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# Dynamic Analysis of Railway Bridges under High Speed Trains

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**Abstract** The interest in dynamic behavior of railway bridges has increased in recent years with the introduction of high speed trains. Higher speeds of the trains have resulted in larger and more complicated loads than earlier, producing significant dynamic effects. The dynamic aspects are of special interest and have often shown to be the governing factor in the structural design. Dynamic analysis of railway bridges is, therefore, required for high train speeds. The objective of this paper is to investigate the dynamic behavior of an existing railway bridge subjected to high speed trains. A railway bridge model has been developed to study dynamic effects such as oscillations produced by moving loads of constant magnitude and to obtain a relation between the velocity, acceleration, load position and deflection of the bridge at any instant of loading, with and without damping, with the help of MATLAB software. The simulation results indicate that the speed of the vehicle is a very important parameter influencing the dynamic response of a railway bridge. The amplitude of bridge deflection has been found to be the highest at speeds between 75 and 85 m/s. Further, the introduction of damping has been found to greatly influence of the amplitude of bridge deflection response. However, the peak deflection values appear at the same speed independent of the damping coefficients.

**Keywords** Railway Bridges, Vibration Control, Dynamic Behavior, Bridge Parameter

## 1. Introduction

The dynamic behavior of railway bridges under the action of moving trains is a complicated phenomenon. Railway bridges are complex structures, consisting of various structural components with different properties. In addition, the dynamic effects are influenced by the interaction between vehicles and the bridge structure. Considering the dynamic effects due to moving vehicles on bridges, the most important parameters that influence the dynamic response of railway bridges have to be considered. Vehicle speed, rail surface roughness, characteristics of the bridge structure and the vehicle, the number of vehicles and their travel paths are different parameters, influence the dynamic behavior [1].

## 2. Oscillations produced by moving loads of constant magnitude

Consider the state of oscillation generated in a girder of uniform mass and section freely supported at its end when subjected to concentrated force W. which moves at a constant speed v in the manner indicated in Fig.1

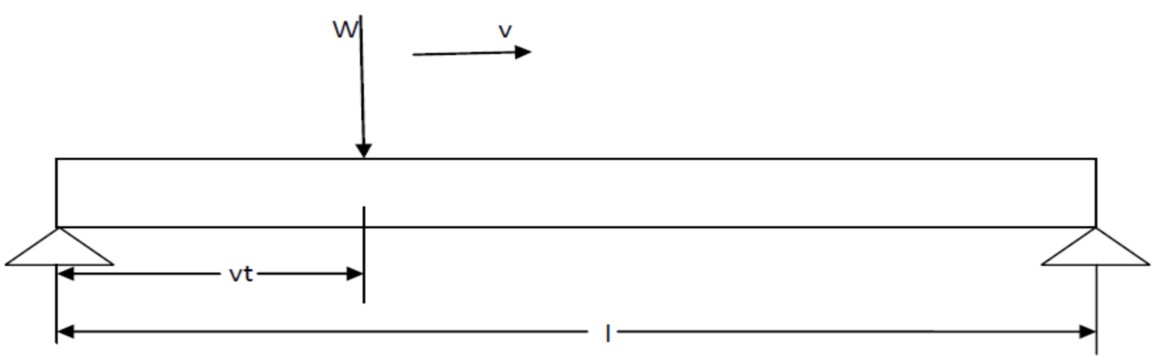


**Figure 1.** Simply supported beam subjected to a moving load [2]

At time t the force is at a distance vt from the left support and the load distribution at this instant can be described by harmonic series [2]

$$\frac{2W}{l}\left[\sin\frac{\pi vt}{l}\sin\frac{\pi x}{l}+\sin\frac{2\pi vt}{l}\sin\frac{2\pi x}{l}+\sin\frac{3\pi vt}{l}\sin\frac{3\pi x}{l}+\cdots\right] \quad (1)$$

Putting $n=\frac{v}{2l}$ the series takes the form

$$\frac{2W}{l}\left[\sin 2\pi nt \sin\frac{\pi x}{l}+\sin 4\pi nt \sin\frac{2\pi x}{l}+\sin 6\pi nt \sin\frac{3\pi x}{l}+\cdots\right] \quad (2)$$

Harmonic analysis reveals the fact that a steady concentrated load ia in effect equivalent to a series of stationary but alternating load distribution of a sinusoidal character. Neglecting the damping the state of oscillation set up given by the differential equation.

$$EI\frac{d^4y}{dx^4}+m\frac{d^2y}{dt^2}=\frac{2W}{l}\left[\sin 2\pi nt \sin\frac{\pi x}{l}+\sin 4\pi nt \sin\frac{2\pi x}{l}+\sin 6\pi nt \sin\frac{3\pi x}{l}+\cdots\right] \quad (3)$$

Forced oscillation or particular integral of this equation takes the form

$$y=\frac{2Wl^3}{\pi^4 EI}\left[\frac{\sin\frac{\pi x}{l}}{1-\left(\frac{n}{n_0}\right)^2}\sin 2\pi nt+\frac{\sin\frac{2\pi x}{l}}{2^4-\left(\frac{n}{n_0}\right)^2}\sin 4\pi nt+\frac{\sin\frac{3\pi x}{l}}{3^4-\left(\frac{n}{n_0}\right)^2}\sin 6\pi nt+\cdots\right] \quad (4)$$

In the case of moving bridges speed of the moving loads is such that $\frac{n}{n_0}$ (nearly $\frac{1}{12}$) is invariably a small fraction. So $\left(\frac{n}{n_0}\right)^2$ is very small in comparison with unity. Accordingly, very close approximation to the force oscillation is given by

$$y=\frac{2Wl^3}{\pi^4 EI}\left[\sin\frac{\pi x}{l}\sin 2\pi nt+\frac{1}{2^4}\sin\frac{2\pi x}{l}\sin 4\pi nt+\frac{1}{3^4}\sin\frac{3\pi x}{l}\sin 6\pi nt+\cdots\right] \quad (5)$$

Since $2\pi nt=\frac{\pi vt}{l}=\frac{\pi a}{l}$ , where a is the momentary distance which W has moved along the span, the above expression can be written as

$$y=\frac{2Wl^3}{\pi^4 EI}\left[\sin\frac{\pi x}{l}\sin\frac{\pi a}{l}+\frac{1}{2^4}\sin\frac{2\pi x}{l}\sin\frac{2\pi a}{l}+\frac{1}{3^4}\sin\frac{3\pi x}{l}\sin\frac{3\pi a}{l}+\cdots\right] \quad (6)$$

Hence to high degree of approximation the force oscillation due to the moving load merely the crawl deflection that is the deflection produced by the load as it moves very slowly along the bridges [2]. Such oscillation as are produced consists of free oscillation which is stimulated into existence at the beginning of the motion to satisfy the condition that the girder shall commence its motion with zero velocity. Introducing the free oscillation necessary to satisfy this condition, the complete solution of the differential equation giving the state of the oscillation is

$$y=\frac{2Wl^3}{\pi^4 EI}\left[\frac{\sin\frac{\pi x}{l}}{1-\left(\frac{n}{n_0}\right)^2}\left(\sin 2\pi nt-\frac{n}{n_0}\sin 2\pi n_0 t\right)+\frac{\sin\frac{2\pi x}{l}}{2^4-\left(\frac{2n}{n_0}\right)^2}\left(\sin 4\pi nt-\frac{n}{2n_0}\sin 2^2(2\pi n_0 t)\right)+\frac{\sin\frac{3\pi x}{l}}{3^4-\left(\frac{3n}{n_0}\right)^2}\left(\sin 6\pi nt-\right.\right.$$

$$\text{n3n0sin322πn0t+}\ldots \quad (7)$$

So far $\left(\frac{n}{n_0}\right)^2$ is very small in comparison with unity. The motion may be viewed as the crawl deflection upon which superimposed a series of free oscillations given by expression

$$y=\frac{-2Wl^3}{\pi^4 EI}\frac{n}{n_0}\left[\sin\frac{\pi x}{l}\sin 2\pi n_0 t+\frac{1}{2^5}\sin\frac{2\pi x}{l}\sin 2^2(2\pi n_0 t)+\cdots\right] \quad (8)$$

The only appreciable dynamic effect is the fundamental free oscillation

$$y=\frac{-2Wl^3}{\pi^4 EI}\frac{n}{n_0}\ \sin\frac{\pi x}{l}\sin 2\pi n_0 t \quad (9)$$

This can be written as

$$y=-\frac{n}{n_0}D_w \sin\frac{\pi x}{l}\sin 2\pi n_0 t \quad (10)$$

where $D_w$ is the steady central deflection ($D_w=\frac{2Wl^3}{\pi^4 EI}\approx\frac{Wl^3}{48EI}$) due to steady central load W.

The risk of dangerous vibrations corresponds to the accelerations of the bridge. Even though the accelerations are low at low speeds, they can reach unacceptable values at higher speeds. In cases with ballasted track bridges, intense acceleration creates the risk of destabilizing of ballast. For track stability and vehicle-bridge contact, it is important to ensure that the maximum accelerations of the bridge remain below [2]. In practical design the acceleration criteria will often be the decisive factor. Furthermore, the maximum dynamic effects occur at resonance peaks. At resonance, a multiple of the load frequency coincide with a natural frequency of the bridge structure, and the dynamic response of the structure increase very rapidly. Resonance may leads to cracks and crumbles of concrete, high ballast attrition due to the high accelerations, and big track irregularities.

# 3. Damping of Railway Bridge

In dynamic analysis, the structural damping is an important key parameter. The damping properties are important in dynamic analysis, but they are often not well known. The response of a bridge structure due to moving loads, and the magnitude of the vibrations of the structure, depends heavily on the structural damping capacity [3]. In risk of resonance, damping is especially important. Damping is a property of building material and structures, which usually reduces the dynamic response. Damping is dependent on the material of the railway bridge and on the state of the structure [4], for example presence of cracks and ballast. The magnitude of damping also depends on the amplitude of vibrations of the bridge. After passages of vehicles, or other excitations of bridges, damping causes the bridges to reach these states of equilibrium. Predicting the exact value of damping of new bridges is unfortunately not possible. In cases of designing new bridges, damping tables are used, which gives the lower limits of the percentage values of critical damping , based on number of past measurements. For already existing bridges, the damping values can be deduced by calculating the logarithmic decrement from free vibration measurements. It is almost impossible to take all sources of damping of vibrations of railway bridges into account in engineering calculations, because of the high number of them. The parameters used for dynamic analysis are shown in Table-5.

**Table 1**. Parameters used for dynamic analysis of bridge [5]

| Parameter | Value |
|---|---|
| Constant moving load, w (KN) | 347 |
| Flexural stiffness, EI ($GNm^2$) | 10.1 |
| Span length, L (m) | 34 |
| Unloaded fundamental frequency, $n_0$ (Hz) | 4.01 |
| Damping coefficient, $n_b$ | 0.12 |

Consider next how the state of oscillation set up by a constant force W moving along a girder at a uniform speed, which discussed in previous section 2 is modified by the influence of damping.

The differential equation for the motion there given is only change by introduction of the damping term $4\pi n_b m\frac{dy}{dt}$ on the left side of the equation,

$$EI\frac{d^4y}{dx^4}+4\pi n_b m\frac{dy}{dt}+m\frac{d^2y}{dt^2}=\frac{2W}{l}\left[sin2\pi nt sin\frac{\pi x}{l}+sin4\pi nt sin\frac{2\pi x}{l}+\cdots\right] \tag{11}$$

And corresponding force oscillation is

$$y=D_w\left[\frac{sin\frac{\pi x}{l}\sin(2\pi nt-\emptyset_1)}{\left[\left(1-\left(\frac{n}{n_0}\right)^2\right)^2+\left(\frac{2n_b n}{n_0^2}\right)^2\right]^{\frac{1}{2}}}+\frac{sin\frac{2\pi x}{l}\sin(4\pi nt-\emptyset_2)}{\left[\left(2^4-\left(\frac{2n}{n_0}\right)^2\right)^2+\left(\frac{4n_b n}{n_0^2}\right)^2\right]^{\frac{1}{2}}}+\cdots\right] \tag{12}$$

$D_w$ is central deflection due to a steady central load W.

In the application of this theory to railway bridges n and $n_b$ are so small in comparison with $n_0$ that $\emptyset_1$ and $\emptyset_2$ are hardly perceptible, and the forced oscillation is given to a high degree of accuracy by the equation.

$$y=D_w\left[sin\frac{\pi x}{l}sin2\pi nt+\frac{1}{2^4}sin\frac{2\pi x}{l}sin4\pi nt+\cdots\right] \tag{13}$$

This is equivalent to say that the force oscillation is indistinguishable from the crawl deflection.

To satisfy the starting conditions free oscillations of the type

$$y=e^{-2\pi n_b t}A_1\sin\frac{\pi x}{l}\sin2\pi n_0 t \tag{14}$$

$$y=e^{-2\pi n_b t}A_2\sin\frac{2\pi x}{l}\sin2^2(2\pi n_0 t) \tag{15}$$

Adjusting the constants of integration to satisfy the starting conditions, the free oscillation which has to be superposed on the crawl deflection is given by

$$y=-\frac{n}{n_0} D_w e^{-2\pi n_b t}\left[sin\frac{\pi x}{l} sin2\pi n_0 t+\frac{1}{2^4} sin\frac{2\pi x}{l} sin2^2(2\pi n_0 t)+\cdots\right] \quad (16)$$

The only part of this which is of any practical importance is the primary component

$$y=-\frac{n}{n_0} D_w e^{-2\pi n_b t}\ sin\frac{\pi x}{l} sin2\pi n_0 t \quad (17)$$

This, even at the start, is quite a small oscillation, and it dies out as the load passes along the bridge, the common ratio of successive oscillations being $e^{-2\pi\frac{n_b}{n_0}}$.

**Parametric plots**

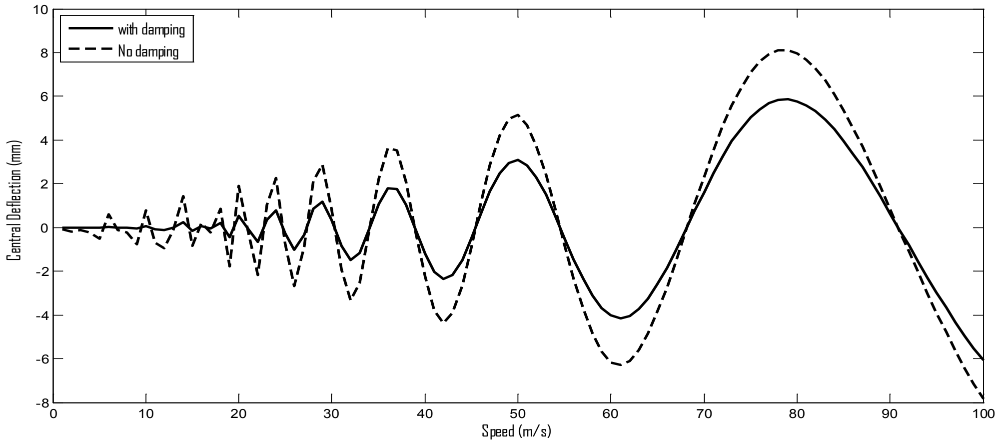


**Figure 2.** Central Deflection versus speed of the train

Fig. 2 shows the bridge under the moving load train as a function of speed with and without the influence of damping. The maximum amplitude of central deflection of the structure without damping is 8.095mm (78m/s). In order to reduce this parameter to control the vibrations, structural damping is taken into account it is analysed that under the influence of damping maximum central deflection reduces from 8.095mm to 5.9mm. So it may be concluded that damping value of the railway bridge has great influence of the amplitude of the response. Higher damping coefficient in the bridge structure gives lower response values. However, the peak values appear at the same speed, independent of different damping coefficient.

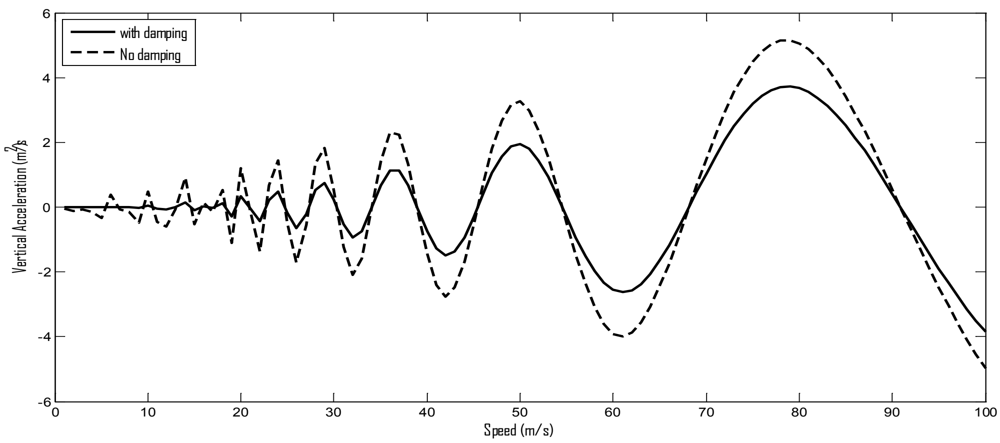


**Figure 3.** Vertical Acceleration versus speed of the train

Fig. 3 shows the bridge under the moving load train as a function of vertical acceleration with and without the influence of damping. Maximum vertical acceleration without damping is about 5.13 m/$s^2$. In order to reduce it to control the vibrations, structural damping is taken into account and results are compared and It is analysed that under the influence of damping maximum vertical acceleration reduces from 5.13 m/$s^2$ to 3.72 m/$s^2$. So it may be concluded that damping value of the railway bridge has great influence of the amplitude of the response.

# 4. Resonance of Railway Bridge

Dynamic analysis of bridge structures is necessary in case of resonance appearance.

Resonance is a dangerous phenomenon, which occurs due to high speeds and regularly spaced axle groups of the trains. In case of resonance due to high accelerations and big track irregularities, excessive bridge deck vibrations may cause loss of wheel-rail contact, destabilization of the ballast, occurrence of cracks and crumbles of concrete, and exceeding the stress limits of the bridge structure [3]. Dynamic effects including the resonance phenomenon always have to been taking into account, when designing railway bridges subjected to high speeds. However, if the traffic speeds remain under 200 km/h, resonance is unlikely to occur and do not need to be taken into account.

The effects of maximum dynamic load occur at the resonant peaks. Risk of resonance arises when the excitation frequency of the loading, or a multiple of it, coincides with a natural frequency of the bridge structure [6]. As the speed of the train increases, the excitation frequency of the train will approach the natural frequency of a mode of vibration of the bridge. When resonance occurs, the dynamic responses of the structure increase very rapidly. Occurrence of resonance depends on the number of groups of regularly spaced loads, the damping of the structure, and the nature of loading and the characteristics of the structure. Especially, the magnitude of the resonant peaks is highly dependent upon structural damping. A low value of the damping of the structure gives high resonance peaks. The peak value of the response occurs due to resonance. Depends on the damping coefficient

**Parametric plots at Resonance**

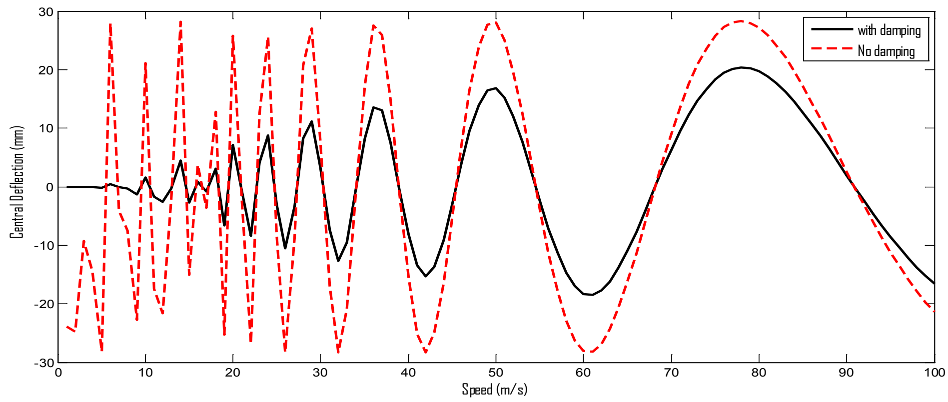


**Figure 4. Central Deflection versus speed of the train at resonance**

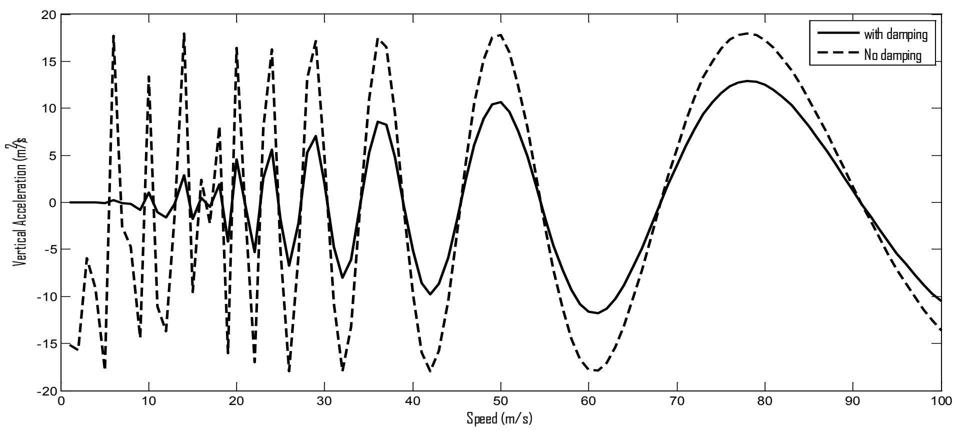


**Figure 5. Vertical Acceleration versus speed of the train at resonance**

Further dynamic response at resonance is also analyses which are shown in Fig. 4 & 5 and It is observe that the speed of the vehicle is a very important parameter, influencing the dynamic responses of the railway bridge and as discussed earlier in previous section that at resonant vibrations it may cause loss of wheel-rail contact, destabilization of the ballast, occurrence of cracks and crumbles of concrete, and exceeding the stress limits of the bridge structure. So it is necessary to control these vibrations that can be done using control strategies up to some extent [7].

# 5. Conclusion

It is observed from Fig 2&3 that the damping value of railway bridges has great influence of the amplitude of the response. Higher damping coefficient in the bridge structure gives lower response values. However, the peak values appear at same speed, independent of different damping coefficients and from Fig 4&5 it is observed that the speed of the vehicle is a very important parameter, influencing the dynamic responses of the railway bridge. Generally, the dynamic responses increase with increasing speed. The response of the railway bridge reaches the greatest resonance peak, which magnified the response several times, at speeds between 75 and 85 m/s.

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