Furthermore, the proposed method is practical and has good operability, it is worth to promote in multi-attribute decision making fields.

5. Conclusions

This study proposed a grey-correlation multi-decision making method based on intuitionistic trapezoidal fuzzy numbers through probing into the multi-attribute decision making problems with the evaluation information being the intuitionistic trapezoidal fuzzy numbers. In addition, it also described the realization steps in details and demonstrated the reasonability of this method via analysis of examples.

The decision making method proposed in this study boasts advantages such as clear logic, simple calculating process and reader friendliness. In addition, this method also enjoys high application, promotion and practical decision making values, and can be applied to related decision making fields such as selection of supply chains, project evaluation, investment decision making and assessment of economic benefit.

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REFERENCES

- [1] Zadeh L A, "Fuzzy sets", Information and Control, vol. 8, no. 3, pp. 338-356, 1965.
- [2] Atanassov K T, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87-96, August 1986.
- [3] Atanassov K T, "More on intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol. 33, no. 1, pp. 37-45, October 1989.
- [4] Atanassov K T, "Operators over interval-valued intuitionistic fuzzy sets", Fuzzy Sets and Systems, vol. 64, no. 2, pp. 159-174, June 1994.
- [5] Xu Z S, "Models for multiple attribute decision-making with intuitionistic fuzzy information", Int J of Uncertainty, Fuzziness and Knowledge-Based Systems, vol. 15, no. 3, pp. 285-297, 2007.
- [6] P. Jayagowri, G. Geetharamani, "A Critical Path Problem Using Intuitionistic Trapezoidal Fuzzy Number", Applied Mathematical Sciences, vol. 8, no. 52, pp. 2555-2562, 2014.
- [7] Wan Shu-ping, Dong Jiu-ying, "Method of intuitionistic trapezoidal fuzzy number for multi-attribute group decision", Control and Decision, vol. 25, no. 5, pp. 773-776, May 2010.
- [8] Wang Jian-qiang, Zhang Zhong, "Multi-criteria decision-making method with incomplete certain information based on intuitionistic fuzzy number", Control and Decision, vol. 24, no. 2, pp. 226-230, February 2009.
- [9] Wang Jian-qiang, Zhang Zhong, "Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems". Journal of Systems Engineering and Electronics, vol. 20, no. 2, pp. 321-326, April 2009.
- [10] Wang Jian-qiang, Nie Rong-rong, "Multi-criteria group decision-making method based on intuitionistic trapezoidal fuzzy information", Systems Engineering-Theory & Practice, vol. 32, no. 8, pp. 1747-1753, August 2012.
- [11] Wan Shu-ping, Zhang Xiao-lu, "Method based on weighted possibility mean for solving matrix games with payoffs of

- intuitionistic trapezoidal fuzzy numbers”, *Control and Decision*, vol. 27, no. 8, pp. 1121-1126,1132, August 2012.
- [12] A.K. Shaw, T.K. Roy, "Trapezoidal intuitionistic fuzzy number with some arithmetic operations and its application on reliability evaluation", *International Journal of Mathematics in Operational Research*, vol. 5, no. 1, pp. 55-73, January 2013.
- [13] Wan Shu-ping, Zhang Xiao-lu, Li Deng-feng, "Possibility mean-variance based method for intuitionistic trapezoidal fuzzy group decision making", *Journal of Systems Engineering*, vol. 27, no. 6, pp. 782-788, December 2012.
- [14] Przemysław Grzegorzewski, "Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric", *Fuzzy Sets and Systems*, vol. 148, no. 2, pp. 319-328, December 2004.
- [15] Liu Si-feng, Dang Yao-guo, Fang Zhi-geng. *Grey System Theory and its Applications*, Beijing: Science Press, 2008
- [16] Gao Yan, Zhou De-qun, Zhang Ling, "Multi-attribute Decision Making with Variable Weight Relationship Based on Intuitionistic Trapezoidal Fuzzy Number", *Systems Engineering*, vol. 29, no. 5, pp. 102-107, May 2011.
- [17] Harish Garg, Nikunj Agarwal, Alka Choubey, "Entropy Based Multi-criteria Decision Making Method under Fuzzy Environment and Unknown Attribute Weights", *Global Journal of Technology and Optimization*, vol. 6, no. 3, pp. 13-20, 2015.
- [18] Yingdong He, Huayou Chen, Ligang Zhou, "Generalized intuitionistic fuzzy geometric interaction operators and their application to decision making", *Expert Systems with Applications*, vol. 41, no. 5, pp. 2484-2495, April 2014.