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Skew - Commuting Derivations of Noncommutative Prime Rings

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Abstract The main purpose of this paper is study and investigate a skew-commuting and skew-centralizing d and g be a derivations on noncommutative prime ring and semiprime ring R, we obtain the derivation d(R)=0 (resp. g(R)=0) .

Keywords Skew-commuting, Derivation, Noncommutative Prime Ring, Semiprime Ring

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1. Introduction

Derivations on rings help us to understand rings better and also derivations on rings can tell us about the structure of the rings. For instance a ring is commutative if and only if the only inner derivation on the ring is zero. Also derivations can be helpful for relating a ring with the set of matrices with entries in the ring (see, [5]). Derivations play a significant role in determining whether a ring is commutative, see ([1],[3],[4],[18],[19] and [20]).Derivations can also be useful in other fields. For example, derivations play a role in the calculation of the eigenvalues of matrices (see, [2]) which is important in mathematics and other sciences, business and engineering. Derivations also are used in quantum physics(see, [18]). Derivations can be added and subtracted and we still get a derivation, but when we compose a derivation with itself we do not necessarily get a derivation. The history of commuting and centralizing mappings goes back to (1955) when Divinsky [6] proved that a simple Artinian ring is commutative if it has a commuting nontrivial automorphism. Two years later, Posner[7]has proved that the existence of a non- zero centralizing derivation on prime ring forces the ring to be commutative (Posner's second theorem).Luch [8]generalized the Divinsky result, we have just mentioned above, to arbitrary prime ring. In[9] M.N.Daif, proved that, let R be a semiprime ring and d a derivation of R with $d^3\neq 0$ .If $[d_{(}x_{)}, d_{(}y_{)}] = 0$ for all x , y $\in$ R , then R contains a non-zero central ideal. M.N.Daif and H.E. Bell [10] proved that, let R be a semiprime ring admitting a derivation d for which either xy+d(xy)= yx+d(yx) for all x , y $\in$ R or xy-d(xy) = yx-d(yx) for all x , y $\in$ R , then R is commutative. V.DeFilippis [11] proved that , when R be a prime ring let d a non-zero derivation of R , U≠(0) a two-sided ideal of R , such that d([x,y])=[x,y]for all x,y $\in$ U , then R is commutative. Recently A.H. Majeed and Mehsin Jabel [12] , give some results as , let R be a 2-torsion free semiprime ring and U a non-zero ideal of R .R admitting a non-zero derivation d satisfying d([d(x),d(y)])=[x,y] for all x,y $\in$ U. If d acts as a homomorphism, then R contains a non-zero central ideal. Our aim in this paper is to investigate skew-commuting d and g be derivations on noncommutative prime ring and semiprime ring R.

2. Preliminaries

Throughout R will represent an associative ring with identity, Z(R) denoted to the center of R ,R is said to be n-torsion free, where n ≠ 0 is an integer, if whenever n x= 0,with x $\in$ R, then x = 0. We recall that R is semiprime if xRx = (0) implies x = 0 and it is prime if xRy = (0) implies x = 0 or y = 0.A prime ring is semiprime but the converse is not true in general. An additive mapping d: R→R is called a derivation if d(xy) = d(x)y+ xd(y) holds for all x ,y $\in$ R , and is said to be n-centralizing on U (resp. n-commuting on U) , if $[x^n ,d(x)] \in Z(R)$ holds for all x $\in$ U (resp. $[x^n ,d(x)] = 0$ holds for all x $\in$ U , where n be a positive integer . Also is called skew-centralizing on subset U of R (resp. skew-commuting on subset U of R)if d(x)x+xd(x) $\in$ Z(R) holds for all x $\in$ U (resp.d(x)x+xd(x)=o holds for all x $\in$ U),and d acts as a homomorphism on U(resp. anti-homomorphism on U) if d(xy)=d(x)d(y)holds for all x,y $\in$ U(resp. if d(xy)= d(y)d(x) holds for all x,y $\in$ U).We write [x,y] for xy – yx and make extensive use of basic commutator identities [xy,z]=x[y,z]+ [x,z]y and [x,yz]=y [x,z] +[x,y]z . In some parts of the proof our theorems(3.1and 3.2), we using same technique in [21].

First we list the lemmas which will be needed in the sequel.

Lemma1[7]

If d is commuting derivation on noncommutative prime ring, then d=0.

Lemma 2 [13:Theorem1.2]

Let S be a set and R a semiprime ring. If functions d and g of S into R satisfy $d(s)xg(t)=g(s)xd(t)$ for all $s,t\in S$, $x\in R$, then there exists idempotents $\alpha_1, \alpha_2, \alpha_3, \in C$ and an invertible element $\lambda\in C$ such that $\alpha_i\alpha_j=0$, for $i\neq j$,

$\alpha_1+\alpha_2+\alpha_3=1$,and $\alpha_1 d(s)=\lambda\alpha_1 g(s)$, $\alpha_2 g(s)=0$, $\alpha_3 d(s)=0$ hold for all $s\in S$.

Lemma 3[14: Theorem 2]

Let R be a 2-torsion free semiprimering. If an additive mapping $g:d\to d$ is skew-commuting on R, then d=0.

Lemma 4 [15: Lemma 4]

Let R be a semiprimering and U a non-zero ideal of R .If d is a derivation of R which is centralizing on U, then d is commuting on U.

3. The Main Results

Theorem 3.1

Let R be a noncommutative prime ring, d and g be a derivations of R. If R admits to satisfy $d(x)x+xg(x) \in Z(R)$ for all $x\in R$, then d(R)=0 (resp. g(R)=0) or wd (resp. wg) is central for all $w\in Z(R)$.

Proof: At first we suppose there exists an element say $w\in R$, such that

$w\in Z(R)$. Let w be a non-zero element of Z(R).By linearizing our relation

$d(x)x+xg(x) \in Z(R)$, we obtain

$d(x)y+d(y)x+xg(y)+yg(x) \in Z(R)$ for all $x,y\in R$. (1)

Taking y=w in (1) , we get

$d(x)w+ d(w)x+xg(w)+wg(x) \in Z(R)$ for all $x\in R$. (2)

Again in (1) replacing y by w^2, we obtain

$d(x)w^2+d(w^2)x+xg(w^2)+w^2g(x) \in Z(R)$ for all $x\in R$, since d and g be a derivations of R and $w \in Z(R)$, we obtain

$d(x)w2+2wd(w)x+x(2wg(w))+w2g(x)\in Z(R)$. (3)

for all $x\in R$, then

$w(d(w)x+xg(w))+w(d(x)w+d(w)x+xg(w)+wg(x)) \in Z(R)$

for all $x\in R$. (4)

According to (2) the relation (4) gives

$w(d(w)x+xg(w)) \in Z(R)$ for all $x\in R$. Thus

$[w(d(w)x+xg(w)),y]=0$ for all $x,y\in R$. Then

$w[d(w)x+xg(w),y]+[w,y](d(w)x+xg(w))=0$

for all $x,y\in R$. (5)

Since $w\in Z(R)$, then (5) gives

$w[d(w)x+xg(w),y]=0$ for all $x,y\in R$. (6)

Also from (2), we obtain

$[d(x)w+d(w)x+xg(w)+wg(x),y]=0$ for all $x,y\in R$. Then

$[d(x)w+wg(x),y]=-[d(w)x+g(w),y]$ for all $x,y\in R$. (7)

Now from (6) and (7), we obtain

$w[d(x)w+wg(x),y]=0$ for all $x,y\in R$. Since $w\in Z(R)$, this relation gives

$w2[d(x)+g(x),y]=0$ for all $x,y\in R$. (8)

Replacing y by zy, with using (8), we get

$w^2z[d(x)+g(x),y]=0$ for all $x,y\in R$, which implies

$wzw[d(x)+g(x),y]=0$ forall$x,y\in R$. (9)

Replacing z by $[d(x)+g(x),y]z$ and since R is prime ring , which implies

$w[d(x)+g(x),y]=0$ for all $x,y\in R$. Then

$[w(d(x)+g(x)),y]=0$ for all $x,y\in R$. Thus

$w(d(x)+g(x)) \in Z(R)$ for all $x \in Z(R)$ for all $x\in R$. (10)

Since $w\in Z(R)$ and d,g are derivations, therefore, wd, wg and w(d+g) are derivations of R.

Further, from (10) we obtain w(d+g) is central and Lemma4,we obtain a commuting derivation. Then by [16:Proposition 2.3] , we get

$(w(d+g))(u)[x,y]=0$ for all $u,x,y\in R$. (11)

From (11) and the fact that $wd(u)+wg(u) \in Z(R)$, we obtain

$[(wd(u)+wg(u))u,y]=(wd(u)+wg(u))[u,y]+[wd(u)+wg(u),y]u=0$

for all $u,y\in R$. Then

$[wd(u)u+wg(u)u,y]=0$ for all $u,y\in R$. (12)

Since $w\in Z(R)$ and $d(u)u+ug(u) \in Z(R)$, therefore,

$wd(u)u+ wug(u) \in Z(R)$, thus (12) implies that

$[wd(u)u+wug(u),y]=0$ for all $u,y\in R$. (13)

Subtracting (13) and (12), we obtain

$[wg(u)u-wug(u),y]=0$ for all $u,y\in R$. Then

$[w(g(u)u-ug(u)),y]=0$ for all $u,y\in R$.

$[w[g(u),u],y]=0$ for all $u,y\in R$.

$[[wg(u),u],y]=0$ for all $u,y\in R$ .Thus, we obtain

$[wg(u),u] \in Z(R)$ for all $u\in R$. Therefore, we obtain

wg is a centralizing derivation. By Lemma 4, we get wg is a commuting derivation. Also by [16:Proposition2.3], we get that $wg(u) \in Z(R)$ for all $u\in R$. Since $wd(u)+wg(u) \in Z(R)$ and $wg(u) \in Z(R)$ for all $u\in R$, therefore, we obtain $wd(u) \in Z(R)$ for all $u\in R$, thus $[wd(u),u]=0$ for all $u\in R$.

Then $w[d(u),u]+[w,u]d(u)=0$ for all $u \in R$. Since we suppose that $w \in Z(R)$, above relation reduces to $w[d(u),u]=0$ for all $u\in R$.

Left-multiplying by r, we get

$wr[d(u),u]=0$ for all $u,r\in R$. Then

$wR[d(u),u]=(0)$.Since R is prime ring and $w\neq 0$,we arrive to

$[d(u),u]=0$ for all $u\in R$. By apply Lemma 1, we obtain

d(R)=0 (resp.g(R)=0).

If w=0, then obviously, we obtain d(R) (resp.g(R)) is central for all $w \in Z(R)$.

Theorem 3.2

Let R be a noncommutative prime ring, d be a skew-centralizing derivation of R (resp. g be a skew-centralizing derivation of R), if R admits to satisfy $d(x)x+xg(x) \in Z(R)$ for all $x \in R$. Then $d(R)=0$ (resp. $g(R)=0$).

Proof: Let $x_o \in R$ and $c=d(x_o)x_o+x_og(x_o)$.Thus , according to our hypothesis, we obtain $c \in Z(R)$.Then by Theorem 3.1, we get cd and cg are commuting , then $[cd(x),y]=0$ for all $x,y \in R$. Then

$cd(x)y= ycd(x)$ for all $x,y \in R$.

Since $c \in Z(R)$, then above relation become

$d(x)yc=cyd(x)$ for all $x,y \in R$. (14)

Now taking S=R, g(x)=c with applying Lemma2 to (14), we obtain that there exist idempotents $\alpha_1, \alpha_2, \alpha_3 \in C$ and an invertible element $\lambda \in C$ such that

$\alpha_i \alpha_j=0$ for $i \neq j$ $\alpha_1+\alpha_2+\alpha_3=1$, and $\alpha_1 d(x)=\lambda \alpha_1 c$, $\alpha_2 c=0$, $\alpha_3 d(x)=0$ for all $x \in R$. (15)

For the first identity of (15) replacing x by xy and using it again, we obtain

$\lambda\alpha_1 c= \alpha_1 d(xy)= \alpha_1 d(x)y+x\alpha_1 d(y)=\lambda\alpha_1 cy+x\lambda\alpha_1 c$ for all $x,y \in R$. Then

$\lambda\alpha_1 c=\lambda\alpha_1 cy+x\lambda\alpha_1 c$ for all $x,y \in R$. (16)

Replacing y by –x in (16), we obtain

$\lambda\alpha_1 c=\lambda \alpha_1 c(-x)+x\alpha_1 c=-x\lambda\alpha_1 c+x \lambda\alpha_1 c=0$ for all $x \in R$. Thus, we get

$\lambda\alpha_1 c=0$. Therefore, the first identity of (15) become

$\lambda\alpha_1 c= \alpha_1 d(x)$ for all $x \in R$. Hence, using (15), we obtain $d(x)=(\alpha_1+\alpha_2+\alpha_3)d(x)=\alpha_2 d(x)$ for all $x \in R$. Then

$cd(x)=c \alpha_2 d(x)= \alpha_2 cd(x)$ for all $x \in R$. Then, from second identity in (15), we obtain $cd(x)=0$ for all $x \in R$. Since cg is commuting, then cg is central, therefore, analogously, it follows that $cg(x)=0$ for all $x \in R$. Hence

$cd(x)x=0$ and $xcg(x)=cxg(x)=0$ for all $x \in R$. Thus from these relations, we obtain $c(d(x_0x+xg(x))=0$ for all $x \in R$.

In particular, $c(d(x_o)x_o+x_og(x_o))=c^2=o$. Since a semiprime ring has no nonzero central nilpotent, therefore, we get c=o, which implies $d(x_o)x_o+x_og(x_o)=0$. Since x_o is an arbitrary element of R, therefore

$d(x)x+xg(x)=0$ for all $x \in R$. (17)

If we taking d(x)=g(x), then

$d(x)x+xd(x)=0$ for all $x \in R$. Then by using Lemma 3, we obtain $d(R)=0$ (resp. $g(R)=0$).

If $d(x) \neq g(x)$,this case lead to $d(x)x+xg(x) \in Z(R)$ for all $x \in R$. By Theorem3.1, we complete our proof.

Theorem 3.3

Let R be a 2-torsion free semiprime ring with cancellation property. If R admits a derivation d to satisfy

(i) d acts as a skew-commuting on R.

(ii) d acts as a skew-centralizing on R . Then d(R) is commuting of R.

Proof: (i) Since d is skew-commuting , then

$d(x)x+xd(x)=o$ for all $x \in R$. (18)

Left –multiplying (18) by x,we obtain

$xd(x)x+x2d(x)=o$ for all $x \in R$. (19)

From (18),we get $d(x2)=o$ for all $x \in R$. (20)

In (20) replacing x by x+y,we obtain

$d(x^2)+d(xy)+d(yx)+d(y^2)=o$ for all $x,y \in R$.

A according to (20) ,a above equation become

$d(xy)+d(yx)=o$ for all $x,y \in R$.Then

$d(x)y+xd(y)+d(y)x+yd(x)=o$ for all $x,y \in R$.

Replacing y by x^2 and according to(20),we arrived to

$d(x)x2+x2d(x)=o$ for all $x \in R$. (21)

Then

$x2d(x)=-d(x)x2$ for all $x \in R$.

By substituting (21) in (19), we get

$xd(x)x-d(x)x^2=o$ for all $x \in R$. Then

$[x,d(x)]x=o$ for all $x \in R$. Then apply the cancellation property on x ,

we get ,we obtain

$[x,d(x)]=o$ for all $x \in R$. Then d(R) is commuting of R.

(ii) We will discuss, when d acts as a skew- centralizing on R.

Then we have $d(x)x+xd(x) \in Z(R)$ for all $x \in R$.

$d(x^2) \in Z(R)$ for all $x \in R$. i.e.

$[d(x2),r]=o$ for all $x,r \in R$. (22)

Also, by replacing r by x in(22) ,we obtain

$(d(x)) x+xd(x)) x=x(d(x)x+xd(x))$ for all $x \in R$

Then $d(x) x^2+xd(x)x= xd(x)x+x^2d(x)$ for all $x \in R$.

Then

$d(x)x^2-x^2 d(x) =o$ for all $x \in R$. Then

$[d(x),x2]=o$ for all $x \in R$. (23)

In(22) ,replacing x by x+y ,we obtain

$[d(x^2)+d(xy) +d(ys)+d(y^2),r]=o$ for all $x,y,r \in R$.

According to(22),we obtain

$[d(x)y+xd(y)+d(y)x+yd(x),r]=o$ for all $x,y,r \in R$.

Replacing y by x^2, we obtain

$[d(x)x^2+xd(x^2)+d(x^2)x+x^2d(x),r]=o$ for all $x,r \in R$.

According to(22) and (23) ,we get

$[x^2d(x)+xd(x^2)+xd(x^2)+x^2d(x),r]=o$ for all $x,r \in R$. Then

$2[x^2d(x)+xd(x^2),r]=o$ for all $x,r \in R$.

Since R is 2-torsion free, we obtain

$[x^2d(x)+xd(x^2),r]=o$ for all $x,r \in R$. Then

$[x(xd(x)+d(x^2)),r]=o$ for all $x, r \in R$. Then

$x[xd(x)+d(x^2),r]+[x,r](xd(x)+d(x^2))=o$ for all $x ,r \in R$.

According to (22), above equation become

$x[xd(x),r]+[x,r](xd(x)+d(x^2))=o$ for all $x,r \in R$. Replacing r by x ,we obtain $x[xd(x),x]=o$ for all $x \in R$.

Then $x^2[d(x),x]=o$ for all $x \in R$. Apply the cancellation property on x^2, we get $[d(x), x]=o$ for all $x \in R$. We complete the proof of theorem.

Theorem 3.4

Let R be a 2-torsion free noncommutative prime ring. If R admits a derivation d to satisfy one of following

(i) d acts as a homomorphism on R. Then d(R)=0.

(ii) d acts as an anti-homomorphism on R. Then d(R)=0.

Proof: (i) d acts as a homomorphism on R. We have d is a derivation, then

d(xy)=d(x)y+xd(y) for all x, y $\in$ R. Then

[d(xy),r]=[d(x)y,r]+[xd(y),r] for all x,y,r$\in$R. Since d acts as a homomorphism, then [d(x)d(y),r]=[d(x)y,r]+[xd(y),r] for all x,y,r$\in$R. Replacing r by d(y),we obtain

[d(x),d(y)]d(y)=[d(x)y,d(y)]+[xd(y),d(y)] for all x,y $\in$ R .Then

[d(x),d(y)]d(y)=d(x)[y,d(y)]+[d(x),d(y)]y+[x,d(y)]d(y) for all x,y$\in$ R .

Replacing y by x,we obtain d(x)[x,d(x)]+[x,d(x)]d(x)=o for all x$\in$ R.

Then $[d(x)^2,x]$=o for all x$\in$ R. Then $d(x)^2 \in Z(R)$ for all x $\in$ R. Since d acts as a homomorphism, then $d(x^2) \in Z(R)$ for all x$\in$ R, i.e. d(x)x+xd(x) $\in$Z(R) for all x$\in$ R, then d is skew-centralizing on R. Then by Theorem3.2, we obtain d(R)=0.

The proof of (ii) is similar. We complete the proof of theorem.

Remark 3.5

In Theorem 3.3 and Theorem 3.4, we can't exclude the condition char.R≠2, as it is shown in the following example.

Example 3.6

Let R be the ring of all 2×2 matrices over a field F with char. R=2, let a=$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, R ={$\begin{pmatrix} a & b \\ b & a \end{pmatrix}$/a,b$\in$ F}.Let d be the inner derivation given by:

d(x)=x$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$-$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$x ,when x$\in$ R ,then x=$\begin{pmatrix} a & b \\ b & a \end{pmatrix}$,

therefore, d(x)=$\begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$. Then

$$d(x)x+xd(x)=\begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}+\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}$$

$$=\begin{pmatrix} b^2 & ab \\ -ba & -b^2 \end{pmatrix}+\begin{pmatrix} -b^2 & ab \\ -ba & b^2 \end{pmatrix}$$

$=\begin{pmatrix} 0 & 2ba \\ -2ba & 0 \end{pmatrix}$.Since char. R=2, then

$=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.Then d is skew- centralizing and skew-commuting on R, i.e. d(R)=o.

Also when we have d acts as homomorphism (resp. acts as an anti-homomorphism).

$$d(x)d(x) =d(\begin{pmatrix} a & b \\ b & a \end{pmatrix})d(\begin{pmatrix} a & b \\ b & a \end{pmatrix})$$

$$=d(\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix})$$

$$=d(\begin{pmatrix} a^2+b^2 & ab+ba \\ ba+ba & a^2+b^2 \end{pmatrix})$$

$=\begin{pmatrix} 0 & 2ba \\ -2ba & 0 \end{pmatrix}$.Since char. R=2, we obtain

$=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Thus d is skew–centralizing and skew-commuting on R, i.e. d(R)=o.

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