

Detection of Structural Changes in Correctly Specified and Misspecified Conditional Quantile Polynomial Distributed Lag (QPDL) Model Using Change-point Analysis

Kwadwo Agyei Nyantakyi*, B. L. Peiris, L. H. P. Gunaratne

Postgraduate Institute of Agriculture, University of Peradeniya, Sri Lanka

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Abstract Change-point analysis is a powerful tool for determining whether a change has taken place or not. In this paper we study the structural changes in the Conditional Quantile Polynomial Distributed Lag (QPDL) model using change-point analysis. We employ both the Binary Segmentation (BinSeg) and Cumulative Sum (Cusum) methods for this analysis. We studied the structural changes in both correctly specified and misspecified QPDL models. As an economic application we considered the production of rubber and its price returns. We observe that both Cusum and BinSeg methods correctly detected the structural changes for both the correctly specified and the misspecified QPDL model. The Cusum method gave the exact positions where the structural changes occurred and the BinSeg gave the approximated positions where the changes occurred. Both methods were able to detect the shift in time for both the mean and variance for the misspecified QPDL model, hence both methods were better for predicting structural stability in a QPDL models. The impact of this is that, when there are changes made to a data knowingly or unknowingly, they can be detected, as well as when these changes were effected. We further observed that both methods were powerful tools that better characterizes the changes, controls the overall error rate, robust to outliers, more flexible and simple to use.

Keywords Binseg, Cusum, Structural Changes, Misspecification, Mean-shift

1. Introduction

The question of structural stability of models is very important in predicting the future in various diverse areas of science such as economics, finance, physics, geology, medicine as well as in quality control and agriculture. Questions like did a change occur? Did more than one

change occur? When did these changes occur? Can be answered by performing a change-point analysis, (Taylor, Wayne 2000a, 2000b)[13, 14].

In order to study the structural changes in models came the evolution of change-points which was introduced by the context of quality control by Csorgo and Horvath, (1997)[3].

Change-points has been employed in finding the possible changes in otherwise independent identically distributed random variables and has been extended to the stability tests of parameters of the regression functions, (Andrews, D. W. K. 1993)[1].

Killick, et al, (2012)[9], considered the common approach of detecting change-points through minimising a cost function over possible numbers and locations of change-points. They applied several established procedures for detecting changing points, such as penalised likelihood and minimum description length. In their study, they observed that Binary Segmentation is quicker as a search method, and believed this would be the case in almost all applications.

Auger and Lawrence (1989)[2] propose an alternative, exact search method for changepoint detection, namely the Segment Neighbourhood (SegNeigh) method which searches the entire segmentation space using dynamic programming. The problem with this exhaustive search method is that it has significant computational cost of $O(Q_n^2)$.

We use both the Binary Segmentation (BinSeg) and Cumulative Sum (cusum) methods for detecting the structural changes in the QPDL model because they are capable of detecting multiple changes (Mueller, 1992)[10]. The advantage of the BinSeg method is that it is computationally efficient, resulting in an $O(n \log n)$ calculation (Eckley et al, 2011)[4]. Both methods are powerful tools that better characterize the changes, control the overall error rate, robust to outliers, more flexible and simple to use, (Efron et al, 1993)[5].

The objectives of this study are, to detect the structural changes in correctly specified QPDL model, to detect the structural changes in misspecified QPDL model, and to study the mean-shift in time of the misspecified QPDL model. This study is important because as the part of our world moves into more sophisticated time periods, the accuracy of predicting the exact occurrences of future events as much as possible to reduce severe losses or impact is very important rather than using approximate estimates. It is also important in modeling to know when the exact change occurred and its impact.

2. Materials and Methods

2.1. Data Source

In order to answer the issues raised above, secondary annual data was collected from FAOSTAT, food balance sheet, price statistics, available with the Department of Census and Statistics Sri Lanka[6], and the World Bank (pink sheet)[15]. These data comprises of the production, imports, exports and prices of rubber. The rubber data ranges from 1961-2011.

2.2. Statistical Software

The R software, with the package ‘Change-point’ was used in analysing the structural changes in the conditional quantile polynomial distributed lag models.

2.3. Methodology

Testing for structural changes

We assume two (2) conditional 2nd degree polynomial $\alpha - quantile$ regimes

$$Y_{1t} = \varphi(\alpha) + a_{1,0}(\alpha)Z_{1,0t} + a_{1,1}(\alpha)Z_{1,1t} + a_{1,2}(\alpha)Z_{1,2t} + \varepsilon_{1t}, \quad t = 1, 2, \dots, T_1 \tag{1}$$

$$Y_{2t} = \varphi(\alpha) + a_{2,0}(\alpha)Z_{2,0t} + a_{2,1}(\alpha)Z_{2,1t} + a_{2,2}(\alpha)Z_{2,2t} + \varepsilon_{2t}, \quad t = 1, 2, \dots, T_2 \tag{2}$$

With $\{\varepsilon_{1t}\}$ and $\{\varepsilon_{2t}\}$ i.i.d unobservable innovations with

$$E(\varepsilon_{1t}) = 0 \text{ and } E(\varepsilon_{2t}) = 0, V(\varepsilon_{1t}) = \sigma_1^2 \text{ and } V(\varepsilon_{2t}) = \sigma_2^2$$

We further assume that $E(\varepsilon_{1t})$ and $E(\varepsilon_{2t})$ are i.i.d innovations which are independent of each other.

Let $\hat{a}_{1,0}(\alpha)$ and $\hat{a}_{2,0}(\alpha)$ be the estimates of $a_{1,0}(\alpha)$ and $a_{2,0}(\alpha)$ respectively. Then we also define

$$Y_{1t}^* = Y_{1t} - \bar{Y}_1 \text{ and } Y_{2t}^* = Y_{2t} - \bar{Y}_1$$

Then the hypothesis can be tested as follows:

Hypothesis:

$H_0: a_{1,0}(\alpha) = a_{2,0}(\alpha)$ and $H_1: a_{1,0}(\alpha) \neq a_{2,0}(\alpha)$
(Assuming $\sigma_1^2 = \sigma_2^2$)

Then under H_0 we have,

$$\left[\frac{\sigma_1^2}{\sum_{t=1}^{T_1} (Y_{1t}^*)^2} + \frac{\sigma_1^2}{\sum_{t=1}^{T_2} (Y_{2t}^*)^2} \right]^{-1/2} (\hat{a}_{1,0}(\alpha) - \hat{a}_{2,0}(\alpha)) \sim N(0, 1) \tag{3}$$

Let $(\hat{\varepsilon}_{1t})$ and $(\hat{\varepsilon}_{2t})$ be the residuals estimated respectively, then $\frac{\sum_{t=1}^{T_1} \hat{\varepsilon}_{1t}^2}{\sigma_1^2} \sim \chi_{T_1-2}^2$ and $\frac{\sum_{t=1}^{T_2} \hat{\varepsilon}_{2t}^2}{\sigma_2^2} \sim \chi_{T_2-2}^2$

and therefore we have

$$\frac{\sum_{t=1}^{T_1} \hat{\varepsilon}_{1t}^2}{\sigma_1^2} + \frac{\sum_{t=1}^{T_2} \hat{\varepsilon}_{2t}^2}{\sigma_2^2} \sim \chi_{T-4}^2 \tag{4}$$

Where we have $T_1 + T_2 = T$ and since (3) and (4) are independent we have

$$\frac{\sqrt{T-4}(\hat{a}_{1,0}(\alpha) - \hat{a}_{2,0}(\alpha))}{\left[\frac{\sigma_1^2}{\sum_{t=1}^{T_1} (Y_{1t}^*)^2} + \frac{\sigma_1^2}{\sum_{t=1}^{T_2} (Y_{2t}^*)^2} \right]^{1/2} \left[\frac{\sum_{t=1}^{T_1} \hat{\varepsilon}_{1t}^2}{\sigma_1^2} + \frac{\sum_{t=1}^{T_2} \hat{\varepsilon}_{2t}^2}{\sigma_2^2} \right]^{1/2}} \sim t_{T-4} \quad \text{and}$$

since $\sigma_1^2 = \sigma_2^2$ we have

$$\frac{(\hat{a}_{1,0}(\alpha) - \hat{a}_{2,0}(\alpha))}{\left[\frac{\sigma_1^2}{\sum_{t=1}^{T_1} (Y_{1t}^*)^2} + \frac{\sigma_1^2}{\sum_{t=1}^{T_2} (Y_{2t}^*)^2} \right]^{1/2}} \sim t_{T-4} \quad \text{where}$$

$$\hat{\sigma}^2 = (T - 4)^2 \left(\sum_{t=1}^{T_1} \hat{\varepsilon}_{1t}^2 + \sum_{t=1}^{T_2} \hat{\varepsilon}_{2t}^2 \right) \tag{5}$$

Hence H_0 can be tested using either one tail or two tail depending on the H_1 .

Now we apply the change point algorithm to see when and where the changes in the QPDL model occurred, using cumulative sum (cusum) and Binary Segmentation (BinSeg) methods.

Description of the test procedure for the detection of the change-point

Let us now consider a conditional 2nd degree polynomial $\alpha - quantile$ model with a change after an unknown time point

$$1 \leq k^* = k^*(n) \leq n$$

$$X_{it} = \begin{cases} Z_{it} & t \leq k^* \\ Y_{it-k^*} & t > k^* \end{cases} \tag{6}$$

Where $\{Y_{it}\}$ is some QPDL which differs distributionally from $\{Z_{it}\}$. The unknown parameter k^* is called the change-point.

We are now interested in the testing problem

$$H_0: k^* = n \text{ vs. } H_1: k^* < n$$

Our testing procedures are based on various functionals of the partial sums of estimated residuals with respect to the model (1)

$$\hat{S}_n(k) = \sum_{t=p+1}^k \hat{\varepsilon}_t = \sum_{t=p+1}^k (X_{it} - f(Z_{it}, \hat{\beta}_n)) \tag{7}$$

where $\hat{\beta}_n$ is the least-squares estimator of β_0 (assuming the null hypothesis holds true).

Precisely we minimize the nonlinear least squares (NLLS) with respect to β .

$$Q_n(\beta) = \sum_{t=p+1}^k (X_t - f(Z_{it}, \hat{\beta}_n))^2 = \sum_{t=p+1}^k q_n(\beta) \quad (8)$$

Thus we consider the nonlinear least squares estimator

$$\hat{\beta}_n = \arg \min_{\beta \in K} Q_n(\beta) \quad (9)$$

for a suitable compact set K . The minimization is usually obtained by solving the nonlinear score function

$$\frac{\partial Q_n(\hat{\beta}_n)}{\partial \beta} = 0. \quad (10)$$

Which yields

$$\sum_{t=p+1}^k \hat{\epsilon}_t = 0. \quad (11)$$

The behaviour of the estimator $\hat{\beta}_n$ is investigated in a variety of situations including the correctly specified case without change as well as possibly misspecified cases with and without change. According to Horvath et al (2004)[8] and Wu, (2004)[16], under appropriate assumptions $\hat{\beta}_n$ is eventually in the interior of the compact set K , so that we can assume for limit considerations.

In fact, if $\hat{\beta}_n$ is not in the interior of K , (Hinkley, D. V. 1971)[7], we will reject the null hypothesis immediately since either a change occurred or the model is not capable of modeling the observed time series sufficiently well.

Test statistics are of the form

$$T_{n1} = \max_{p < k < n} \left(\sqrt{\frac{n-p}{k(n-p-k)}} |\hat{S}_n(k)| \right), \quad (12)$$

$$T_{n2}(q) = \max_{p < k < n} \left(\frac{1}{\sqrt{n-p}q\left(\frac{k}{n-p}\right)} |\hat{S}_n(k)| \right), \quad (13)$$

$$T_{n3}(G) = \max_{p+G < k < n} \frac{1}{\sqrt{G}} (|\hat{S}_n(k) - \hat{S}_n(k-G)|), \quad (14)$$

$$T_{n1} = \max_{p+G < k < n-p-G} \frac{1}{\sqrt{2G}} (|\hat{S}_n(k+G) - 2\hat{S}_n(k) + \hat{S}_n(k-G)|), \quad (15)$$

$$T_{n4}(r) = \frac{1}{n-p} \sum_{k=p+1}^{n-1} \frac{1}{r(k/(n-p))} \left(\frac{1}{\sqrt{n-p}} \hat{S}_n(k) \right)^2, \quad (16)$$

where $q(\cdot)$ and $r(\cdot)$ are weight functions defined on $(0,1)$ specified below and $G < n$. In this case, we obtain a consistent change-point estimator which is related to the test statistics. (Stockis, J.-P et al, 2010, Tadjuidje K., J. et al, 2011)[11, 12]

3. Results and Discussions

Detection of structural Changes for the correctly specified QPDL model

Detection of structural changes in the QPDL model before misspecification are shown on tables 1 - 4, using the methods Cumulative Sum (cusum) and Binary Segmentation (BinSeg) with Schwarz Information Criterion (SIC) penalty.

Changepoint at tau(0.25)

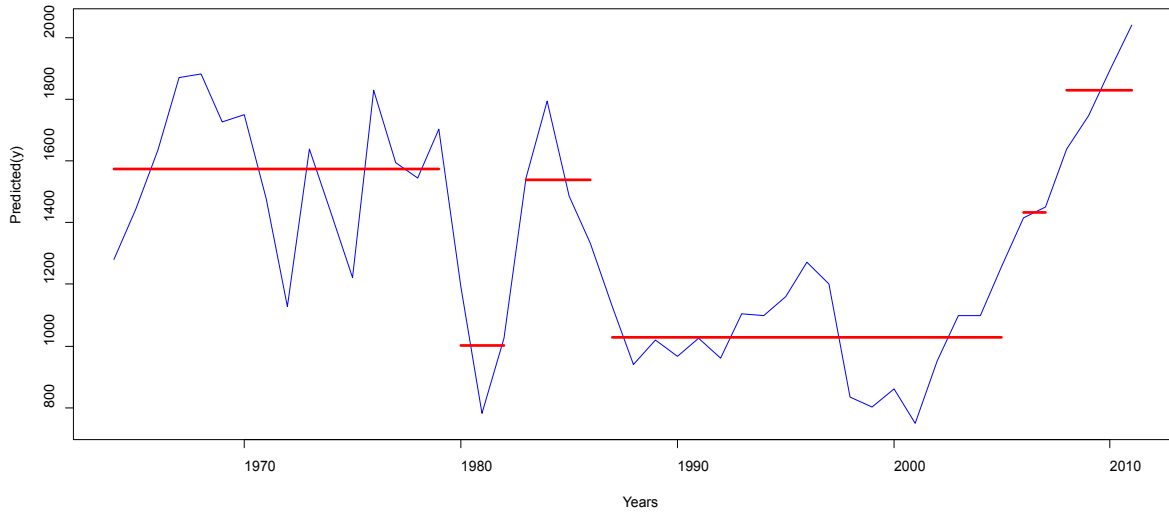


Figure 1. a plot showing the changes in production of Predicted \hat{y} against Years of tau=0.25

Table 1. Conditional quantile polynomial distributed lag (QPDL), tau=0.25 showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. (SIC (Penalty) = 7.742402)

tau	Change type	Positions of the change						Max no. of change
0.25	Cusum	16.00	41.00	44.00	46.00	24.00		5
	BinSeg	16	19	23	42	44	48	5
	Mean	1572.5	1000.9	1539.4	1028.4	1432.9	1829.5	
	Variance	52268.88	42726.38	36481.89	22493.61	570.39	30727.15	

From table 1, we observe that, both the cumulative sum (Cusum) and the Binary Segmentation (BinSeg) methods detected maximum of 5 structural changes. The Cusum gives the exact change at various positions whereas the BinSeg gives the approximated positions. The minimum change for both methods is at the 16th position and the highest maximum for Cusum is 46th and that of BinSeg is 48th position. There changes in mean production with minimum of 1000.9, occurring at 19th position and the maximum mean of 1829.5 with a change at 48th position. The variance changes associated with the changes are shown in the last row with minimum variance of 570.4 and for a mean of 1432.9 at the 44th position and a maximum variance of 52268.9 for a mean of 1572.5 at the 16th position. figure 1 showing the various positions and the time period for which each change occurred.

From figure 1, we observe the 1st change occurred in 1979, showing a high drop in average production to a record low. The 2nd change in production occurred in 1982 with a high increase in average production again. The 3rd change shows another drop in average production which lasted from 1977

to 2005 and then the 4th change in average production occurred. There was an increase in average production from 2005 to 2008 and then a change occurred, and then a subsequent increase till 2011.

Table 2, we observed that, both methods detected 5 optimum changes, with the 1st change occurring at the 16th position for both methods. The minimum average change in production 1013.1 occurred at 19th position with a variance of 148533.5 and the highest average mean production 2579 occurred at 48th position with variance of 42430. The graphical interpretation of the change is shown on figure 2.

Similarly, we observe from figure 2, the 1st change occurred in 1979, showing a high drop in average production to a record low. The 2nd change in production occurred in 1982 with a high increase in average production again. The 3rd change shows another drop in average production which lasted from 1977 to 2005 and then the 4th change in average production occurred. There was an increase in average production from 2005 to 2008 and then a change occurred, and then a subsequent increase till 2011.

Table 2. Conditional quantile polynomial distributed lag (QPDL) $\tau=0.50$ showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. (SIC (Penalty) = 7.742402)

tau	Change type	Positions of the change						Max no. of change
0.50	Cusum	16.00	41.00	45.00	47.00	3.00		5
	BinSeg	16	19	23	42	45	48	5
	Mean	2001.05	1013.08	1953.29	1177.79	1998.99	2579.01	
	Variance	137322.4	148533.48	105870.46	65529.7	27957.45	42430.54	

Changepoint at $\tau(0.50)$

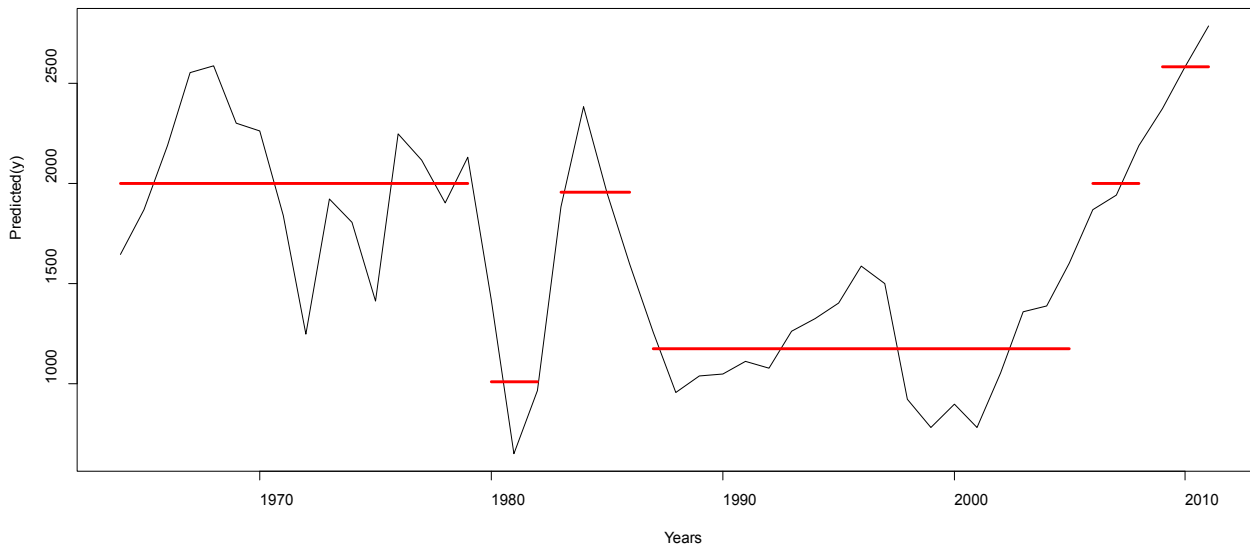


Figure 2 . a plot showing the changes in production of Predicted \hat{y} against Years of $\tau=0.50$

Table 3. Conditional quantile polynomial distributed lag (QPDL) $\tau=0.75$ showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. (SIC (Penalty) = 7.742402)

tau	Change type	Positions of the change						Max no. of change
0.75	Cusum	17.00	42.00	45.00	24.00	19.00		5
	BinSeg	17	23	42	44	48		4
	Mean	2488.207	1987.198	1362.633	2166.128	2965.220		
	Variance	164495.50	533186.09	86739.77	18553.91	146672.70		

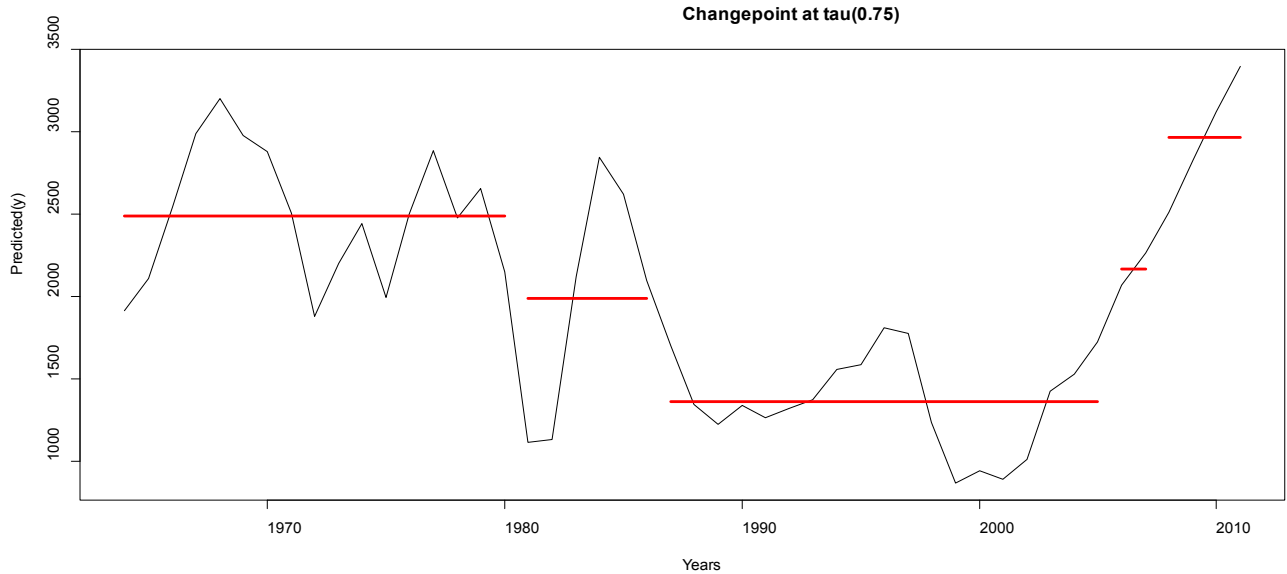


Figure 3. a plot showing the changes in production of Predicted \hat{y} against Years of $\tau=0.75$

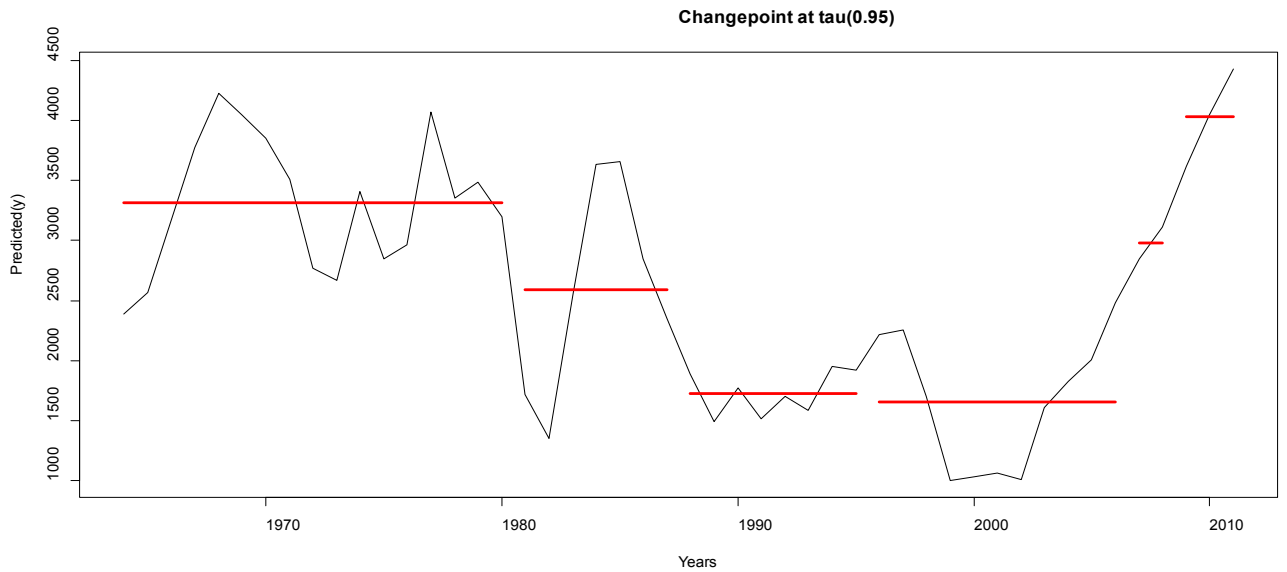


Figure 4. a plot showing the changes in production of Predicted \hat{y} against Years of $\tau=0.95$

Table 4. Conditional quantile polynomial distributed lag (QPDL) $\tau=0.95$ showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. (SIC (Penalty) = 7.742402)

tau	Change type	Positions of the change						Max no. of change
0.95	Cusum	23.000	42.000	45.000	46.000	17.000		5
	BinSeg	17	24	32	43	45	48	5
	Mean	3311.961	2586.547	1728.211	1654.949	2978.641	4032.445	
	Variance	313650.7	773252.0	33320.35	309450.68	33541.0	164135.34	

Table 3 shows an optimum of 5 changes for Cusum and 4 for BinSeg. The minimum average production 1362.6 occurred at the 42nd position with a variance of 86739.8 and the maximum average production 2965.2 occurring at the 48th position with variance of 146672.7. The graphical interpretation is shown by figure 3.

We observe from figure 3, the 1st change occurred in 1981, with a drop in average production between 1981 and 1986. The 2nd change in average production occurred in 1986 and 2005 with a high decrease in average production again. The

3rd change shows another an average increase in production which lasted from 2005 to 2007 and then the 4th change in average production occurred. There was an increase in average production from 2005 till 2011.

Table 4, we observed that, both methods detected 5 optimum changes, with the 1st change occurring at the 23rd position for Cusum and 17th position for BnSeg. The minimum average change in production was 1655.1 and occurred at 43rd position with a variance of 309451 and the highest average mean production was 4032.4 and occurred at

48th position with variance of 164135. The graphical interpretation of the change is shown on figure 4.

Similarly, we observe from figure 2, the 1st change occurred in 1980, showing a drop in average production between 1980 and 1988. The 2nd change in production occurred in 1988 with a further decrease in average production. The 3rd change shows another slight drop in average production which lasted from 1997 to 2006 and then the 4th change in average production occurred. There was an increase in average production from 2006 to 2008 and then a change occurred, and then a subsequent increase till 2011.

Detection of structural Changes for the misspecified QPDL model

Detection of structural changes in the misspecified QPDL model are shown in tables 5 - 8, using the methods Cumulative Sum (cusum) and Binary Segmentation (BinSeg) with Schwarz Information Criterion (SIC) penalty.

From table 5, we observe that, both the cumulative sum

(Cusum) and the Binary Segmentation (BinSeg) methods detected maximum of 5 structural changes. Here there is a shift in position of the 1st change from the 16th position from the correctly specified model to 23rd position for the misspecified model for both methods. The minimum change in mean production was 828.48, occurring at 39th position with variance of 2161. The maximum average production of 1494.7 with a variance of 51195.8 occurred at 23rd position. Below is figure 5 showing the various positions and the time period for which each change occurred.

Similarly, from figure 5, we observe a shift of the change in the average production which occurred in 1979 for the correctly specified QPDL model to 1988 in the misspecified QPDL model. There was high drop in the mean production between 1988 and 1996 and then an increase occurred for a short period in 1997. There was a drop again between 1998 and 2003. There was an increase in average production in 2004. There was a further high increase in production from 2005 till 2011.

Table 5. Misspecified Conditional quantile polynomial distributed lag (QPDL), tau=0.25 showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. SIC (Penalty)= 7.700295

tau	Change type	Positions of the change						Max no. of change
0.25	Cusum	23.000	41.000	45.000	43.000	15.000		5
	BinSeg	23	31	33	39	41	47	5
	Mean	1494.72	974.70	1109.92	828.48	956.63	1463.46	
	Variance	51195.76	7094.85	233.45	2160.53	905.68	49666.63	

Changepoint at ktau(0.25)

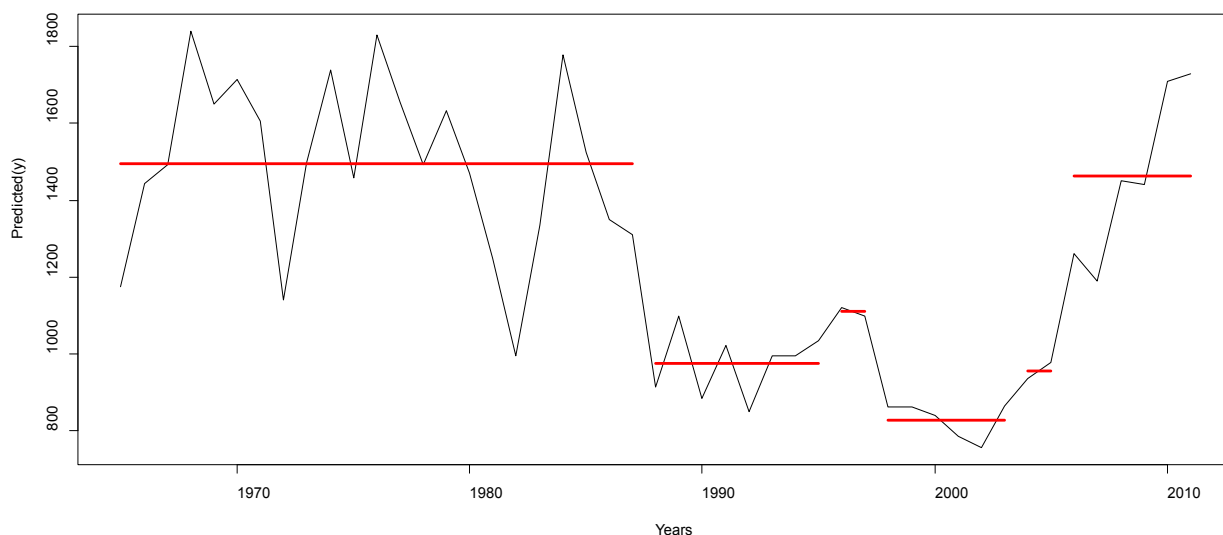


Figure 5. a plot showing the changes in production of Predicted \hat{y} against Years of tau=0.25

Table 6. Conditional quantile polynomial distributed lag (QPDL) misspecified tau=0.50 showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. SIC (Penalty)= 7.700295

tau	Change type	Positions of the change						Max no. of change
0.50	Cusum	21.000	40.000	43.000	41.000	42.000		5
	BinSeg	21	41	47				3
	Mean	1766.641	1311.461	1877.811				
	Variance	111066.87	30018.25	34896.11				

Similarly, from table 6, we observe that, for the misspecified QPDL model, the cumulative sum (Cusum) detected a maximum of 5 structural changes and the Binary Segmentation (BinSeg) methods detected only 3 structural changes at the 50th percentile. Whereas in the correctly specified model, both methods detected 5 structural changes at the 50th percentile. Here there is a shift in position of the 1st change from the 16th position from the correctly specified model to 21st position for the misspecified model for both methods. The change in minimum average production was 1311.461, occurring at 41st position with variance of 30018.25. The maximum average production of 1877.811 with a variance of 34896.11 occurred at 47rd position. Below is figure 6, showing the various positions and the time period for which each change occurred.

Similarly, from figure 6, we observe a shift of the change in the average production which occurred in 1979 for the correctly specified QPDL model to 1987 in the misspecified QPDL model. There was high drop in the mean production between 1988 and 2005 and then followed by a high increase in average production occurred from 2005 till 2011.

From table 7, we observe that, both the cumulative sum (Cusum) and the Binary Segmentation (BinSeg) methods detected maximum of 5 structural changes. Similarly, there is a shift in position of the 1st change from the 16th position from the correctly specified model to 21st position for Cusum and 15th position for BinSeg for the misspecified model. The change in minimum mean production was 1471.1, occurring at 41st position with variance of 56549. The maximum average production of 2594.9 with a variance of 44469.3 occurred at 47th position. Below is figure 7 showing the various positions and the time period for which each change occurred.

Similarly, from figure 7, we observe a shift of the change in the average production which occurred in 1981 for the correctly specified QPDL model to 1979 in the misspecified QPDL model which lasted till 1985 and then there was a further drop in average production 1987. There was high drop in the mean production between 1988 and 2005, and then an increase occurred for a short period in 2006. There was a further high increase in production from 2007 till 2011.

Changepoint at ktau(0.50)

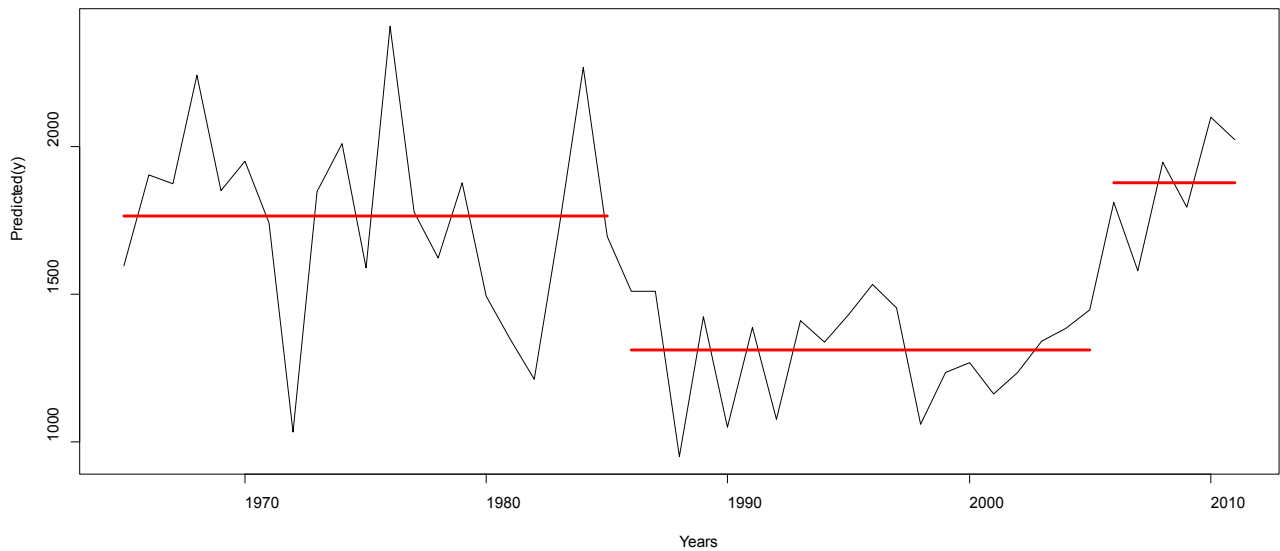


Figure 6. a plot showing the changes in production of Predicted \hat{y} against Years of tau=0.50

Table 7. Conditional quantile polynomial distributed lag (QPDL) misspecified tau=0.75 showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. SIC (Penalty)= 7.700295

tau	Change type	Positions of the change						Max no. of change
0.75	Cusum	21.000	41.000	45.000	43.000	42.000		5
	BinSeg	15	21	23	41	43	47	5
	Mean	2372.246	2048.590	1891.935	1471.076	2141.091	2594.873	
	Variance	200317.63	364103.87	189.03	56549.01	40653.84	44469.31	

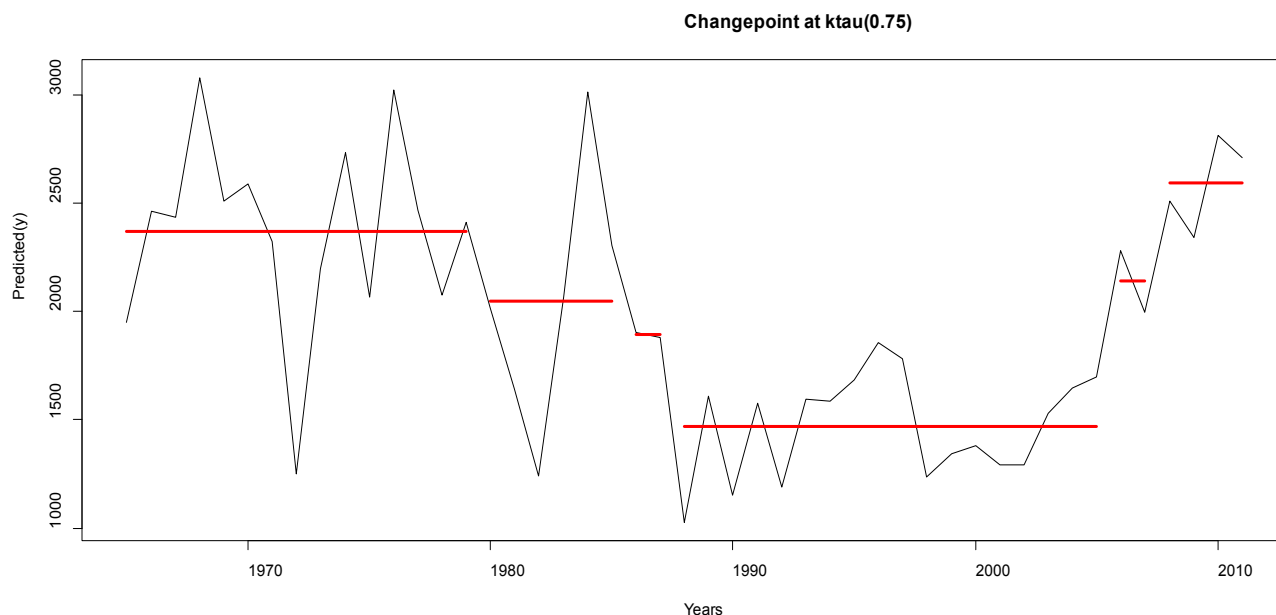


Figure 7. a plot showing the changes in production of Predicted \hat{y} against Years of $\tau=0.75$

Table 8. Conditional quantile polynomial distributed lag (QPDL) misspecified $\tau=0.95$ showing the maximum number of changes, position of the change in both the mean and variance for cusum and BinSeg methods. SIC (Penalty)= 7.700295

tau	Change type	Positions of the change						Max no. of change
		23.000	41.000	45.000	43.000	16.000		
0.95	Cusum	23.000	41.000	45.000	43.000	16.000		5
	BinSeg	16	21	23	41	43	47	5
	Mean	3774.566	3094.037	2968.266	1692.618	2822.483	3979.721	
	Variance	511265.64	1443470.48	7900.62	142103.32	24939.93	273117.81	

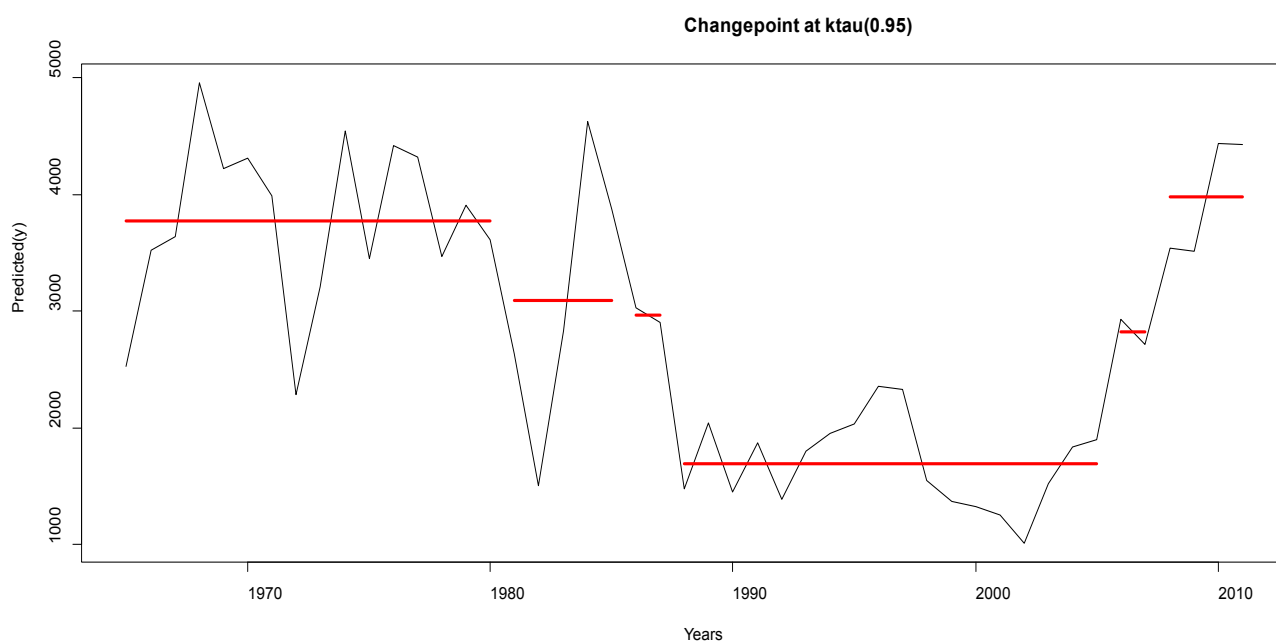


Figure 8. a plot showing the changes in production of Predicted \hat{y} against Years of $\tau=0.95$

From table 8, we observe that, both the cumulative sum (Cusum) and the Binary Segmentation (BinSeg) methods detected maximum of 5 structural changes. Similarly, there is a shift in position of the 1st change from the 23rd position for Cusum and 17th position for BinSeg from the correctly specified model to 23rd position for Cusum and 16th position

for BinSeg for the misspecified model. The change in minimum mean production was 1692.6, occurring at 41st position with variance of 142103.3. The maximum average production of 3979.7 with a variance of 273117.8 occurred at 47th position. Below is figure 8 showing the various positions and the time period for which each change

occurred.

Similarly, from figure 8, we observe a shift of the change in the average production which occurred in 1980 for the correctly specified QPDL model to 1979 in the misspecified QPDL model. There was a drop in the mean production between 1979 and 1986 and then a decrease occurred for a short period in 1987. There was a high drop again between 1988 and 2005. There was an increase in average production in 2006. There was a further high increase in production from 2007 till 2011.

4. Conclusions

From the analysis, we observed that both methods detected the exact time change and the magnitude of the change. We also observed that both the Cusum and the BinSeg methods detected the structural changes for both the correctly specified and the misspecified QPDL model. The Cusum method gives the exact positions where the structural changes occurred and the BinSeg gives the approximated positions where the changes occurred. Both methods were able to detect the shift in time for both the mean and variance for the misspecified QPDL model, hence both methods were better for predicting structural stability in a QPDL models. The impact of this is that, when there are changes made to a data knowingly or unknowingly, they can be detected, as well as when these changes were effected. We further observed that both methods were powerful tools that better characterizes the changes, controls the overall error rate, robust to outliers, more flexible and simple to use.

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