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Prime Number Conjecture

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Abstract This paper builds on Goldbach's weak conjecture, showing that all primes to infinity are composed of 3 smaller primes, suggesting that the modern definition of a prime number may be incomplete, requiring revision. The results indicate that prime numbers should include 1 as a prime number and 2 as a composite number, adding a new dimension to the most fundamental of all integers.

Keywords number theory, prime numbers, Goldbach's weak conjecture, Twin Prime Conjecture

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In a letter to the great mathematician Leonard Euler, Goldbach posited that all prime numbers ($\mathbb{P}'$) greater than five ($\mathbb{P}' > 5$) are the sum of 3 smaller primes, known as Goldbach's weak conjecture (Bruckman (2006); Bruckman (2008); Chang (2013) & Shu-Ping (2013). The conjecture was recently proven true; therefore, all primes to infinity are composed of 3 smaller primes. The following theorem builds on the conjecture, revealing that the modern definition of a prime number may require revision.

Theorem

Premise #1: Assume that *all* prime numbers are the sum of 3 smaller primes, not only those > 5 (as proposed by Goldbach to Euler) with only one exception, the number 1 (1 was assumed prime at the time of Euler and Goldbach).

Premise #2: Assume that the number *two is not, prime.* This claim is intuitive, not one *even* prime has been identified for any number up to 17 million digits in length; so why should it be assumed that the *even number 2* is prime?

Given the premises, a proof follows that resolves the problem in Goldbach's Weak Conjecture regarding primes of less than 6, en passant proving the primality of 1 and non-primality of 2. Contrary to the work of Goldbach, the numbers 5 and 3 are shown to be composed of 3 smaller prime numbers and the number 2 does not fit the revised definition of a prime number:

Proof 1.

$$3 + 1 + 1 = 5$$

$$1 + 1 + 1 = 3$$

$$1 + 1 + ? \neq 2$$

Quod Erat Demonstrandum (Q.E.D.).

Therefore, the modern definition of a prime number is de facto, incomplete. A new prime number definition follows:

Deduction

1. A prime number is an *odd* and natural number,
2. composed of *the sum of three smaller prime numbers*,
3. with *only two factors*: 1 and itself.

Proof 2.

In the second proof, all numbers, including the number 1 are shown to include at least 2 out of 3 of the revised prime number criteria, except the number 2:

1. Odd number;
2. Only factors are 1 and itself,
3. Composed of the sum of 3 smaller primes.

The numbers 5, 3, 2, and 1 are examined for primality:

#5: 3 + 1 + 1 = 5; factors (5, 1); odd number (3 out of 3);

#3: 1 + 1 + 1 = 3; factors (3, 1); odd number (3 out of 3);

#2: 1 + 1 + ? = 2; factors (2, 1); even number (1 out of 3);

#1: 1 + ? + ? = 1; factors (1, 1); odd number (2 out of 3);

Deduction

Given that the number 2 is the only prime number that meets only 1 of the 3 prime number criteria, it is de facto not a prime number. Since the number 1 meets 2 out of 3 of the requirements, it is proposed to be a *weak* prime. In addition, this creates a new prime twin set (1, 3) adding further proof that 2 is not prime, since every twin prime must be separated by an *even* and therefore *composite* number. A logical proof follows (see Equation 1.1):

$$\therefore \{x \mid x \in \mathbb{P}',\ 1 \subset x \wedge 2 \not\subset x\} \Rightarrow \top \qquad (1.1).$$

Hóper édei deîxai (OEΔ)

The findings suggest that the axioms underpinning prime numbers require adjustment: ($2 \neq \mathbb{P}'$), and ($1 = \mathbb{P}'$). Although the proof implies that no even primes exist, in the event that a large even prime is eventually identified, the Prime Number Conjecture proof remains legitimate as long as the new prime is composed of the sum of three smaller primes. The new prime would be classified as a weak prime, like the number one, meeting 2 of the 3 prime criteria.

New Twin Prime

Using the axioms underpinning the Prime Number Conjecture, a new and unrecognized twin prime emerges:

Theorem

Given the Prime Number Conjecture and the twin prime formula ($n = \mathbb{P}'$); $n + 2 =$ twin $\mathbb{P}''$), a previously unknown twin number set includes 1 and 3:

Proof:

$$n = \mathbb{P}';\ n + 2 = \mathbb{P}''$$

$$1 = \mathbb{P}';\ 1 + 2 = \mathbb{P}''$$

$$1 = \mathbb{P}';\ 1 + 2 = 3 = \mathbb{P}''$$

$$\mathbf{1, 3 = \mathbb{P}''}$$

Alternative Proof

Given a specific, positive, even, and natural number, an alternative proof yields similar results:

$$(2\mathbb{Z}):\ n = 2\mathbb{Z};\ n +/- 1 = \mathbb{P}''$$

$$2 + 1 = 3 = \mathbb{P}'$$

$$2 - 1 = 1 = \mathbb{P}'$$

$$\mathbf{1, 3} = \mathbb{P}''$$

Deduction

Given the Prime Number Conjecture and the twin prime formula, a previously undiscovered twin prime set emerges, **1, 3 =** $\mathbb{P}''$. The potential significance of this proof is in the field of number theory, more specifically in the search for proof or a counterexample to the Twin Prime Theory, for which the Clay Mathematics Institute offers a $1,000,000, Millennium Prize.

Discussion

This paper builds on Goldbach's weak conjecture, showing that all primes to infinity are composed of 3 smaller primes, suggesting that the modern definition of a prime number may be incomplete, requiring revision. The results indicate that the axioms underpinning prime numbers should include the number 1 as a prime number and the number 2 as a non-prime number, adding a new dimension to the most fundamental of all integers.

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REFERENCES

[1] Bruckman, P. S. (2006). A statistical argument for the weak twin primes conjecture. International Journal Of Mathematical Education In Science & Technology, 37(7), 838-842.

[2] Bruckman, P. (2008). A proof of the strong Goldbach conjecture. International Journal of Mathematical Education In Science & Technology, 39(8), 1102-1109. doi:10.1080/00207390802136560

[3] Chang, Y. C. (2013). Layman's method to verify Goldbach's conjectures. World Journal of Engineering, 10(4): 401-404. doi: 10.1260/1708-5284.10.4.401

[4] Shu-Ping Sandie Han1, s. (2013). Additive number theory: Classical problems and the structure of sumsets. Mathematics & Computer Education, 47(1), 48-58