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Capital Market Line Based on Efficient Frontier of Portfolio with Borrowing and Lending Rate

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Abstract Capital Asset Pricing Model (CAPM) is a general equilibrium model. It not only allows improved understanding of market behavior, but also practical benefits. However, there exists a risk-free asset in the assumption of the CAPM. Investors are able to borrow and lend freely at the rate may not be a valid representation of the working of the marketplace. Therefore, in this paper, it studies that the efficient frontier of portfolio in different borrowing and lending rate. This paper solves the highly difficult problem by matrix operation method. It first denotes the efficient frontier of Markowitz model with the matrix expression of portfolio. Then it denotes the capital market line (CML) with the matrix expression too. It is easy to calculate by using Excel function. The aim of this study is to develop the mean- variance analysis theory with regard to market portfolio and provide algorithmic tools for calculating the efficient market portfolio. Then explain that the portfolio frontier is hyperbola in mean-standard deviation space. It constructs CML in order to get more returns than that of efficient frontier if risk-free securities are included in the portfolio. A proposed step for CML on efficient frontier of portfolio with borrowing and lending rate is presented. Under these tools, it is easy calculation SML and CML by using Excel function. An example show that proposed method is correct and effective, and can improve the capability of the mean-variance portfolio efficiency frontier model.

Keywords Efficient Frontier, Capital Market Line, Portfolio, Security Market Line, Capital Asset Pricing Model (CAPM)

1. Introduction

Capital market theory represented a major step forward in how investors should think about the investment process. In discussion the Markowitz portfolio is its average covariance with all other assets in the portfolio. The CAPM extends Capital market theory in a way that allows investors to evaluate the risk-return trade-off for both diversified portfolios and individual security. The CPAM redefines the relevant measure of risk from total volatility to just the no diversifiable portion of the total volatility (i.e., systematic risk). This risk measure is call beta coefficient, and it calculates the level of a security's systematic risk compared to that of the market portfolio. One of the first assumptions of the CAPM was that investor could borrow and lend any amount of money at the risk free rate.

CAPM uses the Security Market Line (SML), which is a trade-off between expected return and security's risk (beta risk) relative to market portfolio. Each investor will allocate to the market portfolio and the risk free asset in according to own risk tolerance. Capital Market Line (CML) represents the allocation of capital between risk free securities and risky securities for all investors combined. An investor is only willing to accept higher risk if the return rises proportionally. The optimal portfolio for an investor is the point where the new CML in tangent to the old efficient frontier when only risky securities were graphed.

CAPM discovered by Shap (1964), Lintner (1965), Treynor (1965) and Mossin (1966) is a general equilibrium model for portfolio analysis. It derived from two fund separation theory. Some important researches in CAPM are Mas-celell et al. (1995), Constantinides and malliarias (1995) , Elliott and Kopp (1999) , Merton (1992) , Duffie (1992), and Cochrance (2001).

Markowitz (1952) firstly offers the Portfolio theory. This theory is a model of modern financial economics. In ordering to introduce Markowitz's Portfolio theory, Jarrow (1988) first proposed a general equilibrium mode, and then proposed the concept of mean variance efficient frontier. Markowitz's Portfolio theory has somewhat cumbersome expressions. Levy and Sarmat (1970) study the optimal portfolio under different rate in Markowitz mean-variance model. LeRoy and Werner (2001) use the Hibert Space and orthogonal solution to prove mean variance Portfolio analysis. It denotes the efficient frontier of Markowitz model with the weight vector of portfolio (Constantinides and Malliaris, 1995). Xuemei and Xinshu (2003) use geometric way (linear equation group) in solving the optimal portfolio choice weights. An analysis of Canadian farmland risk and return on investment shows that a Farmland Real Estate Investment Trust (F-REIT) would have been a reasonably good investment over the past 35 years (painter, 2010). Painter (2010) study that shows that for period 1990-2007, international portfolio investment performance was significantly improved with the addition if Canadian farmland. Mangram (2013) found that diversification cannot eliminate all risk, i.e. it cannot eliminate systematic risk but unsystematic risk can be eliminated to a large extent by diversification. Mitra and Khanna (2014) present a simplified perspective of Markowitz's contributions to modern portfolio theory. It is to see the effect of duration of historical data on the risk and return of the portfolio and to see the applicability of risk-reward logic. Chen et al. (2010) introduce the theory and the application of computer program of modern portfolio theory and introduce the mean-variance spanning test which follows directly from the portfolio optimization problem.

Since the assumption of the CAPM that there exists a risk-free asset and investors are able to borrow and lend freely at the rate may not be a valid representation of the working of the marketplace. Therefore, discuss portfolio issues under different borrow and lend rate in theory and financial practice become significance. So it is necessary to discus the portfolio problem in different borrowing and lending rate.

The following statement described the structure of this study. Section 2 deals with Markowitz efficient frontier portfolio. In this section it proposed a procedure for mean-variance efficient portfolio. It will examine the portfolio frontier is a hyperbola in mean-standard deviation space, and finding the minimum variance portfolio of risky assets. Different borrowing and lending rates' capital market line is discussed in section 3. Section 4 gives an illustration. It help in understanding of this procedure, a demonstrative case is given to show the key stages involving the use of the introduced concepts. Section 5 is discussion. Section is conclusion.

2. Markowitz Efficient Frontier Portfolio

The Markowitz portfolios (1959) is that one that gives the highest expected return of all feasible portfolios with the same risk. It is called the mean-variance efficient portfolio. The Markowitz model describes a set of rigorous statistical procedures used to select the optimal project portfolio for wealth maximizing /risk-average investors. This model is described under the framework of risk-return tradeoff graph. Investors are assumed to seek our maximizing their return while minimizing the risk involved.

2.1. Markowitz Portfolio Efficiency Frontier Model

This paper use the following notations

n: number of available assets

r_i : the return (sometimes called rate of return) on asset i

r_p : the (rate of) return of portfolio or the mean of the portfolio return

w_i : the weight (share) of asset i in the portfolio

r_f : the return on the risk free asset

σ_i^2 : variance of the return on asset i

σ_{ij} : covariance between the return on assets i and j

ρ_{ij} : correlation between the returns on assets i and j

Let **r** = $(r_1, r_2, \ldots, r_n)$ denotes the return on asset. **V** denotes covariance matrix. It is the $n \times n$ variance-covariance matrix of the n assets returns.

$$V = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ & & & \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{bmatrix}$$

The Markowitz model (mean- variance analysis method): (Huang and Litzenberger, 1988)

$$\min \quad \sigma_p^2 = W^T V W$$

$$\text{S. t. } W^T r = r_p \tag{1}$$

$$W^T 1 = 1$$

Where 1^T = $(1, 1, \ldots, 1)$, is a $1 \times n$ column vector of 1's. **V** is $n \times n$ matrix, and assumes that **V** is nonsingular matrix and positive definite matrix. Since min (x) = max (-x). Using the Lagrangian method with multipliers λ_1 *and* λ_2, it is

$$La = -W^T V W + \lambda_1 (W^T r - r_p) + \lambda_2 (W^T 1 - 1) \tag{2}$$

It takes the derivate of it with respect to W, λ_1, λ_2, set the derivate equation to zero. The derivates are:

$$\frac{\partial La}{\partial W} = 2VW - \lambda_1 r - \lambda_2 1 = 0 \tag{3}$$

$$\frac{\partial La}{\partial \lambda_1} = r_p - W^T r = 0 \tag{4}$$

$$\frac{\partial La}{\partial \lambda_2} = 1 - W^T 1 = 0 \tag{5}$$

Solving W from Equation (3) yields.

$$W = \frac{1}{2} V^{-1} (\lambda_1 r + \lambda_2 1) = \frac{1}{2} V^{-1} [r\ 1] \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{6}$$

Solving and rearranging from Equation (4) yields.

$$W^T [r \ \ 1] = [r_p\ 1] \tag{7}$$

$$[r \ \ 1]^T W = \begin{bmatrix} r_p \\ 1 \end{bmatrix} \tag{8}$$

Substituting W from Equation (6) into Equation (8) yields.

$$\begin{bmatrix} r_p \\ 1 \end{bmatrix} = \frac{1}{2}[r\ 1]^T V^{-1}[r\ 1]\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{9}$$

Let

$$a = r^T V^{-1} r, \quad b = r^T V^{-1} 1, \quad c = 1^T V^{-1} 1,$$
$$d = ac - b^2$$

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} r^T V^{-1} r & r^T V^{-1} 1 \\ r^T V^{-1} 1 & 1^T V^{-1} 1 \end{bmatrix} = [r\ 1]^T V^{-1}[r\ 1] \tag{10}$$

Equation (9) can be written as

$$\begin{bmatrix} r_p \\ 1 \end{bmatrix} = \frac{1}{2}\mathbf{D}\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{11}$$

Solving the Equation (1) yields.

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = 2\mathbf{D}^{-1}\begin{bmatrix} r_p \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} cr_p - b \\ a - br_p \end{bmatrix} \tag{12}$$

$$W = V^{-1}[r\ 1]\mathbf{D}^{-1}\begin{bmatrix} r_p \\ 1 \end{bmatrix} \tag{13}$$

Substituting W from Equation (13) into σ_p^2 yields

$$\sigma_p^2 = W^T V W = \lfloor r_p\ 1 \rfloor \mathbf{D}^{-1}\begin{bmatrix} r_p \\ 1 \end{bmatrix} =$$

$$\lfloor r_p\ 1 \rfloor \frac{1}{ac - b^2}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}\begin{bmatrix} r_p \\ 1 \end{bmatrix} \tag{14}$$

$$\sigma_p^2 = \frac{a - 2br_p + cr_p^2}{ac - b^2} = \frac{1}{d}(cr_p^2 - 2br_p + a) \tag{15}$$

Equation (15) can be written as:

$$\frac{\sigma_p^2}{1/c} - \frac{(r_p - b/c)^2}{d/c^2} = 1 \tag{16}$$

It explains that the portfolio frontier is a hyperbola in mean-standard deviation space.

2.2. Global Minimum Variance Portfolio Point

Find the minimum variance portfolio of risky assets in this section. The minimum variance portfolio of risky assets denotes as W_g. Differentiating Equation (16) with respect to r_p and setting it equation to zero yields. Figure 1 denotes the minimum variance portfolio hyperbola and efficiency Frontier

$$r_p = (r_g) = \frac{b}{c}, \quad \sigma_p^2 = (\sigma_g^2) = 1/c$$

$$W_g = V^{-1}[r\ 1]\mathbf{D}^{-1}\begin{bmatrix} r_g \\ 1 \end{bmatrix} = \frac{V^{-1}[r\ 1]}{ac - b^2}\begin{bmatrix} c & -b \\ -b & a \end{bmatrix}\begin{bmatrix} b/c \\ 1 \end{bmatrix} =$$

$$\frac{V^{-1}1}{c} \tag{17}$$

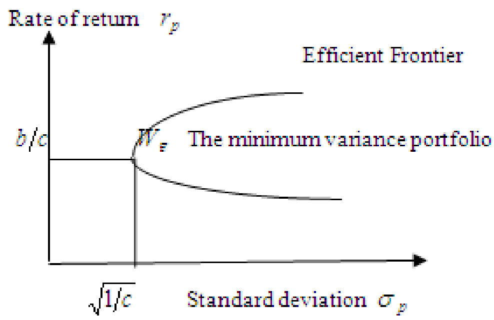

Figure 1. The Minimum Variance Portfolio Hyperbola and Efficiency Frontier

2.3. Tangent Line of Portfolio Efficiency Frontier Model.

The CML leads all investors to invest in the in the sample risky asset portfolio W_T (see Figure 2). It uses the tangent concept to find the weight of tangent point of the portfolio of assets consisting entirely for risk.

Let $r_p = r_f + k\sigma_p$ be a tangent line of portfolio Efficiency Frontier model. K is slop and r_f is borrowing rate or lending rate. Substituting $r_p = r_f + k\sigma_p$ into Equation (16) yields.

$$(d/c - k^2)\sigma_p^2 - 2k(r_f - b/c)\sigma_p - ((r_f - b/c)^2 + d/c^2) = 0 \tag{18}$$

Since Equation (18) is a quadratic equation with one unknown concerning σ_p. σ_p has only one root. According to the extract root formula, it find the values of σ_p, and k

$$\sigma_p = \frac{k(r_f - b/c)}{(d/c - k^2)} \tag{19}$$

$$k^2 = c((r_f - b/c)^2 + d/c^2) \tag{20}$$

The tangent point W_T is

$$\left(\frac{k(r_f - b/c)}{(d/c - k^2)},\ r_f + k\sigma_p\right) \tag{21}$$

Assume that $r_i < r_g$ (= b/c), then the tangent line has only one tangent point W_T. It satisfied $1^T W_T$ = 1, the tangent point of the portfolio of assets consisting entirely for risk. The excess return is ξ_i

$$\xi_i = r_i - r_f = \frac{\xi^T V^{-1}\varsigma}{1^T V^{-1}\varsigma} \tag{22}$$

$$W_T = \frac{V^{-1}\varsigma}{1^Y V^{-1}\xi} \tag{23}$$

Where , $\varsigma_i = r_i - r_f$ Figure 2 denotes the tangent line of portfolio efficient Frontier model

2.4. Capital Market Line and Security Market Line (Sharpe, 1964; 2000)

In this section, it finds Capital market line and Security market line. SML shows that the trade-off between risk and expected return as a straight line intersecting the vertical axis (i.e., zero-risk point) at the risk free rate. CML efficient investment portfolios were those that provided that highest return for chosen level of risk, or conversely, the lowest risk for a chosen level of return.

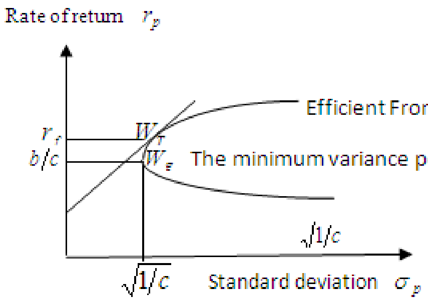

Figure 2. The Tangent Line of Portfolio Efficiency Frontier Model

The risk-return relationship show in Equation (24) holds for every combination of the risk-free asset with any collection of risk assets. The CML (Equation (24)) offers a precise way of calculating the return that investors can expect for providing their financial capital (r_f), and bearing σ_p units of risk $(r_M - r_f)/\delta_M$.

Let (σ_M, r_M) denote the point corresponding to the market portfolio M. The investor will have a point (σ_p, r_p) on the capital market line. The capital market line (CML) is:

$$r_p = r_f + \frac{r_M - r_f}{\sigma_M}\sigma_p \tag{24}$$

$\dfrac{r_M - r_f}{\sigma_M}$ is called price of risk, also is the slope of the CML, which represents the change in expected return r_p per one-unit change in standard deviation σ_p.

Theorem 1 (CAPM formula)

For any asset i,

$$r_i - r_f = \beta_i(r_M - r_f)$$

where

$$\beta_i = \frac{\sigma_{M,i}}{\sigma_M^2} \tag{25}$$

is called the beta of asset i. This beta value serves as an important measure of risk for individual assets (portfolios) that is different from σ_i^2; it measures the no diversifiable part of risk. More generally, for any portfolio $p = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ of risk assets, its beta can be computed as a weighted average of individual assets beta:

$$r_p - r_f = \beta_p(r_M - r_f) \tag{26}$$

where

$$\beta_p = \frac{\sigma_{M,p}}{\sigma_M^2} = \sum_{i=1}^{n}\alpha_i\beta_i \tag{27}$$

Note that when β_p=1 then $r_p = r_M$; the expected rate of return in the same as for the market portfolio.

When $\beta_p > 1$, then $r_p > r_M$; when $\beta_p < 1$, then $r_p < r_M$. Also note that if an asset i is negatively correlation with M, $\sigma_{M,i} < 0$, then $\beta_i < 0$ and $r_i < r_f$; the expected rate of return is less than the risk-free rate. In the space of expected return and beta, the line $r_i - r_f = \beta_i(r_M - r_f)$ is called Security Market Line (SML). Figure 3 denotes Security Market Line with Beta portfolio.

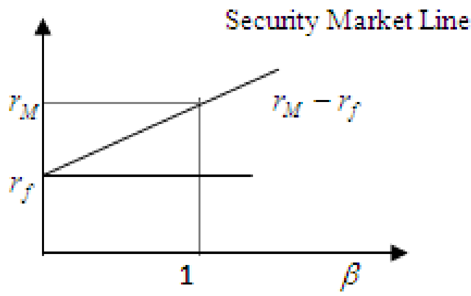

Figure 3. Security Market Line with Beta portfolio

The availability of this zero-beta portfolio will not affect the CML, but it will allow construction of a linear SML. The combination of this zero-beta portfolio and the market portfolio will be a linear relationship in return and risk, because the covariance between the zero-beta portfolio and the market portfolio is similar to what it was with the risk-free assets.

3. Different Borrowing and Lending Rates' Capital Market Line

This section uses the tangent line of portfolio Efficiency Frontier model to calculate Capital Market Line CMLb (borrowing rate capital market line) and CMLl (lending rate capital market line).

In common case, borrowing rate (r_b) is large to lending rate (r_l). The tangent line of portfolio efficient frontier model with borrowing rate and lending rate, which is called Capital Market Line (CML) denoted as CMLb and CMLl.

CMLb: The equation is $r_p = r_b + k_b\sigma_p$, tangent point is T_b.

The tangent line denotes as $D - T_b - B$.

CMLl: The equation is $r_p = r_l + k_l\sigma_p$, tangent point is T_l.

The tangent line denotes as $C - T_l - L$.

Where, k_b and k_l can obtain by Equation (20).
Investor's optimal portfolio may be located at any point on line $T_b - B$ (borrowing funds), may be located at any point on line $C - T_l$ (lending funds), and may be located on the curve $T_l - T_b$, this is neither the lending nor borrowing.
Figure 4 denotes borrowing and lending rates' capital market line

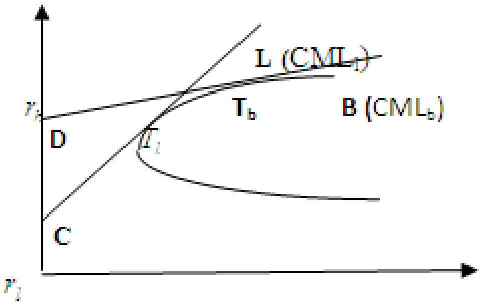

Figure 4. Borrowing and Lending Rates' Capital Market Line

In Figure 4, the CML is made up of $C - T_i - T_b - B$; that is, a line segment ($C - T_i$), a curve segment ($T_i - T_b$); and another line ($T_b - B$). CML_b is slightly kinked as the borrowing rate is higher than the risk-free lending rate and small part of the concave efficient frontier is a segment of the CML_b.

4. Illustration

4.1. The Steps of this Proposed Method

Portfolio optimization involves a mathematical procedure called quadratic programming problem (OPP). There considered two objectives: to maximize return and minimize risk. The OPP can be solved using constrained optimization techniques involving calculus or by computational algorithm applicable to non- linear programming problem. This paper use matrix operation, it includes matrix inverse, matrix multiplication, and matrix transpose. The objective trace the portfolio frontier in mean –standard deviation space and identify the efficient frontier, the minimum variance portfolio and borrowing and lending rates' capital market line.

The steps of this proposed method.

Step 1: Calculate the efficient frontier inputs

Step2: Calculate the efficient frontier by Markowitz portfolios mean- variance analysis method,

Step3: Calculate global minimum variance portfolio point

Step 4: Build the capital market line under different borrowing and lending rate

Step 5: Analyze different borrowing and lending rate in portfolio Efficiency Frontier.

4.2. Example

Selected three stock returns A, B, C. in this illustration.

Step1: Calculate the efficient frontier inputs

Use Excel function AVERAGE ( ) calculation mean return. Use Excel function STDEVP ( ) calculation standard deviation, for variance it uses VARP ( ). Use Excel function CORREL ( ) and COVAR ( ) calculation correlation co-efficient and the covariance. Table 1 denoted as three stocks' mean return standard deviation and covariance.

Table 1. Three Stocks' Mean Return Standard Deviation and Correlation Co-Efficient.

	Mean return (%)	Standard deviation (%)	Correlation co-efficient		
A	15.5	30.3	1.0	0.56	0.22
B	12.3	20.5	0.56	1.0	0.14
C	5.4	8.7	0.22	0.14	1.0

Step 2: Calculate the efficient frontier by Markowitz portfolios mean- variance analysis method

r represents that 1 x 3 mean returns column vector of 3 stocks. V $=[\sigma_{ij}]$, represents that 3 x 3 variance- covariance matrix of 3 assets returns. W represents that 1 x 3 column vector of the portfolio shares. V-1 represents the inverse of matrix V. 1 represents that 1 x 3 column vector of 1's. For calculation V and V^{-1}, it uses Excel function COVAR ( ) and MINVERSE ( )

From table 1, r = [15.5, 12.3, 5.4]T.

$$V = [\sigma_{ij}] = \begin{bmatrix} 918.090 & 347.844 & 57.9942 \\ 347.844 & 420.250 & 24.9690 \\ 57.9942 & 24.9690 & 75.6900 \end{bmatrix}$$

$$V^{-1} = \begin{bmatrix} 0.0016400 & -0.001305 & -0.000823 \\ -0.001305 & 0.0034680 & -0.000144 \\ -0.000823 & -0.000144 & 0.01389 \end{bmatrix}$$

Calculate the value of a, b, c, and d by using Equation (10)

$a = r^T V^{-1} r,$ It uses Excel function MMULT (TRANSPOSE (r), MMULT (V^{-1} $,r)$) and obtain the value of a 0.668237. $b = r^T V^{-1} 1,$ It uses Excel function MMULT (TRANSPOSE (r), MMULT (V^{-1} $,1)$) and obtain the value of b 0.08699. $c = 1^T V^{-1} 1,$ It uses Excel function MMULT (TRANSPOSE (1), MMULT (V^{-1} $,1)$) and obtain the value of b 0.018926.

$d = ac - b^2$ = 0.00508.

From Equation (16), the portfolio frontier is a hyperbola in mean-standard deviation space.

$$\frac{\sigma_p^2}{52.8373} - \frac{(r_p - 4.59632)^2}{14.1817} = 1$$

Step 3: calculate global minimum variance portfolio point

The return on the minimum risk portfolio is r_g and the minimum standard deviation is σ_g

$$r_p = (r_g) = \frac{b}{c} = 4.596\%$$

$$\sigma_p = (\sigma_g) = \sqrt{1/c} = 7.268\%$$

From Equation (17), calculation $W_g = \frac{V^{-1}1}{c}$, it uses Excel function MMULT (V^{-1}, 1), and obtain the weight (22.56%, 4.10%, 73.34%). Therefore, the minimum variance portfolio is characterized by an expected return of 4.596% and risk of 7.268% with portfolio weights of 22.56% for stock A, 4.10% for stock B and 73.34% for stock C.

Step 4: Build the capital market line under different borrowing and lending rate

The capital market line under different borrowing and lending rate are CMLb and CMLl. Calculate the slope of k_b and k_l by using Equation (20).

$$k_b = \sqrt{c((r_b - b/c)^2 + d/c^2)} = 0.813526$$

$$k_l = \sqrt{c((r_l - b/c)^2 + d/c^2)} = 0.81533$$

Calculate the tangent point by using Equation (21). Calculate the portfolio weight by using Equation (23). Therefore, the equation of CML_b is $r_p = r_b + k_b\sigma_p$ = 3.7 + 0.813526 σ_p. The tangent point T_b is $(\delta_p, r_p) = (9.42785\%, 7.70681\%)$. We find that the portfolio weights of 19.9% for stock A, 42.3% for stock B and 37.8% for stock C. The equation of CML_l is $r_p = r_l + k_l\sigma_p$ = 2.0 + 0.815336 σ_p. The tangent point T_l is $(\delta_p, r_p) = (9.4136\%, 7.6952\%)$. We find that the portfolio weights of 10% for stock A, 30.4% for stock B and 59.6.8% for stock C.

Step 5: Analyze different borrowing and lending rate in portfolio Efficiency Frontier.

The line tracing the points from the minimum variance portfolio A to B is the efficient frontiers. A risk adverse investor will never hold a portfolio which is to the south-east of point A. Since any point to the south-east of A such as C would have a corresponding point like D which has a higher expected return than C with the same portfolio risk. More risk adverse investors would choose points on the efficient frontier that is closed to point A. Similarly, an investor who is less risk adverse or can handle more risk would choose a portfolio closed to point B.

It traces the portfolio frontier by selecting different value of expected return and calculating the corresponding standard deviation by the Equation (16). Figure 5 shows the graph of the frontier.

The capital market line under different borrowing and lending rate are CML_b and CML_l. Broken line E -T_1 -T_2 –B in figure 5, it constitutes with line (CML_l and CML_b) and an arc ($T_, T_b$). The investors can lend at one rate but must pay a different and presumably higher rate to borrow. The efficient frontier would become E -T_1 -T_2 –B in Figure 5. Here there is a small range of risky portfolios that would be optional for investors to hold.

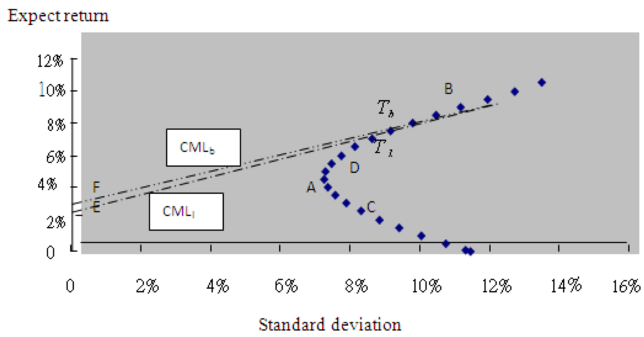

Figure 5. The Efficient Frontier and Capital Market Line

The investors have three available investment options in expected return. (1) When investor's expected return rate is less than r_l, investor can lend riskless assets to invest risk assets. (2) When investor's expected return rate is between r_l with r_b, investor can invest risk assets. (3) When investor's expected return rate is greater than r_b, investor can borrow riskless assets to invest risk assets. From the Equation (25), the beta of r_l is 0.4508 < 1 and the he beta of r_b is 0.4507 < 1. rate of return. Since the tangent portfolio has the maximum slope, one can directly obtain the portfolio weights of the tangent portfolio by maximizing the slope of the line joining a portfolio in the portfolio frontier and the risk free rate of return (Mitra and Khanna, 2014). There are two important differences between the CML and SML. First, the CML measures risk by the standard deviation of the investment while the SML considers only the systematic component of an investment's volatility. Second, as a consequence of the first point, the CML can be applied only to portfolio holdings that are already fully diversified, whereas the SML can be applied to any individual asset or collection of assets.

The contributions of this study: (1) A portfolio with the characteristic is known as an efficiency portfolio and lie in the solution set of the model (see Equation 1). The solution of his model denotes as $\frac{\sigma_p^2}{1/c} - \frac{(r_p - b/c)^2}{d/c^2} = 1$.

5. Findings and Discussions

It is noticed that if the risk free lending and borrowing rates are equal, the optimum risky portfolio is obtained by drawing a tangent to the portfolio frontier from the risk free

Where a, b, c denoted as matrix operation. It is easy calculation by Excel function. (2) The CML is denoted as weight vector too. (3) By the definition of CML, we are able to find the efficient market portfolio. (4) By the concept of tangent line of portfolio efficiency frontier model to calculate Capital Market Line CMLb (borrowing rate capital market line) and CMLl (lending rate capital market line). (5) To find out the weight of securities which are there in the portfolio in order to invest in those securities. (6) To construct CML in order to get more returns than that of efficient frontier if risk-free securities are included in the portfolio.

Xuemet and Xinshu (2003) use the concept of differential geometry to solve an efficient portfolio model. The efficient frontier of Markowitz can represent as linear equation group (simultaneous equation) that is composed by n-2 linear equations. A comparison between this study and Xuemet and Xinshu's research denotes as Table 2.

$$\sum a_{ij} w_j = b_j \ , \ i = 1, 2, ..., n-2, \quad j = 1, 2, ..., n-2$$

Table 2. A Comparison between this Study and Xuemet and Xinshu's Research

	Xuemet and Xinshu (2003)	This study
Design	The efficient frontier of Markowitz can represent as linear equation group (simultaneous equation) that is composed by n-2 linear equations. The efficient frontier of Markowitz can represent as $\sum a_{ij} w_j = b_j$, $i = 1, 2, ..., n-2, \quad j = 1, 2, ..., n-2$	The efficient frontier of Markowitz can represent as $\frac{\sigma_p^2}{1/c} - \frac{(r_p - b/c)^2}{d/c^2} = 1$ By the definition of the CML, the efficient market portfolio thus can be identified.
Methodology	Use different geometry concept to calculate normal vector. Construct a simultaneous equation in order to obtain efficient portfolio.	Use the matrix operation concept to calculate efficient portfolio. Use the concept of tangent line to calculate Capital Market Line CML_b (borrowing rate capital market line) and CML_l (lending rate capital market line)
Algorithmic tool	MATLAB software	EXCEL software
Results	It is difficult to understand the algorithm. The computation of efficient is not easy.	Use Excel software, the computation of the efficient frontier is fairly easy.

6. Conclusions

This paper has discussed an algorithm (matrix operation) to look for the efficient market portfolio of CAPM and a new method (tangent line) to obtain the CML. This algorithm applied in different borrowing and lending rate portfolio problems. Under this algorithmic tool, this paper have finished the following works (1) Prove portfolio frontier is a hyperbola in mean-standard deviation space (2) CML is also expressed as the portfolio weight vector (3) In mean-variance portfolio efficiency frontier model, calculate different borrowing and lending rate's CML (4) calculate the tangent point and slop of CML (5) The steps of this proposed method

Excel is far from the best program for generating the efficient frontier and is no limited in the number of assets it can handle. It finds that Excel, the computation of the efficient frontier is fairly easy. It finds that this paper is helpful to the correct application of the capital market line based on efficient frontier of portfolio with borrowing and rate, enriching the theory and method of invest management. The further research will focus on performing the calculation of this algorithm tool such as Excel and.

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