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Blow up of Solutions for a System of Nonlinear Higher-order Kirchhoff-type Equations

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Abstract In this work, we consider the initial boundary value problem for the Kirchhoff-type equations with damping and source terms

$$\begin{cases} u_{tt} + M\left(\int_{\Omega} \left|(-\triangle)^{\frac{m}{2}} u\right|^2 dx\right)(-\triangle)^m u + |u_t|^{p-1} u_t = f_1(u,v), \\ v_{tt} + M\left(\int_{\Omega} \left|(-\triangle)^{\frac{m}{2}} v\right|^2 dx\right)(-\triangle)^m v + |v_t|^{q-1} v_t = f_2(u,v) \end{cases}$$

in a bounded domain. We prove the blow up of the solution with positive initial energy by using the technique of [26] with a modification in the energy functional due to the different nature of problems. This improves earlier results in the literature [3, 9, 13, 21].

Keywords Blow up, Higher-order Kirchhoff Type Equations, Nonlinear Damping and Source Terms

Mathematics Subject Classification (2010): 35B44, 35G31

1 Introduction

We consider the initial-boundary value problem for the following coupled nonlinear higher-order Kirchhoff-type equations with damping and source terms

$$\begin{cases} u_{tt} + M\left(\left\|P^{\frac{1}{2}} u\right\|^2\right) Pu + |u_t|^{p-1} u_t = f_1(u,v), & (x,t) \in \Omega \times (0,T), \\ v_{tt} + M\left(\left\|P^{\frac{1}{2}} v\right\|^2\right) Pv + |v_t|^{q-1} v_t = f_2(u,v), & (x,t) \in \Omega \times (0,T), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x),\ v_t(x,0) = v_1(x), & x \in \Omega, \\ \frac{\partial^i u}{\partial \nu^i} = \frac{\partial^i v}{\partial \nu^i} = 0,\ i = 0,1,...,m-1, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $P = (-\triangle)^m$, $m \geq 1$ is a natural number, $p, q \geq 1$ are real numbers, Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n, ν is the outer normal, and $M(s) = \beta_1 + \beta_2 s^{\gamma}$, $s, \gamma \geq 0$, $\beta_1, \beta_2 > 0$. Without loss of generality, we can assume that $\beta_1 = \beta_2 = 1$ in the problem (1.1).

In [6], Kirchhoff firstly proposed a model given by the equation

$$\rho h \frac{\partial^2 u}{\partial t^2} + \delta \frac{\partial u}{\partial t} + g\left(\frac{\partial u}{\partial t}\right) = \left\{\rho_0 + \frac{Eh}{2L}\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right\}\frac{\partial^2 u}{\partial x^2} + f(u), \tag{1.2}$$

for $f = g = 0$, $0 < x < L$, $t \geq 0$, where $u(x,t)$ is the lateral displacement, E is the Young modulus, ρ is the mass density, h is the cross-section area, L is the length, ρ_0 is the initial axial tension, δ is the resistance modulus, and f and g are the external forces. Moreover, (1.2) is called a degenerate equation when $\rho_0 = 0$ and nondegenerate one when $\rho_0 > 0$. The problem (1.1) is a generalization of a model introduced by Kirchhoff.

The single higher-order Kirchhoff-type equation of the problem (1.1)

$$u_{tt} + M\left(\int_\Omega |\nabla^m u|^2 dx\right)(-\triangle)^m u + |u_t|^{p-1} u_t = |u|^{r-1} u \tag{1.3}$$

have studied local existence and blow up of the solution [3]. In case of $M(s) = 1$ and $m = 1$, the equation (1.3) becomes a nonlinear wave equation

$$u_{tt} - \triangle u + |u_t|^{p-1} u_t = |u|^{r-1} u. \tag{1.4}$$

Many authors have been established the local existence, blow up and asymptotic behavior, see [4, 7, 8, 11, 15, 22]. The interaction between damping $(|u_t|^{p-1} u_t)$ and the source term $(|u|^{r-1} u)$ makes the problem more interasting. Levine [7, 8] first studied the interaction between the linear damping $(p = 1)$ and source term by using Concavity method. But this method can't be applied in the case of a nonlinear damping term. Georgiev and Todorova [4] extended Levine's result to the nonlinear case $(p > 1)$. They showed that solutions with negative initial energy blow up in finite time. Later, Vitillaro in [26] extended these results to situations where the nonlinear damping and the solution has positive initial energy. When $M(s) = 1$ and $m = 2$, the equation (1.3) becomes a Petrovsky equation which has been extensively studied and several results concerning existence, blow up and asymptotic behavior have been established [12, 25]. In case $M(s) = 1$, $m \geq 1$, Ye [27] obtained the global existence and asymptotic behavior of solutions for the equation (1.3). Also, Zhou et. al. [29] extended the results of [27].

Ono [16] considered equation (1.3) with $M(s) = s^\gamma$, $m = 1$ and showed that the solution blow up if the initial energy is negative. Wu and Tsai [23] showed that the solution blow up under the condition of positive upper bounded initial energy, for $m = 1$ in (1.3). When $M(s) = s^\gamma$, $m \geq 1$ equation (1.3) becomes the higher-order Kirchhoff-type equation which has been discussed by many authors [9, 13, 20, 28].

Recently, Agre and Rammaha [2] studied the existence and blow up of the solution for the problem (1.1) with $M(s) = 1$ and $m = 1$, by using the same techniques as in [4]. After that, Houari [5] showed the global existence and decay of the solution for the problem. Li et. al. [10] showed the global existence, blow up and decay of the solution for the problem (1.1) for $M(s) = 1$ and $m = 2$. In [17, 18], it was shown the global existence, decay and blow up of solutions for the problem (1.1) with $M(s) = 1$, $m \geq 1$ and $p = q = 1$. Later, Pişkin and Polat [19] showed the global existence, decay of solutions for the problem (1.1) with $M(s) = 1$ and $m \geq 1$. Very recently, Pişkin and Polat [21] studied the decay of the solution and blow up the solution with the negative initial energy of the problem (1.1).

Motivated by the above researches, in this work, we analyze the influence of the damping terms and source terms on the solutions of the problem (1.1). In fact, when both nonlinear damping and source terms are present, then the analysis of their interaction is more difficult [2]. Blow up of the solution with positive initial energy was proved for $2(r+2) > \max\{2\gamma+2, p+1, q+1\}$ by using the technique of [26] with a modification in the energy functional.

This work is organized as follows: In the next section, we present some lemmas, and the local existence theorem. In section 3, we show the blow up properties of solutions.

2 Preliminaries

In this section, we shall give some assumptions and lemmas which will be used throughout this work. Let $\|.\|$ and $\|.\|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively.

Concerning the functions $f_1(u,v)$ and $f_2(u,v)$, we take

$$f_1(u,v) = a|u+v|^{2(r+1)}(u+v) + b|u|^r u|v|^{r+2},$$

$$f_2(u,v) = a|u+v|^{2(r+1)}(u+v) + b|u|^{r+2}|v|^r v,$$

where $a, b > 0$ are constants and r satisfies

$$\begin{cases} -1 < r \quad \text{if } n \leq 2m, \\ -1 < r \leq \frac{3m-n}{n-2m} \quad \text{if } n > 2m. \end{cases} \tag{2.1}$$

According to the above equalities one can easily verify that

$$u\, f_1(u,v) + v f_2(u,v) = 2(r+2) F(u,v), \ \forall (u,v) \in R^2, \tag{2.2}$$

where

$$F(u,v) = \frac{1}{2(r+2)}\left[a|u+v|^{2(r+2)} + 2b|uv|^{r+2}\right]. \tag{2.3}$$

We have the following result.

Lemma 1 *[14]. There exist two positive constants c_0 and c_1 such that*

$$c_0\left(|u|^{2(r+2)} + |v|^{2(r+2)}\right) \leq 2(r+2) F(u,v) \leq c_1\left(|u|^{2(r+2)} + |v|^{2(r+2)}\right). \tag{2.4}$$

We define the energy function as follows

$$\begin{aligned} E(t) &= \frac{1}{2}\left(\|u_t\|^2 + \|v_t\|^2\right) + \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right) \\ &+ \frac{1}{2(\gamma+1)}\left(\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) - \int_\Omega F(u,v)\,dx. \end{aligned} \tag{2.5}$$

The next lemma shows that our energy functional (3.3) is a nonincreasing function along the solution of (1.1).

Lemma 2 *$E(t)$ is a nonincreasing function for $t \geq 0$ and*

$$E'(t) = -\left(\|u_t\|_{p+1}^{p+1} + \|v_t\|_{q+1}^{q+1}\right) \leq 0. \tag{2.6}$$

Proof. Multiplying the first equation of (1.1) by u_t and the second equation by v_t, integrating over Ω, using integrating by parts and summing up the product results, we get

$$E(t) - E(0) = -\int_0^t \left(\|u_\tau\|_{p+1}^{p+1} + \|v_\tau\|_{q+1}^{q+1}\right) d\tau \text{ for } t \geq 0.$$

■

Lemma 3 *(Sobolev-Poincare inequality) [1]. If $2 \leq p \leq \frac{2n}{[n-2m]^+}$ $(2 \leq p < \infty$ if $n = 2m)$, then*

$$\|u\|_p \leq C_* \left\|(-\triangle)^{\frac{m}{2}} u\right\| \text{ for } u \in H_0^m(\Omega)$$

holds with some constant C_*, where we put $[a]^+ = \max\{0, a\}$, $\frac{1}{[a]^+} = \infty$ if $[a]^+ = 0$.

Lemma 4 *[11]. Suppose that*

$$p \leq 2\frac{n-1}{n-2},\ n \geq 3$$

holds. Then there exists a positive constant $C > 1$ depending on Ω only such that

$$\|u\|_p^s \leq C\left(\|\nabla u\|^2 + \|u\|_p^p\right)$$

for any $u \in H_0^1(\Omega),\ 2 \leq s \leq p$.

Next, we state the local existence theorem that can be established by combining arguments of [2, 3, 16, 24].

Theorem 5 *(Local existence). Under condition (2.1) there are p, q satisfying*

$$\begin{cases} 1 \leq p, q \quad \text{if } n \leq 2m, \\ 1 \leq p, q \leq \frac{n+2m}{n-2m} \quad \text{if } n > 2m \end{cases} \tag{2.7}$$

and further $u_0, v_0 \in H_0^m(\Omega) \cap H^{2m}(\Omega)$ and $u_1, v_1 \in H_0^m(\Omega)$ such that problem (1.1) has a unique local solution

$$u, v \in C\left([0,T); H_0^m(\Omega) \cap H^{2m}(\Omega)\right),$$

$$u_t \in C\left([0,T); L^2(\Omega)\right) \cap L^{p+1}(\Omega \times [0,T)) \text{ and } v_t \in C\left([0,T); L^2(\Omega)\right) \cap L^{q+1}(\Omega \times [0,T)).$$

Moreover, at least one of the following statements holds

i) $T = \infty$,

ii) $\|u_t\|^2 + \|v_t\|^2 + \left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)} \longrightarrow \infty$ as $t \longrightarrow T^-$.

Remark 6 *We denote by C various positive constants which may be different at different occurrences.*

3 Blow up of solutions

In this section, we are going to consider the blow up of the solution for the problem (1.1).

Lemma 7 *Suppose that (2.1) holds. Then there exists $\eta > 0$ such that for any $(u,v) \in \left(H^{2m}(\Omega) \cap H_0^m(\Omega)\right) \times \left(H^{2m}(\Omega) \cap H_0^m(\Omega)\right)$ the inequality*

$$\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2} \leq \eta\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)^{r+2} \tag{3.1}$$

holds.

Proof. The proof is almost the same that of [14], so we omit it here. ■

For the sake of simplicity and to prove our result, we take $a = b = 1$ and introduce

$$B = \eta^{\frac{1}{2(r+2)}},\ \alpha_1 = B^{-\frac{r+2}{r+1}},\ E_1 = \left(\frac{1}{2} - \frac{1}{2(r+2)}\right)\alpha_1^2, \tag{3.2}$$

where η is the optimal constant in (3.1). Next, we will state and prove a lemma which is similar to the one introduced firstly by Vitillaro in [26] to study a class of a single wave equation.

Lemma 8 *Suppose that (2.1) holds. Let (u,v) be the solution of system (1.1). Assume further that $E(0) < E_1$ and*

$$\left(\left\|P^{\frac{1}{2}}u_0\right\|^2 + \left\|P^{\frac{1}{2}}v_0\right\|^2\right)^{\frac{1}{2}} > \alpha_1. \tag{3.3}$$

Then there exists a constant $\alpha_2 > \alpha_1$ *such that*

$$\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)^{\frac{1}{2}} \geq \alpha_2, \tag{3.4}$$

and

$$\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}\right)^{\frac{1}{2(r+2)}} \geq B\alpha_2, \tag{3.5}$$

for all $t \in [0,T)$.

Proof. We first note that by (2.5), (3.1) and the definition of B, we have

$$\begin{aligned}
E(t) &\geq \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) - \int_\Omega F(u,v)\,dx \\
&= \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) \\
&\quad -\frac{1}{2(r+2)}\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}\right) \\
&\geq \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) \\
&\quad -\frac{1}{2(r+2)}\eta\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)^{r+2} \\
&= \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) \\
&\quad -\frac{B^{2(r+2)}}{2(r+2)}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)^{r+2} \\
&= \frac{1}{2}\alpha^2 - \frac{B^{2(r+2)}}{2(r+2)}\alpha^{2(r+2)} = G(\alpha),
\end{aligned} \tag{3.6}$$

where $\alpha = \left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)^{\frac{1}{2}}$. It is not hard to verify that G is increasing for $0 < \alpha < \alpha_1$, decreasing for $\alpha > \alpha_1$, $G(\alpha) \longrightarrow -\infty$ as $\alpha \longrightarrow \infty$, and

$$G(\alpha_1) = \frac{1}{2}\alpha_1^2 - \frac{B^{2(r+2)}}{2(r+2)}\alpha_1^{2(r+2)} = E_1, \tag{3.7}$$

where α_1 is given in (3.2). Since $E(0) < E_1$, there exists $\alpha_2 > \alpha_1$ such that $G(\alpha_2) = E(0)$.

Set $\alpha_0 = \left(\left\|P^{\frac{1}{2}}u_0\right\|^2 + \left\|P^{\frac{1}{2}}v_0\right\|^2\right)^{\frac{1}{2}}$. Then by (3.6) we get $G(\alpha_0) \leq E(0) = G(\alpha_2)$, which implies that $\alpha_0 \geq \alpha_2$. Now, to established (3.4), we suppose by contradiction that

$$\left(\left\|P^{\frac{1}{2}}u_0\right\|^2 + \left\|P^{\frac{1}{2}}v_0\right\|^2\right)^{\frac{1}{2}} < \alpha_2,$$

for some $t_0 > 0$. By the continuity of $\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)^{\frac{1}{2}}$, we can choose t_0 such that,

$$\left(\left\|P^{\frac{1}{2}}u_0\right\|^2 + \left\|P^{\frac{1}{2}}v_0\right\|^2\right)^{\frac{1}{2}} > \alpha_1.$$

Again, the use of (3.6) leads to

$$E(t_0) \geq G\left(\left\|P^{\frac{1}{2}}u_0\right\|^2 + \left\|P^{\frac{1}{2}}v_0\right\|^2\right) > G(\alpha_2) = E(0).$$

This is impossible since $E(t) \leq E(0)$ for all $t \in [0,T)$. Hence (3.4) is established.

To prove (3.5), we make use of (2.5) to get

$$\begin{aligned}
&\frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2 + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) \\
\leq\ & E(0) + \frac{1}{2(r+2)}\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}\right).
\end{aligned}$$

Consequently, (3.4) yields

$$\begin{aligned}
&\frac{1}{2(r+2)}\left(\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}\right) \\
\geq\ & \frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2+\frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)}+\frac{1}{\gamma+1}\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)-E(0) \\
\geq\ & \frac{1}{2}\alpha_2^2-E(0) \\
\geq\ & \frac{1}{2}\alpha_2^2-G(\alpha_2) \\
=\ & \frac{B^{2(r+2)}}{2(r+2)}\alpha_2^{2(r+2)}.
\end{aligned} \tag{3.8}$$

Therefore, (3.8) and (3.2) yield the desired result. This completes the proof of Lemma 8. ■

Theorem 9 *Assume that (2.1) holds. Assume further that* $2(r+2)>\max\{2\gamma+2,p+1,q+1\}$. *Then any solution of the system (1.1) with initial data satisfying*

$$\left(\left\|P^{\frac{1}{2}}u_0\right\|^2+\left\|P^{\frac{1}{2}}v_0\right\|^2\right)^{\frac{1}{2}}>\alpha_1 \text{ and } E(0)<E_1$$

cannot exist for all time, where constants α_1 *and* E_1 *are defined in (3.2).*

Proof. We suppose that the solution exists for all time and we reach to a contradiction.

For this purpose, we set

$$H(t)=E_1-E(t). \tag{3.9}$$

By using (2.5) and (3.9), we get

$$\begin{aligned}
0 <\ & H(0)\leq H(t)=E_1-\frac{1}{2}\left(\|u_t\|^2+\|v_t\|^2\right) \\
& -\frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right)-\frac{1}{2(\gamma+1)}\left(\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)}+\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) \\
& +\int_\Omega F(u,v)\,dx.
\end{aligned} \tag{3.10}$$

From (2.4) and (3.4) we have

$$\begin{aligned}
& E_1-\frac{1}{2}\left(\|u_t\|^2+\|v_t\|^2\right)-\frac{1}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right) \\
& -\frac{1}{2(\gamma+1)}\left(\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)}+\left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)+\int_\Omega F(u,v)\,dx \\
\leq\ & E_1-\frac{1}{2}\alpha_1^2+\frac{c_1}{2(r+2)}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right) \\
\leq\ & -\frac{1}{2(r+2)}\alpha_1^2+\frac{c_1}{2(r+2)}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right) \\
\leq\ & \frac{c_1}{2(r+2)}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right).
\end{aligned} \tag{3.11}$$

Combining (3.10) and (3.11) we have

$$0<H(0)\leq H(t)\leq\frac{c_1}{2(r+2)}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right). \tag{3.12}$$

We then define

$$\Psi(t)=H^{1-\sigma}(t)+\varepsilon\int_\Omega(uu_t+vv_t)\,dx, \tag{3.13}$$

where ε small to be chosen later and

$$0<\sigma\leq\min\left\{\frac{2(r+2)-(p+1)}{2(r+2)p},\frac{2(r+2)-(q+1)}{2(r+2)q},\frac{r+1}{2(r+2)}\right\}. \tag{3.14}$$

Our goal is to show that $\Psi(t)$ satisfies a differential inequality of the form

$$\Psi'(t) \geq \xi \Psi^{\zeta}(t), \ \zeta > 1.$$

This, of course, will lead to a blow up in finite time.

Taking the time derivative of (3.13) and using Eq. (1.1) we obtain

$$\begin{aligned}\Psi'(t) &= (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|^2 + \|v_t\|^2\right) - \varepsilon\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)\\ &\quad -\varepsilon\left(\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right) + 2\varepsilon(r+2)\int_\Omega F(u,v)\,dx\\ &\quad -\varepsilon\left(\int_\Omega uu_t|u_t|^{p-1}dx + \int_\Omega vv_t|v_t|^{q-1}dx\right). \end{aligned} \tag{3.15}$$

From definition of $H(t)$, it follows that

$$\begin{aligned} &-\left(\left\|P^{\frac{1}{2}}u\right\|^{2(\gamma+1)} + \left\|P^{\frac{1}{2}}v\right\|^{2(\gamma+1)}\right)\\ &= 2(\gamma+1)H(t) - 2(\gamma+1)E_1 + (\gamma+1)\left(\|u_t\|^2 + \|v_t\|^2\right)\\ &\quad + (\gamma+1)\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right) - 2(\gamma+1)\int_\Omega F(u,v)\,dx. \end{aligned} \tag{3.16}$$

Inserting (3.16) into (3.15), we conclude that

$$\begin{aligned}\Psi'(t) &= (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|^2 + \|v_t\|^2\right) - \varepsilon\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right) + 2\varepsilon(\gamma+1)H(t)\\ &\quad -2(\gamma+1)E_1 + \varepsilon(\gamma+1)\left(\|u_t\|^2 + \|v_t\|^2\right) + (\gamma+1)\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)\\ &\quad +\varepsilon\left(1 - \frac{\gamma+1}{r+2}\right)\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}\right)\\ &\quad -\varepsilon\left(\int_\Omega uu_t|u_t|^{p-1}dx + \int_\Omega vv_t|v_t|^{q-1}dx\right). \end{aligned}$$

Then using (3.5), we have

$$\begin{aligned}\Psi'(t) &\geq (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|^2 + \|v_t\|^2\right) - \varepsilon\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right) + 2\varepsilon(\gamma+1)H(t)\\ &\quad +\varepsilon(\gamma+1)\left(\|u_t\|^2 + \|v_t\|^2\right) + (\gamma+1)\left(\left\|P^{\frac{1}{2}}u\right\|^2 + \left\|P^{\frac{1}{2}}v\right\|^2\right)\\ &\quad +\varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)} + 2\|uv\|_{r+2}^{r+2}\right) - \varepsilon\left(\int_\Omega uu_t|u_t|^{p-1}dx + \int_\Omega vv_t|v_t|^{q-1}dx\right), \end{aligned} \tag{3.17}$$

where $c' = 1 - \frac{\gamma+1}{r+2} - 2(\gamma+1)E_1(B\alpha_2)^{-2(r+2)} > 0$, since $\alpha_2 > B^{-\frac{r+2}{r+1}}$. In order to estimate the last two terms in (3.17), we make use of the following Young's inequality

$$XY \leq \frac{\delta^k X^k}{k} + \frac{\delta^{-l}Y^l}{l},$$

where $X, Y \geq 0,\ \delta > 0,\ k, l \in R^+$ such that $\frac{1}{k} + \frac{1}{l} = 1$. Consequently, applying the above inequality we have

$$\begin{aligned}\int_\Omega uu_t|u_t|^{p-1}dx &\leq \frac{\delta_1^{p+1}}{p+1}\|u\|_{p+1}^{p+1} + \frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}\|u_t\|_{p+1}^{p+1}\\ &\leq \frac{\delta_1^{p+1}}{p+1}\|u\|_{p+1}^{p+1} + \frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}H'(t)\end{aligned}$$

and

$$\begin{aligned}\int_\Omega vv_t|v_t|^{q-1}dx &\leq \frac{\delta_2^{q+1}}{q+1}\|v\|_{q+1}^{q+1} + \frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}\|v_t\|_{q+1}^{q+1}\\ &\leq \frac{\delta_2^{q+1}}{q+1}\|v\|_{q+1}^{q+1} + \frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}H'(t),\end{aligned}$$

where $\delta_1,\ \delta_2$ are constants depending on the time t and specified later. Therefore, (3.17) becomes

$$\begin{aligned}\Psi'(t) \geq\ & (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|^2+\|v_t\|^2\right) + \varepsilon\gamma\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right) + 2\varepsilon(\gamma+1)H(t)\\ & +\varepsilon(\gamma+1)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}\right)\\ & -\varepsilon\left(\frac{p\delta_1^{-\frac{p+1}{p}}}{p+1}+\frac{q\delta_2^{-\frac{q+1}{q}}}{q+1}\right)H'(t)-\varepsilon\left(\frac{\delta_1^{p+1}}{p+1}\|u\|_{p+1}^{p+1}+\frac{\delta_2^{q+1}}{q+1}\|v\|_{q+1}^{q+1}\right). \qquad (3.18)\end{aligned}$$

Therefore by choosing δ_1 and δ_2 so that $\delta_1^{-\frac{p+1}{p}} = k_1 H^{-\sigma}(t),\ \delta_2^{-\frac{q+1}{q}} = k_2 H^{-\sigma}(t)$, where $k_1,\ k_2 > 0$ are specified later, we get

$$\delta_1^{p+1} = k_1^{-p} H^{\sigma p}(t) \leq k_1^{-p} c_1^{\sigma p}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)^{\sigma p}, \qquad (3.19)$$

and

$$\delta_2^{q+1} = k_2^{-q} H^{\sigma q}(t) \leq k_2^{-q} c_1^{\sigma q}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)^{\sigma q}, \qquad (3.20)$$

since $H(t) \leq -E(t) \leq \int_\Omega F(u,v)\,dx \leq c_1\left(|u|^{2(r+2)}+|u|^{2(r+2)}\right)$.

Inserting (3.19) and (3.20) into (3.18), we conclude that

$$\begin{aligned}\Psi'(t) \geq\ & \left(1-\sigma-\frac{\varepsilon p k_1}{p+1}-\frac{\varepsilon q k_2}{q+1}\right)H^{-\sigma}(t)H'(t)+2\varepsilon(\gamma+1)H(t)\\ & +\varepsilon\gamma\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right)\\ & +\varepsilon(2+\gamma)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}\right)\\ & -\frac{\varepsilon k_1^{-p}c_1^{\sigma p}}{p+1}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)^{\sigma p}\|u\|_{p+1}^{p+1}\\ & -\frac{\varepsilon k_2^{-q}c_1^{\sigma q}}{q+1}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)^{\sigma q}\|v\|_{q+1}^{q+1}. \qquad (3.21)\end{aligned}$$

Since $2(r+2) > \max\{p+1, q+1\}$, we obtain

$$\|u\|_{p+1}^{p+1} \leq C\|u\|_{2(r+2)}^{p+1} \leq C\left(\|u\|_{2(r+2)}+\|v\|_{2(r+2)}\right)^{p+1}$$

and

$$\|v\|_{q+1}^{q+1} \leq C\|v\|_{r+1}^{q+1} \leq C\left(\|u\|_{2(r+2)}+\|v\|_{2(r+2)}\right)^{q+1}.$$

Thus,

$$\begin{aligned}\Psi'(t) \geq\ & \left(1-\sigma-\frac{\varepsilon p k_1}{p+1}-\frac{\varepsilon q k_2}{q+1}\right)H^{-\sigma}(t)H'(t)+2\varepsilon(\gamma+1)H(t)\\ & +\varepsilon\gamma\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right)\\ & +\varepsilon(2+\gamma)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}\right)\\ & -\frac{\varepsilon k_1^{-p}c_1^{\sigma p}C}{p+1}\left(\|u\|_{2(r+2)}+\|v\|_{2(r+2)}\right)^{2\sigma(r+2)p+p+1}\\ & -\frac{\varepsilon k_2^{-q}c_1^{\sigma q}C}{q+1}\left(\|u\|_{2(r+2)}+\|v\|_{2(r+2)}\right)^{2\sigma(r+2)q+q+1}, \qquad (3.22)\end{aligned}$$

where $(a+b)^\lambda \leq C\left(a^\lambda+b^\lambda\right),\ a,b>0$ is used.

From (3.14), we have $2 \leq 2\sigma(r+2)p+p+1 \leq 2(r+2),\ 2 \leq 2\sigma(r+2)q+q+1 \leq 2(r+2)$. By using Lemma 4 and Sobolev-Poincare inequality, we have

$$\begin{aligned}\|u\|_{2(r+2)}^{2\sigma(r+2)p+p+1} &\leq C\left(\|\nabla u\|^2+\|u\|_{2(r+2)}^{2(r+2)}\right)\\ &\leq C\left(\left\|P^{\frac{1}{2}}u\right\|^2+\|u\|_{2(r+2)}^{2(r+2)}\right)\end{aligned}$$

and

$$\begin{aligned}\|v\|_{2(r+2)}^{2\sigma(r+2)q+q+1} &\leq C\left(\|\nabla v\|^2+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &\leq C\left(\left\|P^{\frac{1}{2}}v\right\|^2+\|v\|_{2(r+2)}^{2(r+2)}\right).\end{aligned}$$

Thus,

$$\begin{aligned}\Psi'(t) \geq\ & \left(1-\sigma-\frac{\varepsilon pk_1}{p+1}-\frac{\varepsilon qk_2}{q+1}\right)H^{-\sigma}(t)\,H'(t)+2\varepsilon(\gamma+1)H(t)\\ &+\varepsilon\gamma\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right)\\ &+\varepsilon(2+\gamma)\left(\|u_t\|^2+\|v_t\|^2\right)+\varepsilon c'\left(\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}\right)\\ &+\varepsilon\left(-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &+\varepsilon\left(-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}\right)\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right). \qquad (3.23)\end{aligned}$$

By using the $c_0\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\leq\|u+v\|_{2(r+2)}^{2(r+2)}+2\|uv\|_{r+2}^{r+2}$ in (3.23) we obtain

$$\begin{aligned}\Psi'(t) \geq\ & \left(1-\sigma-\frac{\varepsilon pk_1}{p+1}-\frac{\varepsilon qk_2}{q+1}\right)H^{-\sigma}(t)\,H'(t)\\ &+2\varepsilon(\gamma+1)H(t)+\varepsilon(2+\gamma)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &+\varepsilon\left(c_0c'-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}\right)\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &+\varepsilon\left(\gamma-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}\right)\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right). \qquad (3.24)\end{aligned}$$

We choose $k_1,\ k_2$ large enough so that

$$c_0c'-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}>\frac{c_0c'}{2}$$

and

$$\gamma-\frac{k_1^{-p}c_1^{\sigma p}C}{p+1}-\frac{k_2^{-q}c_1^{\sigma q}C}{q+1}>\frac{\gamma}{2}.$$

Then, we choose ε small enough so that $1-\sigma-\frac{\varepsilon pk_1}{p+1}-\frac{\varepsilon qk_2}{q+1}\geq 0$. Thus, we have

$$\begin{aligned}\Psi'(t) &\geq \varepsilon(2+\gamma)\left(\|u_t\|^2+\|v_t\|^2\right)+2\varepsilon(\gamma+1)H(t)\\ &\quad+\varepsilon\frac{\gamma}{2}\left(\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right)+\varepsilon\frac{c_0c'}{2}\left(\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right)\\ &\geq \eta\left(\|u_t\|^2+\|v_t\|^2+H(t)+\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2+\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}\right), \qquad (3.25)\end{aligned}$$

where $\eta=\min\left\{\varepsilon(2+\gamma),\ 2\varepsilon(\gamma+1),\ \varepsilon\frac{\gamma}{2},\ \varepsilon\frac{c_0c'}{2}\right\}$. Consequently we have

$$\Psi(t)\geq\Psi(0)=H^{1-\sigma}(0)+\varepsilon\left(\int_\Omega u_0u_1dx+\int_\Omega v_0v_1dx\right)>0,\ \forall t\geq 0. \qquad (3.26)$$

On the other hand, applying Hölder inequality, we obtain

$$\begin{aligned}\left|\int_\Omega uu_tdx+\int_\Omega vv_tdx\right|^{\frac{1}{1-\sigma}} &\leq \|u\|^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}+\|v\|^{\frac{1}{1-\sigma}}\|v_t\|^{\frac{1}{1-\sigma}}\\ &\leq C\left(\|u\|_{2(r+2)}^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}+\|v\|_{2(r+2)}^{\frac{1}{1-\sigma}}\|v_t\|^{\frac{1}{1-\sigma}}\right). \qquad (3.27)\end{aligned}$$

Young's inequality gives

$$\left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u\|_{2(r+2)}^{\frac{\mu}{1-\sigma}}+\|u_t\|^{\frac{\theta}{1-\sigma}}+\|v\|_{2(r+2)}^{\frac{\mu}{1-\sigma}}+\|v_t\|^{\frac{\theta}{1-\sigma}}\right), \tag{3.28}$$

for $\frac{1}{\mu}+\frac{1}{\theta}=1$. We take $\theta=2(1-\sigma)$, to get $\mu=\frac{2(1-\sigma)}{1-2\sigma}\leq 2(r+2)$ by (3.14). Therefore (3.28) becomes

$$\left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|v_t\|^2+\|u\|_{2(r+2)}^{\frac{2}{1-2\sigma}}+\|v\|_{2(r+2)}^{\frac{2}{1-2\sigma}}\right). \tag{3.29}$$

By using Lemma 4, we obtain

$$\left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|v_t\|^2+\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}+\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right). \tag{3.30}$$

Thus

$$\begin{aligned}\Psi^{\frac{1}{1-\sigma}}(t) &= \left[H^{1-\sigma}(t)+\varepsilon\left(\int_\Omega uu_t dx+\int_\Omega vv_t dx\right)\right]^{\frac{1}{1-\sigma}}\\ &\leq 2^{\frac{\sigma}{1-\sigma}}\left(H(t)+\varepsilon^{\frac{1}{1-\sigma}}\left|\int_\Omega uu_t dx+\int_\Omega vv_t dx\right|^{\frac{1}{1-\sigma}}\right)\\ &\leq C\left(\|u_t\|^2+\|v_t\|^2+H(t)+\|u\|_{2(r+2)}^{2(r+2)}+\|v\|_{2(r+2)}^{2(r+2)}+\left\|P^{\frac{1}{2}}u\right\|^2+\left\|P^{\frac{1}{2}}v\right\|^2\right). \end{aligned} \tag{3.31}$$

By combining of (3.25) and (3.31) we arrive

$$\Psi'(t)\geq \xi\Psi^{\frac{1}{1-\sigma}}(t), \tag{3.32}$$

where ξ is a positive constant.

A simple integration of (3.32) over $(0,t)$ yields $\Psi^{\frac{\sigma}{1-\sigma}}(t)\geq \frac{1}{\Psi^{-\frac{\sigma}{1-\sigma}}(0)-\frac{\xi\sigma t}{1-\sigma}}$. Therefore $\Psi(t)$ blows up in a finite time $T^*\leq \frac{1-\sigma}{\xi\sigma\Psi^{\frac{\sigma}{1-\sigma}}(0)}$. ■

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