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Bayesian Multiperiod Forecasting for Arma Model under Jeffrey's Prior

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Abstract The main purpose of this study is to find the Bayesian forecast of ARMA model under Jeffrey's prior assumption with quadratic loss function. The point forecast model is obtained based on the mean of the marginal conditional posterior predictive in mathematical expression. Furthermore, the point forecast model of the Bayesian forecasting compared to the traditional forecasting. The simulation shows that the forecast accuracy of Bayesian forecasting is better than the traditional forecasting and the descriptive statistics of Bayesian forecasting are closer to the true value than the traditional forecasting.

Keywords ARMA Model, Bayes Theorem, Jeffrey's Prior, Multiperiod Forecast

1. Introduction

The Bayesian approach in general requires explicit formulation of a model and conditioning on known quantities in order to draw inferences about unknown ones. The main difference between the Bayesian approach and the classical approach is that in the Bayesian approach, the parameters supposed as random variables, which are described by their probability density function, whereas the classical approach considers the parameters to be fixed but unknown. The classical forecasting has been developed by Box and Jenkins [4]. There are three steps are accomplished in the process of fitting the ARMA *(p, q)* model to a time series identification of the model, estimation of the parameters, and model checking to conclude whether the models obtained are adequate for forecasting. Several of works relating to Bayesian forecasting in the ARMA model are Fan & Yao [8] and Uturbey [13] using ARMA with normal-gamma prior. Kleibergen & Hoek [11], Liu [13], and Mohamed et al. [14] using ARMA model with Jeffrey's prior. This paper focuses on the Bayesian multiperiod forecasting for ARMA model using Jeffrey's prior with quadratic loss function.

2. Materials and Methods

The materials in this paper are a set time series data in the ARMA model for application and simulation. The method is study of literature by applying a set of theories of mathematical statistics such as the ARMA model, likelihood function, Jeffrey's prior, posterior distribution, Bayes theorem, conditional predictive density, conditional posterior predictive density, marginal conditional posterior predictive density, posterior mean and point forecast. Stages of discussion are presented in Figure 1 as follows:

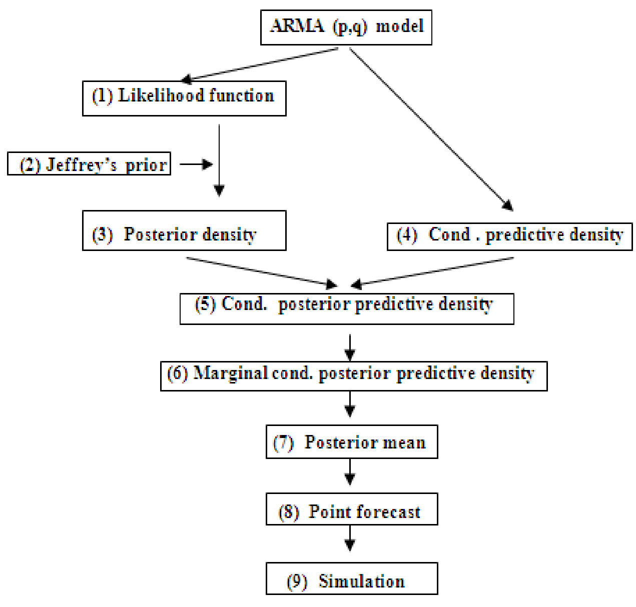

Figure 1. Stages of discussion

3. Results

3.1. Likelihood Function

The one–step–ahead point forecast of y_{n+1} based data is observed $S_n = (y_1, y_2, \ldots, y_n)$ denoted by $\hat{y}_n(1)$, and defined by:

$$\hat{y}(1) = E(y_{n+1} \mid S_n) \tag{3.1}$$

This case is expandable for the k–step–ahead point forecast of y_{n+k}, that is:

$$\hat{y}(k) = E(y_{n+k} \mid S_n^*) \tag{3.2}$$

where $S_n^* = (y_1, y_2, \ldots, y_{n+k-1})$

The ARMA (p, q) model defined by:

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + e_t \tag{3.3}$$

where $\{e_t\}$ is sequence of $i\ i\ d$ normal random variables with $e_t \sim N(0, \tau^{-1})$, $\tau > 0$ and unknown, ϕ_i and θ_j are parameters.

Residuals are as:

$$e_t = y_t - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j e_{t-j} \tag{3.4}$$

By conditioning the first p observations and letting $e_p = e_{p-1} = \ldots = e_r = 0$, where r = min(0, p+1–q), one may approximate (Box & Jenkins, [4]), the likelihood function for $\Psi = (\phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q)$ and τ based S_n^* is :

$$L(\Psi, \tau \mid S_n^*) \propto \tau^{((n+k--p)|2}$$

$$\exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1}\left(y_t - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j e_{t-j}\right)^2\right]\right\} \tag{3.5}$$

The equation (3.5) can be expressed as:

$$L(\Psi, \tau \mid S_n^*) \propto \tau^{((n+k-1)-p)|2}$$

$$\times \exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} y_t^2 - 2\Psi^T \sum_{t=p+1}^{n+k-1} y_t \mathbf{B}_{t-1} + \sum_{t=p+1}^{n+k-1}\left(\Psi^T B_{t-1}\right)^2\right]\right\} \tag{3.6}$$

where $B_t = (y_t, y_{t-1}, \ldots, y_{t+1-p}, e_t, e_{t-1}, \ldots, e_{t+1-q})$

By allowing

$$U = \begin{bmatrix} y_p & y_{p+1} & \cdots & y_{n+k-2} \\ y_{p-1} & y_p & \cdots & y_{n+k-3} \\ \vdots & \vdots & \vdots & \vdots \\ y_1 & y_2 & \cdots & y_{n+k-1-p} \\ \hat{e}_p & \hat{e}_{p+1} & \cdots & \hat{e}_{n+k-2} \\ \hat{e}_{p-1} & \hat{e}_p & \cdots & \hat{e}_{n+k-3} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{e}_{p+1-q} & \hat{e}_{p+2-q} & \cdots & \hat{e}_{n+k-1-q} \end{bmatrix},$$

$$x_0 = \begin{bmatrix} y_{p+1} \\ y_{p+2} \\ \vdots \\ y_{n+k-1} \end{bmatrix}, \quad W = UU^T \text{ and } V = Ux_0$$

where $\hat{e}_t = y_t - \sum_{i=1}^{p} \tilde{\phi}_i y_{t-i} - \sum_{j=1}^{q} \tilde{\theta}_j \hat{e}_{t-j}$, t = p+1, p+2, … , n, $\tilde{\phi}_i$ and $\tilde{\theta}_j$ are maximum likelihood estimator of ϕ_i and θ_j, $\hat{e}_t, \hat{e}_{t-1}, \ldots, \hat{e}_{t-q}$ be obtained via:

$$\hat{e}_t = y_t - \tilde{\Psi}^T \boldsymbol{B}_{t-1} \tag{3.7}$$

and the likelihood function can be expressed as:

$$L(\Psi, \tau \mid S_n^*)$$

$$\propto \tau^{((n+k-1)-p)|2} \exp\left\{-\frac{\tau}{2}\left[\sum_{t=p+1}^{n+k-1} y_t^2 - 2\Psi^T V + \Psi^T W \Psi\right]\right\} \tag{3.8}$$

3.2. Posterior Distribution under Jeffrey's Prior Assumption

Based the likelihood function in equation (3.8), the Jeffrey's prior is:

$$\pi_{Jeff}(\tau^{-1}) \propto \tau^{-1} \tag{3.9}$$

By applying the Bayes theorem to equation (3.8) and (3.9), the posterior of Ψ and τ^{-1} is:

$$\pi(\Psi, \tau^{-1} \mid S_n^*) \propto \tau^{\frac{(n+k-1-2p)+p}{2}-1} \times$$

$$\times \exp\left\{-\frac{\tau}{2}\left[\Psi^T W \Psi - 2\Psi^T V + \sum_{t=p+1}^{n+k-1} y_t^2\right]\right\} \tag{3.10}$$

3.3. Conditional Predictive Density

Based on $e_t = y_t - \sum_{i=1}^{p} \phi_i y_{t-i} - \sum_{j=1}^{q} \theta_j e_{t-j}$ with $e_t \sim N(0, \tau^{-1})$, be obtained:

$$f(e_t \mid S_n^*, \Psi, \tau^{-1}) = \left(2\pi\tau^{-1}\right)^{-\frac{1}{2}} \exp\left\{-\frac{\tau}{2}[e_t]^2\right\}, \text{ if}$$

expressed in y_t as:

$$f(y_t \mid S_n^*, \Psi, \tau^{-1}) = \left(2\pi\tau^{-1}\right)^{-\frac{1}{2}} \times$$

$$\times \exp\left\{-\frac{\tau}{2}\left[y_t - \sum_{i=1}^{p} \varphi_i y_{t-i} - \sum_{j=1}^{q} \theta_j e_{t-j}\right]^2\right\}, \text{ such}$$

that the conditional predictive density of Y_{n+k} based S_n^*, Ψ and τ^{-1} is:

$$f(y_{n+k} \mid S_n^*, \Psi, \tau^{-1}) = \left(2\pi\tau^{-1}\right)^{-\frac{1}{2}} \times \exp\left\{-\frac{\tau}{2}\left[y_{n+k} - \sum_{i=1}^{p} \varphi_i y_{n+k-i} - \sum_{j=1}^{q} \theta_j e_{n+k-j} \right]^2\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k} - \left(\sum_{i=1}^{p} \phi_i y_{n+k-i} + \sum_{j=1}^{q} \theta_j e_{n+k-j}\right)\right]^2\right\} \quad (3.11)$$

By changing $\sum_{i=1}^{p} \phi_i y_{n+k-i} + \sum_{j=1}^{q} \theta_j e_{n+k-j}$ to:

$$\begin{matrix} \varphi_1 y_{n+k-1} + \varphi_2 y_{n+k-2} + \ldots + \varphi_p y_{n+k-p} + \\ + \theta_1 e_{n+k-1} + \theta_2 e_{n+k-2} + \ldots + \theta_q e_{n+k-q} \end{matrix} =$$

$$\begin{pmatrix} \phi_1 & \phi_2 & \ldots & \phi_P & \theta_1 & \theta_2 & \ldots & \theta_q \end{pmatrix} \begin{pmatrix} y_{n+k-1} \\ y_{n+k-2} \\ \vdots \\ y_{n+k-p} \\ e_{n+k-1} \\ e_{n+k-2} \\ \vdots \\ e_{n+k-q} \end{pmatrix} =$$

$$\Psi^T B_{n+k-1}$$

where $B_{n+k-1} = \begin{pmatrix} y_{n+k-1}, & y_{n+k-2}, \ldots, & y_{n+k-p}, \\ e_{n+k-1}, & e_{n+k-2}, \ldots, & e_{n+k-q} \end{pmatrix}$, the equation (3.11) can be written as:

$$f(y_{n+k} \mid S_n^*, \Psi, \tau^{-1}) \propto \tau^{\frac{1}{2}} \times \exp\left\{-\frac{\tau}{2}\left[y_{n+k} - \Psi^T B_{n+k-1} \right]^2\right\}$$

$$\propto \tau^{\frac{1}{2}} \exp\left\{-\frac{\tau}{2}\left[y_{n+k}^2 - 2\Psi^T B_{n+k-1}\, y_{n+k} + \Psi^T R \Psi \right]\right\} \quad (3.12)$$

where $B_{n+k-1} \otimes B_{n+k-1}^T = R$ and $\left(\Psi^T B_{n+k-1}\right)^2 = \Psi^T R \Psi$.

3.4. Conditional Posterior Predictive Density

Based equation (3.10) and equation (3.12) be obtained the conditional posterior predictive density:

$$f_p(y_{n+k} \mid S_n^*, \Psi, \tau^{-1}) \propto \tau^{\frac{(n+k-1-2p)+p+1}{2}-1} \times \exp\left\{-\frac{\tau}{2}\left[\begin{matrix} \Psi^T Z \Psi - \Psi^T (V + B_{n+k-1}\, y_{n+k}) - \\ (V^T + B_{n+k-1}^T\, y_{n+k})\Psi + y_{n+k}^2 + \sum_{t=p+1}^{n+k-1} y_t^2 \end{matrix}\right]\right\} \quad (3.13)$$

where Z = W + R

3.5. Marginal Conditional Posterior Predictive Density

The marginal conditional posterior predictive density of Y_{n+k} be obtained by integrating equation (3.13) with respect to Ψ and τ^{-1}, that is:

$$f_p(y_{n+k} \mid S_n^*) = \int_0^\infty \int_0^\infty f_p(y_{n+k} \mid S_n^*, \Psi, \tau^{-1})\, d\psi\, d\tau$$

$$\propto \int_0^\infty \tau^{\frac{(n+k-1-2p)+p+1}{2}-1} \exp\left\{-\tau \times \left[\frac{y_{n+k}^2 + \sum_{t=p+1}^{n+k-1} y_t^2 - \left(V + B_{n+k-1} y_{n+k}\right)^T Z^{-1} \left(V + B_{n+k-1} y_{n+k}\right)}{2}\right]\right\} d\tau \quad (3.14)$$

By using the formula of gamma-distribution, based equation (3.14) be obtained:

$$f_p(y_{n+k} \mid S_n^*) \propto \left[n + k - 1 - p + \left\{ \frac{\left[y_{n+k} - \left(1 - B_{n+k-1}^T Z^{-1} B_{n+k-1}\right)^{-1} \left(B_{n+k-1}^T Z^{-1} V\right)\right]^2}{\frac{\sum_{t=p+1}^{n+k-1} y_t^2 - V^T Z_0 V}{(n+k-1-p)\left(1 - B_{n+k-1}^T Z^{-1} B_{n+k-1}\right)}} \right\} \right]^{-\frac{(n+k-1-p)+1}{2}} \quad (3.15)$$

The marginal conditional posterior predictive density of Y_{n+k} is a univariate student's t−distribution on *(n+k−1−p)* degrees of freedom with mean $\mu = \left(1 - B_{n+k-1}^T Z^{-1} B_{n+k-1}\right)^{-1} \left(B_{n+k-1}^T Z^{-1} V\right)$.

3.6. Point Forecast

For quadratic loss function, the point forecast of Y_{n+k} is the posterior mean of the marginal conditional posterior predictive, that is:

$$\hat{y}(k) = E\left(Y_{n+k} \mid S_n^*\right) = \\ = \left(1 - B_{n+k-1}^T Z^{-1} B_{n+k-1}\right)^{-1} \left(B_{n+k-1}^T Z^{-1} V\right) \quad (3.16)$$

4. Simulation

A simulation study was conducted to compare between the Bayesian forecasting and the traditional forecasting of the ARMA (*1, 1*) model with $\tilde{\phi}$ =0.93176 and $\tilde{\theta}$ =0.56909 for series lengths of 70, 90, 110, 130, 150, 180. 210, 240, 270 and 300 each to forecast the 10 steps ahead. The result of forecast by using the traditional method is:

$$\hat{y}(k) = \tilde{\phi}\hat{y}_{60+k-1} + \tilde{\theta}\hat{e}_{60+k-1} = \\ = 0.93176\hat{y}_{60+k-1} + 0.56909\hat{e}_{60+k-1} \quad (3.17)$$

where $\hat{e}_k = \hat{y}_k - 0.93176\ \hat{y}_{k-1} - 0.56909\ \hat{e}_{k-1}$, k=1,2,3, …, 10.

4.1. Comparison of Forecast Accuracy and Descriptive Statistics

There are measures to determine the accuracy of a forecasting model. Assis [1] in their paper present four measures, namely Root Mean square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and U-statistic defined respectively as follows:

$$RMSE = \sqrt{\frac{ESS}{n}} \quad (3.18)$$

$$MAE = \frac{\sum_{t=1}^{n} \left|Y_t - \hat{Y}_t\right|}{n} \quad (3.19)$$

$$MAPE = \frac{\sum_{t=1}^{n} \left|\frac{Y_t - \hat{Y}_t}{Y_t}\right|}{n} x100\% \quad (3.20)$$

$$U\text{-statistics} = \frac{RMSE}{\sqrt{\sum_{t=1}^{n} \frac{\hat{Y}^2}{n}} + \sqrt{\sum_{t=1}^{n} \frac{Y^2}{n}}} \quad (3.21)$$

where Y_t = the factual value at time t, $\hat{Y}_t$ = the forecast value at time t, n= the number of observations, and ESS = the error sum of square.

The comparison of forecast accuracy between Bayesian method in equation (3.16) with traditional method in equation (3.17) is presented in the Table 1. The comparison of descriptive statistics is presented in the Table 2, Columns 3 through 9 contain the minimum (Min), first quartile (Q1), median, mean, third quartile (Q3), maximum (Max), and standard deviation for N factual data, N-10 factual data and the result of Bayesian forecasting for the 10 steps ahead, and N-10 factual data and the result of traditional forecasting for the 10 steps ahead.

Table 1. Comparison of forecast accuracy

N	Bayesian				Traditional			
	RMSE	MAE	MAPE	U-STAT	RMSE	MAE	MAPE	U-STAT
70	1.2462	0.9816	0.1871	0.1239	1.6826	1.4202	0.3562	0.1501
90	3.7513	3.2171	0.3578	0.2806	4.8687	4.1934	0.4677	0.3905
110	0.6994	0.5659	0.7852	0.2188	0.8794	0.7826	0.6264	0.3084
130	0.9393	0.7804	2.5616	0.3878	0.9455	0.7961	3.0440	0.3745
150	1.1932	0.9763	0.1962	0.0847	1.5356	1.3354	0.2877	0.1068
180	2.3678	1.9732	0.2687	0.1916	4.7827	3.5014	0.5130	0.3956
210	2.0662	1.5278	0.2130	0.1884	3.2460	2.6855	0.3975	0.3314
240	0.4899	0.3047	0.7064	0.3023	0.8176	0.7675	4.8140	0.3330
270	0.4137	0.3818	1.5597	0.2640	0.6570	0.6069	1.0924	0.5617
300	0.8170	0.7052	0.6543	0.4283	0.9923	0.9278	1.0468	0.3003

Table 2. Comparison of descriptive statistics

N		Min.	Q1	Median	Mean	Q3	Max.	Stan. dev.
70	Factual data	-7.147	-4.289	-1.322	-0.9628	0.9795	8.493	4.009674
	Bayesian forecast	-7.147	-4.289	-1.322	-0.9735	0.9795	8.493	4.011988
	Traditional forecast	-7.147	-4.289	-1.322	-0.7801	0.9795	8.493	4.279630
90	Factual data	-4.65	-2.057	0.7315	0.713	5.913	10.97	4.557475
	Bayesian forecast	-4.65	-2.057	0.7315	0.355	4.955	8.424	4.097466
	Traditional forecast	-4.65	-2.057	0.7315	0.247	4.470	8.424	4.019370
110	Factual data	-7.312	-4.369	-1.465	-0.619	1.938	10.77	4.583494
	Bayesian forecast	-7.312	-4.369	-1.465	-0.648	1. 481	10.77	4.565613
	Traditional forecast	-7.312	-4.369	-1.465	-0.680	1.242	310.77	4.552436
130	Factual data	-8.151	-2.755	-0.4623	-0.2581	1.971	10.9	3.887779
	Bayesian forecast	-8.151	-2.755	-0.4623	-0.2581	1.971	10.9	3.889712
	Traditional forecast	-8.151	-2.755	-0.4623	-0.2680	1.971	10.9	3.881269
150	Factual data	-10.55	-0.686	1.298	1.0190	3.362	10.46	3.963828
	Bayesian forecast	-10.20	-0.686	1.298	0.9642	3.362	10.46	4.063910
	Traditional forecast	-10.17	-0.686	1.298	0.9532	3.362	10.46	4.107567
180	Factual data	-8.273	-2.465	0.3198	0.2020	2.994	8.262	3.915096
	Bayesian forecast	-7.341	-2.465	0.3198	0.3078	2.994	8.262	3.742356
	Traditional forecast	-7.341	-2.465	0.3198	0.3233	2.994	8.262	3.720305
210	Factual data	-11.97	-6.403	-3.571	-2.955	-0.535	8.739	4.760351
	Bayesian forecast	-11.97	-6.403	-3.571	-3.025	-0.535	8.088	4.626985
	Traditional forecast	-11.97	-6.403	-3.571	-3.083	-0.535	8.088	4.537499
240	Factual data	-4.269	-1.011	-0.0007	0.0034	0.9542	4.094	1.396144
	Bayesian forecast	-4.269	-1.011	0.03994	-0.006	0.8485	4.094	1.388480
	Traditional forecast	-4.269	-1.011	0.03994	0.03275	1.055	4.094	1.414801
270	Factual data	-1.98	0.2856	1.191	1.186	1.89	5.231	1.309002
	Bayesian forecast	-1.98	0.2866	1.182	1.179	1.89	5.231	1.310926
	Traditional forecast	-1.98	0.2383	1.176	1.166	1.89	5.231	1.317332
300	Factual data	-3.527	-1.104	-0.02213	-0.0013	0.9715	3.548	1.391662
	Bayesian forecast	-3.527	-1.156	-0.02213	-0.0322	0.9715	3.548	1.380253
	Traditional forecast	-3.527	-1.017	-0.02213	0.02062	0.9715	3.548	1.426892

5. Conclusions

The results in the Table 2 shows that the forecast accuracy value of the Bayesian method is smaller than the traditional method, it is indicates that the forecast accuracy the Bayesian forecasting is better than the traditional forecasting. The results in the Table 3 shows that descriptive statistics of Bayesian forecasting is closer to the factual data as compared to traditional forecasting, so it can be concluded that Bayesian forecasting is better than traditional forecasting.

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