Mathematics and Statistics 2(6): 205-213, 2014
DOI: 10.13189/ms.2014.020602 http://www.hrpub.org

Characterization of Generalized Uniform Distribution Through Expectation of Function of Order Statistics

Bhatt Milind B.

Department of Statistics, Sardar Patel University, Vallabh Vidyanagar-388120, Anand, Gujarat, India

*Corresponding Author: bhattmilind_b@yahoo.com

Abstract Normally the mass of a root has a uniform distribution but some of have different uniform distribution named generalized uniform distribution (GUD). The characterization result based on expectation of function of order statistics has been obtained for generalized uniform distribution. Applications are given for illustrative purpose.

Keywords Generalize Uniform Distribution, Uniform Distribution, Probability Density Function

1 Introduction

Plant develops into the reproductive phase of growth, a mat of smaller roots grows near the surface to a depth of approximately $(\frac{1}{6})^{th}$ of maximum depth achieve [See G. Ooms and K. L. Moore [1]]. Dixit [2] studied problem of efficient estimation of parameters of a uniform distribution in the presence of outliers. He assumed that a set of random variables $X_1, X_2, , X_n$ represents the masses of roots where out of n random variables some of these roots (say k) have different masses therefore, those masses have different uniform distribution with unknown parameters and these k observations are distributed with Generalize Uniform Distribution (GUD) with probability density function (pdf)

$$f(x;\theta) = \begin{cases} \frac{\alpha+1}{\theta^{\alpha+1}} x^{\alpha}; & a < x < \theta < b; \alpha > -1, a \equiv 0, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $-\infty < a < b < \infty$ are known constants, x^{α} is positive absolutely continuous function and $(\frac{1}{\theta})^{\alpha+1}$. ince derivative of $x^{\alpha+1}$ and since range is truncated by θ from left $a \equiv 0$. Dixt [3] obtained Maximum Likelihood Estimator (MLE) and the Uniformly Minimum Variance (UMVU) estimator of reliability functional; $P\lfloor X > Y \rfloor$ hat the UMVUE is better than MLE when one parameter of GUD is known, where as both parameters of the GUD are unknown, $P\lfloor X > Y \rfloor$ estimated by using mixture estimate and is consistent.

In this paper characterizing property of GUD with pdf given in (1) has been studied which also holds for uniform distribution on interval $(0, \theta)$ when $\alpha = 0$ in (1). Various approaches used to characterize uniform distribution, few of them have used coefficient of correlation of smaller and the larger of a random sample of size two, Bartoszyn'ski [4], Terreel [5], Lopez -Bldzquez [6] were as Kent [7], has used independence of sample mean and variance, Lin [8], Too [9], Arnold [10], Driscoll [11], Shimizu [12] ,Abdelhamid [13] have used moment conditions, n-fold convolution modulo one and inequalities of chernoff-type also used see Chow [14] and Sumrita [15].

In contrast to all above brief research background and application of characterization of member of Pearson family, this research not provide unified approach to characterize generalize uniform distribution.

The aim of the present research note is to give path breaking new characterization for generalize uniform distribution through expectation of function of order statistics, using identity and equality of expectation. Characterization theorem derived in section 2 with method for characterization as remark and section 3 devoted to applications for illustrative purpose including special case of uniform distribution.

2 Characterization theorem

Theorem

Let $X_1, X_2, , X_n$ be a random sample of size n from distribution function F. Let $X_{1:n} < X_{2:n}, ..., < X_{n:n}$ be the set of corresponding order statistics.Assume that F is continuous on the interval (a, b) where $-\infty < a < b < \infty$. Let $g(X_{n:n})$ and $\phi(X_{n:n})$ be be two distinct differentiable and intregrable functions of n^{th} order statistic; $X_{n:n}$ on the interval (a, b) where $-\infty < a < b < \infty$ and moreover $g(X_{n:n})$ be non-constant function of $X_{n:n}$. Then

$$E\Big[g(X_{n:n}) + \Big(\frac{X_{n:n}}{n(\alpha+1)}\Big)\frac{d}{dX_{n:n}}g(X_{n:n})\Big] = g(\theta), \tag{2}$$

is the necessary and sufficient condition for pdf $f(x;\theta)$ of F to be $f(x;\theta)$ defined in (1).

Proof

Given $f(x;\theta)$ defined in (1), for necessity of (2) if $\phi(X_{n:n})$ is such that $g(\theta) = E[\phi(X_{n:n})]$ where $g(\theta)$ is differentiable function then using $f(X_{n:n};\theta)$; pdf of n^{th} order statistic one gets,

$$g(\theta) = \int_a^\theta \phi(x_{n:n}) f(x_{n:n};\theta)\mathrm{d}x_{n:n}. \tag{3}$$

Differentiating (2) with respect to θ on both sides and replacing $X_{n:n}$ for θ, and simplifying one gets

$$\phi(x_{n:n}) = g(x_{n:n}) + \Big(\frac{x_{n:n}}{n(\alpha+1)}\Big)\frac{d}{dx_{n:n}}g(x_{n:n}), \tag{4}$$

which establishes necessity of (2). Conversely given (2), let $k(X_{n:n};\theta)$ be the p.d.f. of pdf of n^{th} order statistic such that

$$g(\theta) = \int_a^\theta \Big[g(x_{n:n}) + \Big(\frac{x_{n:n}}{n(\alpha+1)}\Big)\frac{d}{dx_{n:n}}g(x_{n:n})\Big]k(x_{n:n};\theta)\mathrm{d}x_{n:n}, \tag{5}$$

Since $a = 0$, the following identity holds

$$g(\theta) \equiv \frac{1}{\theta^{n(\alpha+1)}}\int_a^\theta \Big[\frac{d}{dx_{n:n}}g(x_{n:n})x_{n:n}^{n(\alpha+1)}\Big]\mathrm{d}x_{n:n}. \tag{6}$$

Differentiating integrand $g(x_{n:n})x_{n:n}^{n(\alpha+1)}$ with respect to $x_{n:n}$ and simplifying after taking $\frac{d}{dx_{n:n}}x_{n:n}^{n(\alpha+1)}$ as one factor one gets (6)

$$\begin{aligned} g(\theta) \equiv \int_a^\theta &\Big[g(X_{n:n}) + \Big(\frac{x_{n:n}^{n(\alpha+1)}}{\frac{d}{dX_{n:n}}x_{n:n}^{n(\alpha+1)}}\Big)\frac{d}{dX_{n:n}}g(X_{n:n})\Big] \\ &.\Big(\frac{\alpha+1}{\theta^{\alpha+1}}\frac{d}{dx_{n:n}}x_{n:n}^{n(\alpha+1)}\Big)\mathrm{d}x_{n:n}. \end{aligned} \tag{7}$$

Substituting derivative of $x_{n:n}^{n(\alpha+1)}$ (7) one gets (7) as

$$g(\theta) \equiv \int_a^\theta \phi(x_{n:n})\Big(\frac{\alpha+1}{\theta^{\alpha+1}}x_{n:n}^{n(\alpha+1)}\Big)\mathrm{d}x_{n:n}, \tag{8}$$

where $\phi(X_{n:n})$ is as derived in (4). By uniqueness theorem from (5) and (8)

$$k(x_{n:n};\theta) = \frac{\alpha+1}{\theta^{\alpha+1}}x_{n:n}^{n(\alpha+1)}. \tag{9}$$

Since $\frac{1}{a^\alpha}$ is decreasing function for $-\infty < a < b < \infty$ and $a^\alpha = 0$ is satisfy only when range of $x_{n:n}$ is truncated by θ from right and integrating (6) on the interval (a,θ) on both sides, one gets (8) as

$$1 = \int_a^\theta k(x_{n:n};\theta)\mathrm{d}x_{n:n}. \tag{10}$$

For $n = 1$, $[k(x_{n:n};\theta)]_{n=1}$ reduces to $f(x;\theta)$ defined in (1). Hence sufficiency of (2) establishes.

Remark 2.1

Using $\phi(X_{n:n})$ derived in (4), the $f(x;\theta)$ given in (1) can be determined by

$$M(X_{n:n}) = \frac{\frac{d}{dX_{n:n}}g(X_{n:n})}{\phi(X_{n:n}) - g(X_{n:n}))}. \tag{11}$$

and pdf is given by

$$f(x;\theta)=\Big[\frac{\frac{d}{dX_{n:n}}T(X_{n:n})}{T(X_{n:n})}\Big]_{n=1}. \tag{12}$$

where $T(X_{n:n})$ is increasing function for $-\infty<a<b<\infty$ with $T(a)=0$ such that it satisfies

$$M(X_{n:n})=\frac{d}{dX_{n:n}}\log T(X_{n:n}). \tag{13}$$

Examples

Using method describe in remark generalize uniform distribution (GUD) through expectation of non-constant function of order statistics such as mean, rth raw moment, e^{θ} ,$e^{-\theta}$, pth quantile, distribution function, reliability function and hazard function is given to illustrate application and significant of unified approach of characterization result.

Examples 3.1 Characterization of generalize uniform distribution (GUD) through Uniformly Minimum Variance Unbiased Estimator (UMVUE); $\widehat{\lambda}(t)$ of $\lambda(t)$ the hazard function is given to illustrate application and significant of unified approach of characterization result

$$g(X_{n:n})=-\frac{1}{n}\Big(\frac{t^{\alpha}}{t^{\alpha}-X_{n:n}^{\alpha}}\Big)\Big[\frac{\alpha X_{n:n}^{\alpha}}{t^{\alpha}-X_{n:n}^{\alpha}}+n(\alpha+1)\Big]=\widehat{\lambda}(t). \tag{14}$$

Using (4) one gets

$$\begin{aligned}\phi(x_{n:n})&=g(x_{n:n})+\Big(\frac{x_{n:n}}{n(\alpha+1)}\Big)\frac{d}{dx_{n:n}}g(x_{n:n})\\&=-t^{\alpha}\Big(\frac{2\alpha X_{n:n}^{\alpha}+n(\alpha+1)}{n(t^{\alpha}-X_{n:n}^{\alpha})^2}\\&+\frac{\alpha^2X_{n:n}^{\alpha}(t^{\alpha}-X_{n:n}^{\alpha})}{n^2(\alpha+1)(t^{\alpha}-X_{n:n}^{\alpha})^3}\Big)\end{aligned} \tag{15}$$

and (11) of remark 2.1 will be

$$M(X_{n:n})=\frac{\frac{d}{dX_{n:n}}g(X_{n:n})}{\phi(X_{n:n})-g(X_{n:n}))}=\frac{\alpha+1}{X_{n:n}}. \tag{16}$$

By characterizing method given in remark 2.1

$$\frac{d}{dX_{n:n}}\log(X_{n:n}^{n(\alpha+1)})=\frac{\alpha+1}{X_{n:n}}=M(X_{n:n}), \tag{17}$$

then

$$T(X_{n:n})=X_{n:n}^{n(\alpha+1)}, \tag{18}$$

and

$$f(x;\theta)=\Big[\frac{\frac{d}{dx_{n:n}}T(x_{n:n})}{T(x_{n:n})}\Big]_{n=1}=\frac{\alpha+1}{\theta^{\alpha+1}}x^{\alpha}. \tag{19}$$

Examples 3.2 Characterization of generalize uniform distribution (GUD) through Uniformly Minimum Variance Unbiased Estimator (UMVUE); $\widehat{\theta}^r$ of θ^r is given to illustrate application and significant of unified approach of characterization result

$$g(X_{n:n})=\frac{n\alpha+n+r}{n(\alpha+n+r)}X_{n:n}^{r}=\widehat{\theta}^r. \tag{20}$$

Using (4) one gets

$$\begin{aligned}\phi(x_{n:n})&=g(x_{n:n})+\Big(\frac{x_{n:n}}{n(\alpha+1)}\Big)\frac{d}{dx_{n:n}}g(x_{n:n})\\&=\frac{(n\alpha+n+r)^r}{n^2(\alpha+1)(\alpha+r+1)}X_{n:n}^{r}\end{aligned} \tag{21}$$

and repeating (16) to (19), (1) is characterized.

Examples 3.3 Using the uniformly minimum variance unbiased (UMVU) estimator $\widehat{g}(\theta)$ and maximum likelihood estimator (MLE) $\widetilde{g}(\theta)$ of $g(\theta)$ such as mean; $\mu_1^{'}(\theta)$, rth moment; $\mu_r^{'}(\theta)$, e^{θ}, $e^{-\theta}$, pth quantile; $Q_p(\theta)$, distribution function; $F(t)$; reliability function; $\bar{F}(t)$, hazard rate; $\lambda(t)$, GUD characterized to illustrate application and significant of unified approach of characterization result. The UMVU estimators

$$\widehat{g}(\theta) = \begin{cases} \widehat{\mu_1^{'}(\theta)} = \frac{n\alpha+n+1}{\alpha+1} X_{n:n};, & \text{for i = 1,} \\ \widehat{\mu_r^{'}(\theta)} = \frac{n\alpha+n+r}{n(\alpha+r+1)} X_{n:n}^r, & \text{for i = 2,} \\ \widehat{e^{\theta}} = \left[1 + \frac{X_{n:n}}{n(\alpha+1)}\right] e^{X_{n:n}}; & \text{for i = 3,} \\ \widehat{e^{-\theta}} = \left[1 - \frac{X_{n:n}}{n(\alpha+1)}\right] e^{-X_{n:n}}; & \text{for i = 4,} \\ \widehat{Q_p}(\theta) = p^{\frac{1}{\alpha+1}} \left[1 + \frac{1}{n(\alpha+1)}\right] X_{n:n}; & \text{for i = 5,} \\ \widehat{F}(t) = \left[1 - \frac{1}{n}\right] \left(\frac{t}{X_{n:n}}\right)^{\alpha+1}; & \text{for i = 6,} \\ \widehat{\bar{F}}(t) = 1 - \left[1 - \frac{1}{n}\right] \left(\frac{t}{X_{n:n}}\right)^{\alpha+1}, & \text{for i = 7,} \\ \widehat{\lambda}(t) = \frac{(\alpha+1)t^{\alpha}}{X_{n:n}^{\alpha+1} - t^{\alpha+1}} \left[1 - \frac{1}{n(X_{n:n}^{\alpha+1} - t^{\alpha+1})}\right] & \text{; for i = 8,} \end{cases} \tag{22}$$

and MLE

$$\widetilde{g}(\theta) = \begin{cases} \widetilde{\mu_1^{'}(\theta)} = \dfrac{\alpha+1}{\alpha+2} X_{n:n}; \quad \text{for i = 9,} \\ \widetilde{\mu_r^{'}(\theta)} = \dfrac{\alpha+1}{(\alpha+r+1)} X_{n:n}^r; \quad \text{for i = 10,} \\ \widetilde{e^{\theta}} = e^{X_{n:n}}; \quad \text{for i = 10,} \\ \widetilde{e^{-\theta}} = e^{-X_{n:n}} \quad \text{for i = 10,} \\ \widetilde{Q_p}(\theta) = p^{\alpha+1} X_n : n; \quad \text{for i = 13,} \\ \widetilde{F}(t) = \left(\dfrac{t}{X_{n:n}}\right)^{\alpha+1}; \quad \text{for i = 14,} \\ \widetilde{\bar{F}}(t) = 1 - \left(\dfrac{t}{X_{n:n}}\right)^{\alpha+1}; \quad \text{for i = 15,} \\ \widetilde{\lambda}(t) = \dfrac{(\alpha+1)t^{\alpha}}{X_{n:n}^{\alpha+1} - t^{\alpha+1}}, \quad \text{for i = 16,} \end{cases} \tag{23}$$

one gets

$$[\phi_i(X_{n:n}) - \widehat{g}_i(\theta)] = \begin{cases} \frac{n\alpha+n+1}{n^2(\alpha+1)(\alpha+2)} X_{n:n}; & \text{for i = 1,} \\ \frac{r(n\alpha+n+r)}{n^2(\alpha+1)(\alpha+r+1)} X_{n:n}^r; & \text{for i = 2,} \\ \frac{X_{n:n}}{n(\alpha+1)} \Big[1 + \frac{X_{n:n}}{n(\alpha+1)} \\ \quad \frac{1}{n(\alpha+1)} \Big] e^{X_{n:n}}; & \text{for i = 3,} \\ \Big[\Big(\frac{X_{n:n}}{n(\alpha+1)} \Big)^2 - \frac{X_{n:n}}{n(\alpha+1)} \\ \quad - \frac{X_{n:n}}{(n(\alpha+1))^2} \Big] e^{-X_{n:n}}; & \text{for i = 4,} \\ \frac{X_{n:n} P^{\frac{1}{\alpha+1}}}{n(\alpha+1)} \Big[1 + \frac{1}{n(\alpha+1)} \Big]; & \text{for i = 5,} \\ -\frac{1}{n} \Big[1 - \frac{1}{n} \Big] \Big(\frac{t}{X_{n:n}} \Big)^{\alpha+1}; & \text{for i = 6,} \\ \frac{1}{n} \Big[1 - \frac{1}{n} \Big] \Big(\frac{t}{X_{n:n}} \Big)^{\alpha+1}; & \text{for i = 7;} \\ A; & \text{for i = 8,} \end{cases} \tag{24}$$

where

$$A = \Big[n^2 \Big(t^{\alpha+1} - X_{n:n}^{\alpha+1} \Big)^3 \Big]^{-1} \Big[(\alpha+1) t^{\alpha} X_{n:n}^{\alpha} \\ \Big((n+1) t^{\alpha+1} - (n-1) X_{n:n}^{\alpha} \Big) \Big]$$

and

$$[\phi_i(X_{n:n}) - \widetilde{g}(\theta)] = \begin{cases} \dfrac{X_{n:n}}{n(\alpha+2)} \text{ ; for i = 9,} \\[2ex] \dfrac{rX_{n:n}}{n(\alpha+r+1)} \text{ ; for i = 10,} \\[2ex] \dfrac{X_{n:n}}{n(\alpha+1)}e^{X_{n:n}} \text{ ; for i = 11,} \\[2ex] -\dfrac{X_{n:n}}{n(\alpha+1)}e^{-X_{n:n}} \text{ ; for i = 12,} \\[2ex] \dfrac{X_{n:n}P^{\frac{1}{\alpha+1}}}{n(\alpha+1)} \text{ ; for i = 13,} \\[2ex] -\dfrac{1}{n}\Big(\dfrac{t}{X_{n:n}}\Big)^{\alpha+1} \text{ ; for i = 14,} \\[2ex] \dfrac{1}{n}\Big(\dfrac{t}{X_{n:n}}\Big)^{\alpha+1} \text{ ; for i = 15,} \\[2ex] \dfrac{(\alpha+1)t^{\alpha}X_{n:n}^{\alpha+1}}{n(X_{n:n}^{\alpha+1}-t^{\alpha+1})} \text{ ; for i = 16,} \end{cases} \tag{25}$$

respectively.

Then by defining $M(X_{n:n})$ given in (11) and substituting $T(X_{n:n})$ as appeared in (13) for (12), $f(x;\theta)$ given in (1)is characterized.

Examples 3.4 In context of remark 2.2 uniform distribution on interval $(0,\theta)$ with pdf given in (15) characterized through (UMVU) estimator;

$$g(X_{n:n}) = \Big[1 - \frac{1}{n}\Big]\Big(\frac{X_{n:n}}{n}\Big) = \widehat{Q_p}, \tag{26}$$

of Q_p the p^{th} quantile is given to illustrate application and significant of unified approach of characterization result given in remark 2.2.

For $\alpha = 0$, using (4) one gets

$$\begin{aligned} \phi(X_{n:n}) &= g(X_{n:n}) + \Big(\frac{X_{n:n}}{n}\Big)\frac{d}{dX_{n:n}}g(X_{n:n}) \\ &= p\Big(1+\frac{1}{n}\Big)^2 X_{n:n} \end{aligned} \tag{27}$$

and for $\alpha = 0$, (11) will be

$$M(X_{n:n}) = \frac{\frac{d}{dX_{n:n}}g(X_{n:n})}{\phi(X_{n:n}) - g(X_{n:n}))} = \frac{n}{X_{n:n}}. \tag{28}$$

By characterizing method given in remark 2.2 for $f(x;\theta)$ given in (15) for $\alpha = 0$, one gets

$$\frac{d}{dX_{n:n}}\log(X_{n:n}^n) = \frac{n}{X_{n:n}} = M(X_{n:n}), \tag{29}$$

then

$$T(X_{n:n}) = X_{n:n}^n, \tag{30}$$

and

$$f(x;\theta) = \Big[\frac{\frac{d}{dx_{n:n}}T(x_{n:n})}{T(x_{n:n})}\Big]_{n=1} = \frac{1}{\theta}. \tag{31}$$

given in (15) characterized.

Examples 3.5

Using method given in remark 2.2 for $f(x;\theta)$ given in (15), characterized through (UMVU) estimator $\widehat{g}(\theta)$ and maximum likelihood estimator (MLE) $\widetilde{g}(\theta)$ of $g(\theta)$ such as mean; $\mu_1^{'}(\theta)$, rth moment; $\mu_r^{'}(\theta)$, e^{θ}, $e^{-\theta}$, pth quantile; $Q_p(\theta)$, distribution function; $F(t)$; reliability function; $\bar{F}(t)$, hazard rate; $\lambda(t)$. The UMVU estimators

$$\widehat{g}(\theta) = \begin{cases} \widehat{\mu_1^{'}(\theta)} = \left[1 + \frac{1}{n}\right]\frac{X_{n:n}}{2} & \text{; for i = 1,} \\ \widehat{\mu_r^{'}(\theta)} = \left[1 + \frac{r}{n}\right]\frac{rX_{n:n}^r}{r+1} & \text{; for i = 2,} \\ \widehat{e^{\theta}} = \left[1 + \frac{X_{n:n}}{n}\right]e^{X_{n:n}} & \text{; for i = 3,} \\ \widehat{e^{-\theta}} = \left[1 - \frac{X_{n:n}}{n}\right]e^{-X_{n:n}} & \text{; for i = 4,} \\ \widehat{Q_p}(\theta) = p\left[1 + \frac{1}{n}\right]X_{n:n}; & \text{for i = 5,} \\ \widehat{F}(t) = \left[1 - \frac{1}{n}\right]\left(\frac{t}{X_{n:n}}\right) & \text{; for i = 6,} \\ \widehat{\bar{F}}(t) = 1 - \left[1 - \frac{1}{n}\right]\left(\frac{t}{X_{n:n}}\right) & \text{; for i = 7,} \\ \widehat{\lambda}(t) = \frac{1}{X_{n:n}-t}\left[1 - \frac{1}{n(X_{n:n}-t)}\right] & \text{; for i = 8,} \end{cases} \tag{32}$$

and MLE

$$\widetilde{g}(\theta) = \begin{cases} \widetilde{\mu_1^{'}(\theta)} = \dfrac{X_{n:n}}{2}. \text{; for i = 9,} \\ \widetilde{\mu_r^{'}(\theta)} = \dfrac{X_{n:n}^r}{r+1} \text{; for i = 10,} \\ \widetilde{e^{\theta}} = e^{X_{n:n}} \text{; for i = 11,} \\ \widetilde{e^{-\theta}} = e^{-X_{n:n}} \text{; for i = 12,} \\ \widetilde{Q_p}(\theta) = pX_{n:n} \text{; for i = 13,} \\ \widetilde{F}(t) = \dfrac{t}{X_{n:n}} \text{;for i = 14,} \\ \widetilde{\bar{F}}(t) = 1 - \left(\dfrac{t}{X_{n:n}}\right) \text{; for i = 15,} \\ \widetilde{\lambda}(t) = \dfrac{1}{X_{n:n}-t} \text{; for i = 16,} \end{cases} \tag{33}$$

one gets

$$[\phi_i(X_{n:n}) - \widehat{g_i}(\theta)] = \begin{cases} \left[1+\frac{1}{n}\right]\frac{X_{n:n}}{2n} \text{ ; for i = 1,} \\ \left[1+\frac{r}{n}\right]\frac{rX^r_{n:n}}{n(r+1)} \text{ ; for i = 2,} \\ \frac{X_{n:n}}{n}\left[1+\frac{X_{n:n}}{n} + \frac{1}{n}\right]e^{X_{n:n}}; \text{ for i = 3,} \\ \frac{X_{n:n}}{n}\left[\frac{X_{n:n}}{n} - 1 - \frac{1}{n}\right]e^{-X_{n:n}} \text{ ; for i = 4,} \\ \frac{X_{n:n}P.}{n}\left[1+\frac{1}{n.}\right] \text{ ; for i = 5,} \\ -\frac{n-1}{n^2}\left(\frac{t}{X_{n:n}}\right) \text{ ; for i = 6,} \\ \frac{n-1}{n^2}\left(\frac{t}{X_{n:n}}\right) \text{ ; for i = 7,} \\ \left(n^2(t-X_{n:n})^3\right)^{-1}\left[X_{n:n}\left(t+nt+X_{n:n}-nX_{n:n}\right)\right] \text{ ; for i = 8,} \end{cases} \quad (34)$$

and

$$[\phi_i(X_{n:n}) - \widetilde{g}(\theta)] = \begin{cases} \frac{X_{n:n}}{2n} \text{ ; for i = 9,} \\ \frac{rX^r_{n:n}}{n(r+1)} \text{ ; for i = 10,} \\ \frac{X_{n:n}}{n}e^{X_{n:n}} \text{ ; for i = 11,} \\ -\frac{X_{n:n}}{n}e^{-X_{n:n}} \text{ ; for i = 12,} \\ \frac{X_{n:n}}{n}p \text{ ; for i = 13,} \\ -\frac{1}{n}\left(\frac{t}{X_{n:n}}\right) \text{ ; for i = 14,} \\ \frac{1}{n}\left(\frac{t}{X_{n:n}}\right) \text{ ; for i = 15,} \\ -\frac{X_{n:n}}{n(X_{n:n}-t)^2}; \text{ ; for i = 16,} \end{cases} \quad (35)$$

respectively.

Then using characterizing method descibed in remark 2.2, for $\alpha = 0$ define $M(X_{n:n})$ given in (11) and substituting $T(X_{n:n})$ as appeared in (13) for (12), $f(x;\theta)$ is characterized.

Acknowledgements

This work was supported by UGC Major Research Project : F.No.42-39/2013(SR) dated 12-3-2013

REFERENCES

[1] G. Ooms, and K. L. Moore. A model assay for genetic and environmental changes in the architecture of intact roots systems of plants grown in vitro, Plant Cell, Tissue and Organ Culture, 27, 129-139, 1991.

[2] U. J. Dixit and Masoom, Ali and J. Woo. Efficient estimation of parameters of a uniform distribution in the presence of outliers, Soochow Journal of mathematics, Volume 29, No. 4, pp. 363-369, October 2003.

[3] U. J. Dixit and K. D. Phal. Estimation of P (X Y) for the uniform distribution in the Presence of Outliers, Journal of Probability and Statistical Science, 7(1), 11-18, Feb, 2009.

[4] R. Bartoszynski. Personal communication, 1980.

[5] G. R. Terrel. A characterization of rectangular distributions. The Annals of Probability, Vol. 11, N 3, 823-826, 1983.

[6] F. Lopez -Bldzquez and B. Salamanca Milno. On Terrers characterization of uniform distribution Statistical Papers 40, 335-342, 1999.

[7] J. T. Kent, K. V. Mardia and J. S. Rao. Characterization of the uniform distribution on the circle. The Annals of Statistics, Vol. 7, No. 4, 882-889, 1979.

[8] G. D. Lin. Characterizations of uniform distributions and of exponential distributions. Sankhya A, 50: 64-69, 1988.

[9] Y. H. Too and G. D. Lin. Characterizations of uniform and exponential distributions. Statistics and Probability Letters 7 357-359, 1989.

[10] B. C. Arnold and G. Meeden. A characterization of the uniform distribution based on summation modulo one, with application to fractional backlogs. Austral. J. Statist., 18, (3), 173-175, 1976.

[11] M. F. Driscoll. On pairwise and mutual independence: characterizations of rectangular distributions, J. Amer. Statist. Assoc. 73, 432-433, 1978.

[12] R. Shimizu and J. S. Huang. On a characteristic property of the uniform distribution. Ann. Inst. Statist. Math., 35, 91-94, 1983.

[13] S. N. Abdelhamid. On a characterization of rectangular distributions. Statistics and Probability Letters, 3, 235-238, 1985.

[14] Y. Chow and S. Huang. A characterization of the uniform distribution via moments of n-fold convolution modulo one. Sankhya : The Indian Journal of Statistics, Volume 61, Series A, Pt. 1, pp. 148-151, 1999.

[15] P. Sumitra. and B. Subir. Characterization of uniform distributions by inequalities of chernoff-type. Sankhya ? : The Indian Journal of Statistics 1990, Volume 52, Series A, Pt. 3, pp. 376-382, 1990.