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Dynamics of Transversely Vibrating Pipes under Non-classical Boundary Conditions

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Abstract The transverse of free vibration of pipes conveying fluid has been examined by using Euler-Bernoulli beam theory to show the effect of varied boundary conditions on pipes dynamic behaviors. The equation of motion of the pipe conveying fluid is obtained with a new approach with the assumptions of ideal fluid, which moves in the vertical direction with pipe and the pipe makes small oscillations, by Hamilton's variation principle. The flow in the pipe was modeled by considering well-known Euler equation. The dimensionless equations are solved for two different set of non-classical boundary conditions. The natural frequency equations and the critical flow velocity equations are obtained and the relation between the mass ratio and vibration frequency is examined by solving the differential equations. The values of natural frequencies caused by the fluid velocity are presented graphically.

Keywords Pipes Conveying Fluid, Natural Frequency, Non-classical Boundary Conditions

1. Introduction

The dynamic behavior and stability of pipes conveying fluid has been investigated thoroughly by many researchers due to technological importance. As early as 1878 a series of experiments have been done on travelling chains and elastic cords by Aitken. Then Brillouin studied on cantilevered pipe conveying fluid (1885). Benjamin (1961) was the first to provide a comprehensive study on pipe vibrations. Housner (1952) was the first to derive the correct governing equation of motion of a pipe conveying fluid. There are many studies on pipe dynamic especially modeled as Euler Bernoulli beam in the literature. By using the Euler Bernoulli theory following studies are done: nonlinear vibration of the pipe Paidoussis (1988), analysis of vibration and stability of slightly curved pipes Sinir (2010), full curved pipes Chen (1972-1973) etc. There are also studies which used different beam theories. For example, Ergut et al.(2011) examined the pipe dynamic behavior using Rayleigh beam theory.

The linear equations of motion using Timoshenko beam theory was improved and non-dimensionalized by Paidoussis (1976)- and Laithier (1981) taking plug-flow into account for the first time.

In the present paper, dynamical behavior of pipes which modeled by using Euler–Bernoulli theory are investigated by considering two set of non-classical boundary conditions.

The equations of motion of beam have been achieved with the variational approach, which accounts for the exchange of energy between a flowing fluid and a pipe. (Han et al.,1999) Then the equations have been rendered dimensionless to obtain universal results. Non-dimensional velocity u, stiffnesses of translational springs k_1, k_3; stifnesses of rotational springs k_2, k_4, damping coefficient μ and also mass ratio β is obtained for Euler–Bernoulli pipe model. Solutions of linear differential equations have been studied by analytical methods by applying two set of non-classical boundary conditions. The non-classical boundary conditions take into account the shape deflection curve and the additional mass, the damper, as well as the translational and rotational springs at the boundaries. As a result of the analysis the effect of stifnesses of translation-rotational springs and damping coefficient on the fluid velocity and natural frequency is given in figures for each boundary condition. Besides, some critical velocities for the pipe instability have been obtained by considering Euler Theory.

2. Assumptions and Equation of Motion

The system which is studied in the present paper consists of a pipe of length L, cross-sectional area A_p, mass per unit length m, the modulus of elasticity E, density ρ_p and area moment of inertia I. The fluids terms are represented as shown; mass M , per unit length with axial flow velocity u and density of the fluid as ρ_f . The fluid is incompressible and inviscid, pipe is composed by uniform and homogeneous material. System's motion is planar. Under these assumptions the Lagrangian for Euler Bernoulli beam can be written as

$$\mathscr{L} = \frac{1}{2}\int_0^L \rho_p A_p \left(\frac{\partial v^*(x^*,t^*)}{\partial t^*}\right)^2 dx^* - \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v^*(x^*,t^*)}{\partial x^{*2}}\right)dx^* + \delta W_{nc} \tag{1}$$

The first term in equation (1) is the kinetic energies and the following term is the potential energies of the beam. Where, $v^*(x^*,t^*)$ is the transverse deflection at the axial location x^* and time t^*, δW_{nc} is virtual work done by non-conservative forces (which involves structural damping and forces) [2]. In this study, the virtual work consists of only damping of pipe and the damping is assumed to follow the Kelvin-Voigt model. The work done by non-conservative forces can be expressed as

$$\delta W_{nc} = -\int_0^L \eta I \dot{v}'' \delta v'' dx \tag{2}$$

Where η is the damping coefficient.

Using the Hamilton's principle for single dependent and two independent variables,

$$\delta \int_{t_1}^{t_2} \mathscr{L}\, dt \tag{3}$$

the well-known governing equation for Euler Bernoulli model is obtained as follows;

$$\rho_p A_p \ddot{v}^* + EIv^{*IV} + \eta I\dot{v}^{*IV} = 0 \tag{4}$$

Representation of the derivatives with respect to the spatial variable is shown by $(\ )'$ and the derivatives with respect to time shown by $(\dot{\ })$. Superscript $*$ symbols are used to indicate the dimensional quantities.

To derive the equation of motion of Euler–Bernoulli pipe, flow in lateral direction is investigated by using Euler equation. The equation for ideal fluids can be written as;

$$\frac{\partial P}{\partial y^*} = \rho_f Y - \rho_f a_y \tag{5}$$

Where, y is the vertical deflection of the pipe, P is the pressure, vertical components internal force is Y, and its acceleration of fluid in motion is a_y. The fluid and the pipe move together in vertical direction. Thus, the equation of motion of the Euler- Bernoulli pipe conveying fluid will be obtained by superposing the acceleration terms of the beam with the fluid term. The differential equation of a pipe can be expressed as

$$\rho_p A_p \ddot{v}^* + \rho_f A_f \left(\ddot{v}^* + 2u^*\dot{v}^{*\prime} + u^{*2} v^{*\prime\prime}\right) + EIv^{*IV} + \eta I\dot{v}^{*IV} = 0 \tag{6}$$

The equation is made dimensionless though the definitions

$$v = \frac{v^*}{r} \quad x = \frac{x^*}{L} \quad t = t^*\sqrt{\frac{EI}{L^4(m+M)}}$$

$$\eta = \eta^* \frac{I}{L^2\sqrt{EI(m+M)}} \quad u = u^*\frac{\sqrt{M}}{\sqrt{EI}}L \tag{7}$$

Here, r indicates the radius of gyration which is used to non-dimensionalize the displacement. Introducing dimensionless parameters, Eq. (6) becomes:

$$v^{iv} + u^2 v'' + 2\sqrt{\beta}u\dot{v}' + \ddot{v} + \eta\dot{v}^{iv} = 0 \tag{8}$$

Where β is the relative mass of fluid and can be written as

$$\beta = \frac{M}{(m+M)} \tag{9}$$

3. Analytical Solutions

In this section, we search for exact solution of homogeneous, linear ordinary differential equations with the associated boundary conditions. The dynamic behavior of the pipe can be assumed to be harmonic because the equations of motion is linear. In this case, the general solution can be given as

$$v(x,t) = X_n(x)e^{i\omega_n t} + \overline{X}_n(x)e^{-i\omega_n t} \tag{10}$$

Where, X_n represents a complex function because of the Coriolis term and ω_n. is the natural frequency.

The substitution of equation (10) into the dimensionless equation of motion gives

$$(1+i\omega_n\eta)X_n^{iv} + u^2X_n'' + 2\sqrt{\beta}ui\omega_n X_n' - \omega_n^2 X_n = 0 \tag{11}$$

The general solution of the homogeneous differential equations with the arbitrary constants has the form

$$X(x) = \sum_{j=1}^{4} C_j e^{m_j x} \tag{12}$$

In this expression, m_j and C_j are unknown terms. The equation,

$$a_4 m^4 + a_3 m^3 + a_2 m^2 + a_1 m + a_0 = 0 \tag{13}$$

is called the characteristic equation and a_i coefficients have been described for Euler-Bernoulli theory as shown below; transactions have been made suitable for the computer program.

$$a_0 = -\omega_n^2 \quad a_1 = +2\sqrt{\beta}ui.\omega_n \quad a_2 = +u^2 \quad a_3 = 0$$
$$a_4 = 1 + i\omega_n\eta \tag{14}$$

3.1. Non-classical Boundary Conditions

In this study two different cases of support at the ends of the pipe are investigated, as shown in Table 1.

The boundary conditions pairs are expressed as

$$\begin{aligned} &\text{at } x=0 && v \rightarrow \quad \eta I\dot{v}''' + EIv''' = -Mu(\dot{v} + uv') \\ & && v' \rightarrow \quad \eta I\dot{v}'' + EIv'' = 0 \\ &\text{at } x=L && v \rightarrow \quad \eta I\dot{v}''' + EIv''' = 0 \\ & && v' \rightarrow \quad \eta I\dot{v}'' + EIv'' = 0 \end{aligned} \tag{15}$$

The shear natural boundary condition at x=0 is different than at x=L because of the flow out-release effect. [2]

Parameters k_1, k_3 are stiffnesses of translational springs; k_2, k_4 are the stifnesses of rotational springs.

In the first case, pipe conveying fluid is considered with transversal and rotational springs at the left and right-hand ends. At that rate the dimensionless boundary conditions can be given as

$$\begin{aligned} x = 0 &\rightarrow \eta\dot{v}''' + v''' + \sqrt{\beta}u\dot{v} + u^2v' - k_1v = 0 \\ &\rightarrow v'' + \eta\dot{v}'' - k_2v' = 0 \\ x = L &\rightarrow \eta\dot{v}''' + v''' + k_3v = 0 \\ &\rightarrow v'' + \eta\dot{v}'' + k_4v' = 0 \end{aligned} \tag{16}$$

In the second case, pipe with transversal and rotational springs at the left- hand and elastic clamped with a concentrated mass at the right-hand end, is investigated. The dimensionless boundary conditions corresponding to Case II. Become

$$\begin{aligned} x = 0 &\rightarrow v = 0 \\ &\rightarrow \eta\dot{v}'' + v'' - k_2v' = 0 \\ x = L &\rightarrow \eta\dot{v}''' + v''' = 0 \\ &\rightarrow \eta\dot{v}'' + v'' + k_4v' = 0 \end{aligned} \tag{17}$$

Where the dimensionless parameters are

$$k_1 = k_1^* \frac{L^3}{EI}, \quad k_2 = k_2^* \frac{L}{EI}$$
$$k_3 = k_3^* \frac{L^3}{EI}, \quad k_4 = k_4^* \frac{L}{EI} \tag{18}$$

As a result of the substitution the boundary conditions into the dimensionless equation of motion, four algebraic equations are obtained to determine the arbitrary constants. The equations which gives the arbitrary constants can be represented by a matrix form as

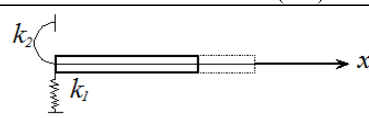
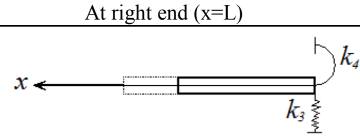
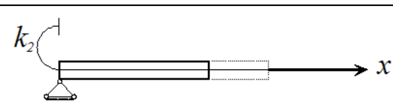
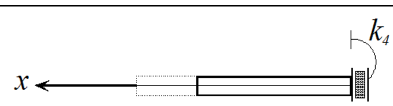
Case I.

$$\begin{bmatrix} (\eta i\omega+1)m_1^3 + u^2m_1 + \sqrt{\beta}ui\omega - k_1 & (\eta i\omega+1)m_1^3 + u^2m_1 + \sqrt{\beta}ui\omega - k_1 & (\eta i\omega+1)m_1^3 + u^2m_1 + \sqrt{\beta}ui\omega - k_1 & (\eta i\omega+1)m_1^3 + u^2m_1 + \sqrt{\beta}ui\omega - k_1 \\ (\eta i\omega+1)m_1^2 - k_3m_1 & (\eta i\omega+1)m_2^2 - k_3m_2 & (\eta i\omega+1)m_3^2 - k_3m_3 & (\eta i\omega+1)m_4^2 - k_3m_4 \\ (\eta i\omega+1)m_1^3e^{m_1} + k_3e^{m_1} & (\eta i\omega+1)m_1^3e^{m_2} + k_3e^{m_2} & (\eta i\omega+1)m_1^3e^{m_3} + k_3e^{m_3} & (\eta i\omega+1)m_1^3e^{m_4} + k_3e^{m_4} \\ (\eta i\omega+1)m_1^2e^{m_1} + k_4m_1e^{im_1} & (\eta i\omega+1)m_2^2e^{m_2} + k_4m_2e^{im_2} & (\eta i\omega+1)m_3^2e^{m_1} + k_4m_3e^{im_3} & (\eta i\omega+1)m_4^2e^{m_1} + k_4m_4e^{im_4} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

Case II.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ (\eta i\omega+1)m_1^2 - k_2m_1 & (\eta i\omega+1)m_2^2 - k_2m_2 & (\eta i\omega+1)m_3^2 - k_2m_3 & (\eta i\omega+1)m_4^2 - k_2m_4 \\ (\eta i\omega+1)m_1^3e^{m_1} & (\eta i\omega+1)m_2^3e^{m_2} & (\eta i\omega+1)m_3^3e^{m_3} & (\eta i\omega+1)m_4^3e^{m_4} \\ (\eta i\omega+1)m_1^2e^{m_1} + k_4m_1e^{m_1} & (\eta i\omega+1)m_2^2e^{m_2} + k_4m_2e^{m_2} & (\eta i\omega+1)m_3^2e^{m_3} + k_4m_3e^{m_3} & (\eta i\omega+1)m_4^2e^{m_4} + k_4m_4e^{m_4} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

Table 1. Non-classical bounday conditions

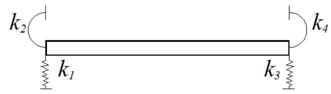
Boundary Conditions	At left end (x=0)
Case I. Elastic Clamped end	 At right end (x=L)  At left end (x=0)
Case II. Pinned end with torsional spring & sliding end with torsional spring	 At right end (x=L) 

The determinant of the matrix of coefficients must be equal to zero to obtain a non-trivial solution. In this case, the problem becomes the well-known eigenvalue-eigenvector problem. The eigenvalues represent natural frequencies and the eigenvectors are corresponded to the mode shapes.

4. Numerical Results

Numerical values of the natural frequencies are obtained for the initial three modes in this section. In Table 2. the first three frequencies are presented for elastic clamped- elastic clamped boundary condition (Case I) for various stiffnesses of translational and rotational springs. To verify the effect of fluid velocity on the natural frequencies five values of *u=0.10, 0.50, 1.00, 1.50* and *2.00* are assumed.

Table 2. Non-classical bounday conditions β=0.2, η=0.01



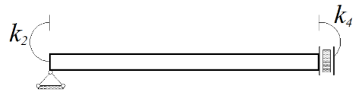
u	k_1	ω_1	ω_2	ω_3	k_2	ω_1	ω_2	ω_3
0.10	0.00	4.0804	25.2907	62.3429	0.00	1.6827	23.6354	60.4502
	10.00	1.9274	24.5766	62.0327	10.00	5.8557	29.4037	68.9634
	100.00	1.4857	20.6727	59.5307	100.00	6.4416	31.2083	71.0620
	1000.00	1.4471	18.0014	52.7894	1000.00	6.5109	31.4355	71.1243
0.50	0.00	3.8967	25.1142	62.0935	0.00	1.0363	23.4416	60.1920
	10.00	1.7829	24.3973	61.7828	10.00	5.7531	29.2561	68.7329
	100.00	1.3977	20.4966	59.2763	100.00	6.3472	31.0593	70.8149
	1000.00	1.3639	17.8759	52.5594	1000.00	6.4170	31.2854	70.8743
1.00	0.00	3.2794	24.6321	61.5267	0.00	-	22.9065	59.6020
	10.00	1.3130	23.9064	61.2146	10.00	5.4349	28.8733	68.2283
	100.00	1.0873	20.0310	58.6978	100.00	6.0638	30.6923	70.3046
	1000.00	1.0670	17.5401	52.0584	1000.00	6.1367	30.9189	70.3624
1.50	0.00	1.9226	23.8431	60.6697	0.00	-	22.0257	58.7075
	10.00	0.0001	23.1026	60.3552	10.00	4.8704	28.2621	67.4782
	100.00	-	19.2835	57.8228	100.00	5.5734	30.1181	69.5659
	1000.00	-	16.9973	51.3160	1000.00	5.6527	30.3474	69.6243
2.00	0.00	1.6189	22.7166	59.5111	0.00	-	20.7603	57.4955
	10.00	-	21.9551	59.1931	10.00	3.9705	27.4085	66.4745
	100.00	-	18.2383	56.6396	100.00	4.8219	29.3260	68.5919
	1000.00	-	16.2321	50.3256	1000.00	4.9133	29.5603	68.6532
u	k_3	ω_1	ω_2	ω_3	k_4	ω_1	ω_2	ω_3
0.10	0.00	4.0734	25.2907	62.3430	0.00	1.7038	23.6366	60.4512
	10.00	1.9224	24.5758	62.0320	10.00	5.8526	29.4021	68.9585
	100.00	1.4792	20.6480	59.5259	100.00	6.4388	31.2118	71.0746
	1000.00	1.4403	18.0065	52.8115	1000.00	6.5082	31.4401	71.1405
0.50	0.00	3.9106	25.1149	61.0939	0.00	1.2999	23.4598	60.2014
	10.00	1.6312	24.3904	61.7790	10.00	5.6613	29.2122	68.6852
	100.00	1.2058	20.4809	59.2466	100.00	6.2426	31.0272	70.8407
	1000.00	1.1685	17.8453	52.6387	1000.00	6.3113	31.2573	70.9173
1.00	0.00	3.3632	24.6345	61.5277	0.00	-	22.9749	59.6316
	10.00	0.0134	23.8837	61.2057	10.00	5.0259	28.6940	68.0744
	100.00	-	19.9160	58.6235	100.00	5.5935	30.5014	70.2632
	1000.00	-	17.3336	52.1385	1000.00	5.6604	30.7319	70.3524
1.50	0.00	2.2821	23.8486	60.6715	0.00	-	22.1793	58.7687
	10.00	-	23.0513	60.3394	10.00	3.7333	27.8484	67.1577
	100.00	-	18.9643	57.6864	100.00	4.3038	29.6319	69.3622
	1000.00	-	16.4483	51.3151	1000.00	4.3696	29.8607	69.4629
2.00	0.00	1.1356	22.7272	59.5139	0.00	-	21.0404	57.6009
	10.00	-	21.8546	59.1684	10.00	-	26.6457	65.9237
	100.00	-	17.5623	56.4197	100.00	-	28.3886	68.1262
	1000.00	-	15.1279	50.1547	1000.00	1.1590	28.6134	68.2374

As shown in Table 2, while k_1 increases and other parameters keep constant (k_2, k_3, k_4 =1), the values of the natural frequencies for the first, second and third mode are lower. When *u= 1.50*, the numerical results cannot be calculated for the values 100 and 1000 of k_1 and also natural frequencies cannot be obtained for $k_1=10$ assuming the fluid velocity as 2 at first mode. Due to the different k_3, the values of natural frequencies are similar to the values obtained with various k_1. On the other hand, natural frequencies increase with increase in value of the stiffnesses of rotational springs k_2, k_4

Table 3 shows the correlation between the values of natural frequencies and stiffnesses *(k_2, k_4)* for pinned end with torsional spring - sliding end with torsional spring. As the support translational stiffness increases, natural frequencies increase. Stiffness parameter gives less change on the natural frequencies between the values of 100 and 1000. According to Table 3, it is verified that stiffness of rotational spring slightly changes natural frequencies. When k_4 increases, values of the first, second and third dimensionless natural frequencies increase too. . The result cannot be taken at the first mode for k_2=0.00 assuming the fluid velocity as 1.50 and 2.00, for k_4=10.00, 100.00, 1000.00 in case of *u*=2.00.

The pipes conveying fluid under nonclassical boundary conditions behave as pipes with classical supports for special values of stiffnesses of translational and rotational springs. In this section variation of natural frequencies investigated for four special cases. For the analysis, k_1, k_2, k_3, k_4 are assumed as 0, ∞, ∞, 0 respectively for Case I. Under these values of coefficients, Case I corresponded to guided-pinned pipe. For free-clamped condition, k_1, k_2, k_3, k_4 were used as 0, 0, ∞, ∞ for Case İI. In Case III, k_1, k_2, k_3, k_4 were considered as ∞, ∞, 0, ∞ to correspond clamped-guided condition. The last one was designed as clamped-pinned condition assuming k_1, k_2, k_3, k_4 as ∞, ∞, ∞, 0 for Case IV. As shown in Fig. 1, the values of natural frequencies decrease with increasing fluid velocity. Due to the different pipe ends conditions, the natural frequencies are very different to each other. For Case I and II, the stifnesses of the supports give no visible change to the critical velocity. For Case IV, the natural frequency and critical velocity approach maximum values. According to the results the natural frequencies are higher for a pipe under tensile effects due to the immovable boundary conditions.

Table 3. Non-classical bounday conditions β=0.2, η=0.01



u	k_2	ω_1	ω_2	ω_3	k_4	ω_1	ω_2	ω_3
0.10	0.00	1.4155	16.9254	50.1793	0.00	1.5563	16.2004	49.3388
	10.00	3.6106	20.9099	56.0241	10.00	2.9777	21.3322	57.1109
	100.00	4.1388	23.1840	60.1037	100.00	3.1690	22.7646	59.4501
	1000.00	4.2079	23.5402	60.5189	1000.00	3.1912	22.9462	59.6483
0.50	0.00	1.3106	16.8052	49.9831	0.00	1.5329	16.0896	49.1467
	10.00	3.5756	20.8055	55.8408	10.00	2.8808	21.1979	56.8978
	100.00	4.1041	23.0729	59.9017	100.00	3.0633	22.6300	59.2656
	1000.00	4.1729	23.4264	60.3106	1000.00	3.0845	22.8122	59.4714
1.00	0.00	0.9092	16.4796	49.5437	0.00	1.4580	15.7904	48.7157
	10.00	3.4710	20.5290	55.4342	10.00	2.5598	20.8303	56.4240
	100.00	4.0060	22.7942	59.4761	100.00	2.7121	22.2461	58.8128
	1000.00	4.0747	23.1447	59.8778	1000.00	2.7298	22.4270	59.0269
1.50	0.00	-	15.9501	48.8837	0.00	1.3252	15.3040	48.0679
	10.00	3.2957	20.0847	54.8263	10.00	1.9181	20.2319	55.7151
	100.00	3.8452	22.3552	58.8541	100.00	2.0034	21.6126	58.1085
	1000.00	3.9144	22.7031	59.2495	1000.00	2.0134	21.7898	58.3293
2.00	0.00	-	15.1996	47.9949	0.00	1.1158	14.6152	47.1955
	10.00	3.0449	19.4624	54.0107	10.00	-	19.3856	54.7637
	100.00	3.6194	21.7474	58.0299	100.00	-	20.7108	57.1449
	1000.00	3.6896	22.0932	58.4198	1000.00	-	20.8817	57.3706

The natural frequencies are obtained by assuming m=0.01and m=0.1 to find out the effect of varied damping coefficients on pipes dynamic behaviors .According to Fig. 1, it is verified that the values of natural frequencies decrease as m increases for all cases. For Case I and II, variation of damping coefficient has no visible effect on the natural frequencies and critical velocities. However, lower values of critical velocities are obtained with increasing damping coefficient in Case III. According to analysis results one observes much change in critical velocity in Case IV. At that case, the ratio between two natural frequencies (i.e .the value of natural frequency when m=0.01 / the value of natural frequency when m=0.1) is $\approx$ 66.56%. Due to the pipe end conditions, importance of the damping coefficient greatly changes.

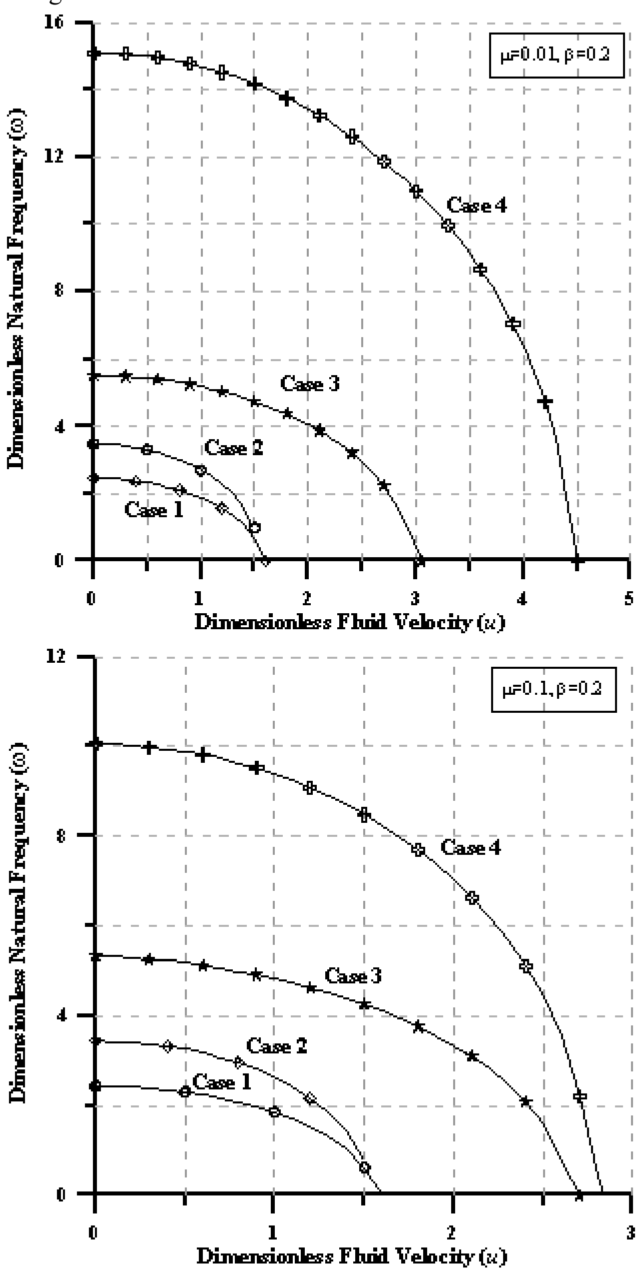


Figure 1. The natural frequency value versus fluid velocity for different mass ratios and damping coefficients

5. Conclusions

The dynamic behavior of a pipe conveying incompressible fluid supported by two different boundary conditions has been investigated in this study. The equations of motion of the pipe were obtained with a new approach with the assumptions of ideal fluid, which moves in the vertical direction with pipe and the pipe makes small harmonic oscillations. Differential equations obtained by applying the elastic clamped end, pinned end with torsional spring - sliding end with torsional spring conditions on solutions have been solved by using software program.

The correlation between the values of natural frequencies and stiffnesses has been presented as tables. According to the results the values of natural frequencies are higher under tensile effects due to the immovable end conditions. For the analysis, five different values fluid velocity were assumed to confirm the effect of fluid velocity on the natural frequencies. As shown in results, this study has proved that the frequency values decrease with increasing fluid velocity.

In addition, the study developed with analysis of special values of stiffnesses of translational and rotational springs. The critical fluid velocities for the system instability are obtained by assuming m=0.01and m=0.1. The values of critical velocity decrease with increasing damping coefficient. The rate of decrease of the natural frequencies and critical velocities due to the effect of damping coefficent has a small value for less rigid pipe systems.

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