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On the Optimization Problem of Stochastic Observations of Random Walks

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Abstract The optimal control problem for the intensity of observation events of the process of random walk is considered for the case of counting Poisson process in semimartingale terms. The linear function of the intensity as a cost of observations and the expected value of the quadratic form of errors of estimation as a cost of an error are reckoned in a loss function. The analogues result for the problem of the optimal intensity of stochastic approximation is presented.

Keywords Random Walk, Poisson Process, Optimal Control, Estimation, Semimartingale

1. Introduction and Definitions

Let $\mathcal{B}=(\Omega,\mathcal{F},\mathbf{F}=(\mathcal{F}_t)_{t\geq 0},\mathcal{P})$ be a stochastic basis satisfying the «usual conditions» of Dellacherie, [1]. We consider the following model. On $\mathcal{B}$ a process $X=(X_t)_{t\geq 0}$ is a random walk on the lattice $\mathbf{Z}=\{\dots,-1,0,1,2,\dots\}$ with trajectories in the Skorokhod space and $\Delta X_t=X_t-X_{t-}\in\{-1,0,1\}$, [2]. Along with $X=(X_t)_{t\geq 0}$ we shall consider on $\mathcal{B}$ a Poisson process $N=(N_t)_{t\geq 0}$ with intensity $\lambda>0$. For all $k=1,2,\dots$ define stopping times $\tau(k)$ such, that $\Delta N_{\tau(k)}=1$:

$$\tau(k)=\inf(t:\ t>0, N_t=k) \quad (1)$$

and denote $\tau(0)=0$ for $k=0$. Then random variables $\{\theta(k)\}_{k=1,2,\dots}$ with $\theta(k)=\tau(k)-\tau(k-1)$ are independent and identically distributed with the density function $\rho(x)$: $\rho(x)=0$ for $x<0$ and

$$\rho(x)=\lambda\cdot\exp\{-\lambda\cdot x\} \text{ for } x\geq 0. \quad (2)$$

Mathematical expectations and variances of $\theta(k)$ and $\tau(k)$ for all $k\geq 1$ are equal to $\mathbf{E}\,\theta(k)=1/\lambda$, $\mathbf{D}\,\theta(k)=1/\lambda^2$ and $t\in[\tau(k),\tau(k+1))$, $\mathbf{D}\,\tau(k)=k/\lambda^2$ respectively. Suppose that processes X and N are independent and hence times of their jumps cannot be simultaneous:

$$\mathcal{P}\{\sum_{0\leq s\leq t}\Delta X_s\cdot\Delta N_s=0\}=1 \quad \text{for all } t\geq 0.$$

In the model N is supposed to be the counting process of observation events. The initial value X_0 is observable: $Y_0(\omega)=X_0(\omega)$ for all $\omega\in\Omega$ and hence we $\tau(0)=0$. Because the jump of counting process ΔN_t is equal zero on the interval $t\in[0,\tau(1))$, the observation Y_t remains equal to $X_{\tau(0)}=X_0$ for any time t from this interval: $Y_t=Y_{\tau(0)}=X_0$. The observation Y_t can change its value only at stopping time $\tau(1)$ (the time of the first observation): as it follows from the definition (1) $Y_{\tau(1)}=X_{\tau(1)}$. Hence $Y_{\tau(1)}=Y_{\tau(1)-}+\Delta Y_{\tau(1)}=Y_{\tau(0)}+(X_{\tau(1)}-Y_{\tau(0)})\cdot 1=Y_{\tau(0)}+(X_{\tau(1)}-Y_{\tau(1)-})\cdot\Delta N_{\tau(1)}$. Analogously we can describe the algorithm of observation at times $\tau(k)$ for all $k\geq 1$ $Y_{\tau(k)}=X_{\tau(k)}$ and $Y_t=Y_{\tau(k)}$ for times $t\in[\tau(k),\tau(k+1))$. Hence the process of observations $Y=(Y_t)_{t\geq 0}$ is a solution of the following stochastic equation:

$$Y_t=X_0+\int_0^t\left(X_s-Y_{s-}\right)dN_s .$$

Note, that a process of random walk X can be represented as a difference of two counting processes:

$$X=X_0+A-B, \quad (3)$$

where $A=(A_t)_{t\geq 0}$ and $B=(B_t)_{t\geq 0}$ are the counting processes of the numbers of positive and negative jumps of X respectively:

$$A_t=\sum_{0<s\leq t}\mathrm{I}(\Delta X_s=1),\ B_t=\sum_{0<s\leq t}\mathrm{I}(\Delta X_s=-1),$$

$$\sum_{0<s\leq t}\Delta A_s\cdot\Delta B_s=0 \quad \text{for all } t\geq 0$$

with $A_0=B_0=0$. This representation of random walks with trajectories from Skorokhod space on the lattice Z is the most general possible, but in this article we shall restrict the model by the assumption of very simple distributions of processes A and B. Let A and B be independent Poisson processes with intensities $\alpha \geq 0$ and $\beta \geq 0$ respectively. According to the well-known Doob–Meyer decomposition of submartingales, [3], A and B can be represented as

$$A_t = \alpha \cdot t + m_t^A,\ B_t = \beta \cdot t + m_t^B, \qquad (4)$$

where $m^A = (m_t^A)_{t \geq 0}$ and $m^B = (m_t^B)_{t \geq 0}$ are square integrable martingales on $\mathcal{B}$ with quadratic characteristics:

$$\langle m^A \rangle_t = \alpha \cdot t, \langle m^B \rangle_t = \beta \cdot t, \langle m^A, m^B \rangle_t = 0.$$

It is clear that the process $Y = (Y_t)_{t \geq 0}$ is a stochastic discrete time approximation of the process $X = (X_t)_{t \geq 0}$. In this simple case it is possible to estimate the value of X_t given the observations $\{Y_t, 0 \leq s \leq t\}$ for all $t \geq 0$.

The more is the rate of observations, the better is approximation and the less could be the error of estimation. In many applications the cost of observations is not negligible. So the problem is in finding such a compromise intensity of observations λ, which results in minimization of a loss function reckoning in a cost of observations and a cost of an error of estimation.

In the model a cost of observations is supposed to be linearly depending on its averaged and normalized number N_T in a space of time $T > 0$, and therefore on the intensity λ of the Poisson process N. A cost of errors of observations is the expected value of a quadratic form of errors. The process of estimation is considered here as a continuous-time construction of a (discontinuous) process $\hat{X} = (\hat{X}_t)_{t \geq 0}$ with random variable $\hat{X}_t$, defined as an optimal mean square estimate of X_t given $\{Y_s, 0 \leq s \leq t\}$ for all $t \geq 0$ (i.e. given a discreet set of observations $Y_{\tau(k)} = X_{\tau(k)}$ for all $\tau(k) \leq t, k = 0,1,2,...$): $\hat{X}_t = \mathcal{E}(X_t | \mathcal{F}_t^Y)$, where a σ-algebra $\mathcal{F}_t^Y$ = $\sigma(Y_s; s \leq t)$ of a non–decreasing family $\mathcal{F}^Y = (\mathcal{F}_t^Y)_{t \geq 0}$ is completed by sets from $\mathcal{F}$ of $\mathcal{P}$ measure zero. Thus a cost of error of estimation is a normalized and expected value of an integral of the variance $\gamma_t = \mathcal{E}\varepsilon_t^2$ of the error of estimation $\varepsilon_t = X_t - \hat{X}_t$ for $t \geq 0$. Along with a consideration of errors of estimations given the observations $\mathcal{F}^Y$ it is possible to examine proper errors of approximation $\delta_t = X_t - Y_t$ and the expected value of the quadratic form of errors with $\Gamma_t = \mathcal{E}\delta_t^2$.

2. Results

Let us define the loss process $\varphi = (\varphi(t))_{t \geq 0}$ as a linear function of numbers of observations (as a cost of observations) and the quadratic form of errors of estimation (as a cost of an error) with positive constants h and g:

$$\phi(t) = h \cdot \int_0^t (\varepsilon_s)^2\, ds + g \cdot N_t, \qquad (5)$$

The expected and normalized value $\psi(\lambda)$ of the loss function $\phi(t)$ corresponding to the intensity λ is

$$\psi(\lambda) = \lim_{T \to \infty} \frac{1}{T} \mathcal{E}\phi(T), \qquad (6)$$

In terms of optimal control the problem is in finding such intensity λ^* that

$$\psi(\lambda^*) = \min_{\lambda \geq 0} \psi(\lambda). \qquad (7)$$

In order to investigate the variances of errors $\gamma_t = \mathcal{E}\varepsilon_t^2$ we at first formulate preliminary results for auxiliary random variables $\gamma_t^{(u)}$ being conditional variances $\gamma_t^{(u)} = \mathcal{E}((\varepsilon_t^{(u)})^2 | \mathcal{F}_u^Y)$ of the error $\varepsilon_t^{(u)} = X_{t+u} - \hat{X}_t^{(u)}$ of the estimate $\hat{X}_t^{(u)} = \mathcal{E}(X_{t+u} | \mathcal{F}_u^Y)$ for $t \geq 0$ and $u \geq 0$ (times t and u can be considered as arbitrary random $\mathcal{F}^Y$ -adapted stopping times). The properties of $\gamma_t^{(u)}$ are studied in the following lemma:

Lemma 1. For all $k = 1,2,...$ and stopping times $\tau(k)$ for $\lambda > 0$ the following equality holds:

$$\mathcal{E} \int_{\tau(k-1)}^{\tau(k)} \gamma_t^{(\tau(k-1))} dt = \frac{\alpha + \beta}{\lambda^2}.$$

Proof. According to the semimartingale presentation (4) of components (3) of X and the equality $Y_{\tau(k-1)} = X_{\tau(k-1)}$ the estimate $\hat{X}_t^{(\tau(k-1))} = \mathcal{E}\left(X_{t+\tau(k-1)} | \mathcal{F}_{\tau(k-1)}^Y\right)$ is equal to $X_{\tau(k-1)} + (\alpha - \beta) \cdot t$. Therefore the conditional error can be presented as $\varepsilon_t^{(\tau(k-1))}$ = $(m_{\tau(k-1)+t}^A - m_{\tau(k-1)}^A)$ - $(m_{\tau(k-1)+t}^B - m_{\tau(k-1)}^B)$. Because the martingales m^A and m^B in (4) are independent, the conditional variance $\gamma_t^{(\tau(k-1))} = \mathcal{E}\{(\varepsilon_t^{(\tau(k-1))})^2 | \mathcal{F}_{\tau(k-1)}^Y\}$ is equal to

$$(\langle m^A \rangle_{\tau(k-1)+t} - \langle m^A \rangle_{\tau(k-1)}) + (\langle m^B \rangle_{\tau(k-1)+t} - \langle m^B \rangle_{\tau(k-1)}) =$$
$$= t \cdot (\alpha + \beta). \qquad (8)$$

Random variables $\{\theta(k)\}_{k=1,2,...}$ are independent and their distribution density $\rho(x)$ is exponential, (2), then for all $k \ge 1$

$$\mathcal{E}\int_{\tau(k-1)}^{\tau(k)} \gamma_t^{(\tau(k-1))} dt = \mathcal{E}\int_{\tau(k-1)}^{\tau(k)} t\cdot(\alpha+\beta)dt =$$

$$\mathcal{E}\int_0^{\theta(k)} t\cdot(\alpha+\beta)dt = \int_0^{\infty} \rho(x)\cdot\int_0^{x} t\cdot(\alpha+\beta)\,dt\,dx = \frac{\alpha+\beta}{\lambda^2}.$$

Lemma 1 is proved.

The following lemma gives the way of calculation of $\psi(\lambda)$ in (6) in terms of $\phi(t)$ and $\tau(k)$:

Lemma 2. For the loss process Φ and $k=[\lambda\cdot T]$ the following convergence takes place

$$\psi(\lambda) = \lim_{k\to\infty} (\lambda/k)\mathcal{E}\phi(\tau(k)),$$

where $[\cdot]$ is a greatest integer function.

Proof. As it follows from the definition (1) the equality $(\lambda/k)\cdot g\cdot N_{\tau(k)} = g\cdot\lambda$ holds for all $k \ge 1$. The equality $\mathcal{E}(1/T)\cdot g\cdot N_T = g\cdot\lambda$ takes place for all $T>0$ because the intensity of the Poisson process N is equal to λ Hence for the normalized and expected right summand in (5) at time $\tau(k)$ the following equality holds:

$$\lim_{k\to\infty}\frac{\lambda}{k}\mathcal{E}N_{\tau(k)} = \lim_{T\to\infty}\frac{1}{T}\mathcal{E}\{g\cdot N_T\} = g\cdot\lambda. \quad (9)$$

Consider the normalized and expected left summand in (5) at time $\tau(k)$ at time $T=k/\lambda$, $k \ge 1$. It is clear that

$$\mathcal{E}\int_0^{\tau(k)} (\varepsilon_t)^2\,dt = \mathcal{E}\int_0^{\tau(k)} \gamma_t dt = \sum_{i=1}^{k}\mathcal{E}\int_{\tau(i-1)}^{\tau(i)} \gamma_t^{(\tau(i-1))} dt$$

and, as it follows from ***Lemma 1,*** therefore holds

$$h\cdot\frac{\lambda}{k}\cdot\mathcal{E}\int_0^{\tau(k)} (\varepsilon_t)^2\,dt = h\cdot\frac{\lambda}{k}\cdot k\cdot\frac{\alpha+\beta}{\lambda^2} = h\cdot\frac{\alpha+\beta}{\lambda}$$

Hence the proof of ***Lemma 2*** is reduced to the verification of the statement (9):

$$\lim_{k\to\infty}\frac{1}{T}\cdot\mathcal{E}\int_0^{T} (\varepsilon_t)^2\,dt = \frac{\alpha+\beta}{\lambda}. \quad (10)$$

Let us consider the auxiliary random variable $\zeta(T)$ with values in $[0,T]$ for $T>0$ defined as

$$\zeta(T) = \sup\{s : s \le T, \Delta N_s = 1\} =$$

$$= \inf\{s : s\in[0,T], N_s = N_T\}.$$

It is clear that $\zeta(t)$ is not a stopping time on basis $\mathcal{B}$ (because in general case the set $\{\omega : \zeta(t) \le u\} \notin \mathcal{F}_u$ for $u<t$). Nevertheless it is possible to investigate the set $\{\Theta(T)\}_{T>0}$ of random variables

$$\Theta(T) = \int_0^{\zeta(T)} \gamma(t)dt \text{ for } T>0$$

in terms of inverse time. It is clear, that from the convergence $\tau(k)/(\lambda\cdot k) \to 1$ $\mathcal{P}$–a.s. (as $k\to\infty$) it follows that (for $k\to\infty$ and $T=\lambda\cdot k\to\infty$) $\mathcal{P}-a.s$

$$\frac{\Theta(T)}{T/\lambda} = \sum_{i=1}^{\infty} \mathrm{I}\{\tau(i)=\Theta(T)\}\cdot\left(\frac{1}{\lambda\cdot T}\cdot\int_0^{\tau(i)}\gamma_t\,dt\right) =$$

$$\sum_{i=1}^{\infty} \mathrm{I}\{\tau(i)=\Theta(T)\}\cdot\frac{i}{T/\lambda}\cdot\frac{\alpha+\beta}{\lambda^2} \to \frac{\alpha+\beta}{\lambda}, \quad (11)$$

where $\mathrm{I}\{\cdot\}$ is an indicator function ($\mathrm{I}\{true\}=1$, $\mathrm{I}\{false\}=0$). So the proof of the lemma would follow from the convergence

$$\mathcal{E}\frac{1}{T}\cdot\left(\int_0^{T}\gamma_t\,dt - \Theta(T)\right) \to 0. \quad (12)$$

Note, that

$$\int_0^{T}\gamma_t\,dt - \Theta(T) = \int_{\zeta(T)}^{T}\gamma_t\,dt \quad (13)$$

and in inverse time presentation the process $R=(R_u)_{u\in[0,T]}$ for $u=T-t$ with $R_u = N_{T-t}$ is $\overline{\mathcal{F}}^R$ -adapted, where $\overline{\mathcal{F}}^R = (\overline{\mathcal{F}}_u^R)_{0\le u\le T}$ and $\overline{\mathcal{F}}_u^R$ = $\sigma\{R_v ; 0\le v\le u\}$ = $\sigma\{N_t, T-u\le t\le T\}$. The process R is a supermartingale and therefore admits the following Doob – Meyer decomposition

$$R_u = N_T - r_u = N_T - \tilde{r}_u + \overline{m}_u^r,$$

where $\tilde{r} = (\tilde{r}_u)_{0\le u\le T}$ is a compensator and $\overline{m}^r = \left(\overline{m}_u^r\right)_{0\le u\le t}$ is a square – integrable martingale. The compensator $\tilde{r}_u$ is equal to

$$\tilde{r}_u = \int_0^{u}\frac{r_u}{T-u}du \quad (14)$$

(note that formula (14) is similar to that of semimartingale presentation of a Brownian bridge and results from the infinitesimal semimartingale presentation of Poisson process in inverse time). According to (14) and from the well-known formula of Dellacherie $d\tilde{r}_x = dF_{\overline{\zeta}}(x)/\left(1-F_{\overline{\zeta}}(x-)\right)$ (see, e.g. [1]) it follows that for the conditional distribution function $F_{\overline{\zeta}}(x)$ and the density $\rho_{\overline{\zeta}}(x)$ of the stopping time $\overline{\zeta}$ of the first jump of the process r given the random value N_T hold

$$F_{\bar{\zeta}}(x)=1-(1-x/T)^{N_T},$$

$$\rho_{\bar{\zeta}}(x)=N_T\cdot(T-x)^{N_T}/T^{N_T} \tag{15}$$

Because R coincides with N in inverse time, then $\bar{\zeta}=\zeta$. From the formula (8) and independence of the processes $X=(X_t)_{t\ge 0}$ and $N=(N_t)_{t\ge 0}$ it follows that

$$\mathcal{E}\frac{1}{T}\cdot\int_{\zeta(T)}^{T}\gamma_t\,dt=\mathcal{E}\frac{1}{T}\cdot\int_{0}^{\bar{\zeta}(T)}\bar{\gamma}_u\,du=$$

$$\frac{1}{T}\cdot\int_0^T\rho_{\bar{\zeta}}(x)\cdot\frac{x^2}{2}dx,$$

where $\bar{\gamma}_u=\gamma_{T-t}$ for $u=T-t$. From (15) we receive

$$\frac{1}{T}\cdot\int_0^T\rho_{\zeta}(x)\cdot\frac{x^2}{2}dx=\frac{1}{N_T}\cdot\mathrm{I}\{N_T\ge 1\}.$$

Because $\mathcal{P}\{1\le N_T\le T/2\}\to 0$ for $T\to\infty$ and

$$\mathcal{E}\{\mathrm{I}\{N_T\ge 1\}/N_T\}\le$$

$$\le\mathcal{E}\left\{1\cdot\mathrm{I}\{1\le N_T\le T/2\}+\frac{1}{T/2}\cdot\mathrm{I}\{T/2<N_T\}\right\}\le$$

$$\le\mathcal{P}\{1\le N_T\le T/2\}+\frac{1}{T/2},$$

then

$$\mathcal{E}\frac{1}{T}\cdot\int_{\zeta(T)}^{T}\gamma_t\,dt\to 0. \tag{16}$$

The convergences (11) and (16) along with the equality (13) result in (12) and in the statement of ***Lemma 2.*** Lemma is proved.

The value of $\psi(\lambda)$ in (6) is obtained in the following lemma.

Lemma 3. *Under assumptions of the Theorem the function* $\psi(\lambda)$ *is equal to*

$$\psi(\lambda)=h\cdot(\alpha+\beta)/\lambda+g\cdot\lambda.$$

Proof of the lemma follows from the statements (9) and (10). Here is the solution of the problem (6)-(7):

Theorem 1. Let $h\ge 0$, $g>0$. *Then the value* λ^* *in the problem (7) for the loss function (6) is equal to*

$$\lambda^*=\sqrt{h/g}\cdot\sqrt{\alpha+\beta}, \tag{17}$$

and the value of loss function is

$$\psi(\lambda^*)=2\sqrt{h\cdot g}\cdot\sqrt{\alpha+\beta}. \tag{18}$$

Proof of the theorem follows from the statement of ***Lemma 3*** and the equation $d\psi(\lambda)/d\lambda=0$, resulting in (17) and (18).

Now we study an intensity of observations λ as a rate of stochastic approximation of X with Y. In order to investigate the rates of approximation we define functions $\Phi(t)$ and $\Psi(\lambda)$ by analogy with $\phi(t)$ and $\psi(t)$ substituting the estimates $\hat{X}_s$ by the observations Y_s (and hence substituting ε_s by δ_s) in (5)-(6):

$$\Phi(t)=h\cdot\int_0^t(\delta_s)^2\,ds+g\cdot N_t,$$

$$\Psi(\lambda)=\lim_{T\to\infty}\frac{1}{T}\mathcal{E}\Phi(T), \tag{19}$$

and we consider the optimal control problem of finding such intensity of stochastic approximation Λ that

$$\Psi(\Lambda)=\min_{\lambda\ge 0}\Phi(\lambda) \tag{20}$$

By analogy with $\gamma_t^{(u)}$ we define $\Gamma_t^{(u)}=\mathcal{E}((\delta_t^{(u)})^2|\mathcal{F}_u^Y)$ with $\delta_t^{(u)}=X_{t+u}-Y_u$.

Then the following result for $\Gamma_t^{(u)}$ is true:

Lemma 4. *For all* $k=1,2,\ldots$ *and stopping times* $\tau(k)$ *for* $\lambda>0$ *there holds*

$$\mathcal{E}\int_{\tau(k-1)}^{\tau(k)}\Gamma_t^{(\tau(k-1))}dt=\frac{\alpha+\beta}{\lambda^2}+\frac{2\cdot(\alpha-\beta)^2}{\lambda^3}$$

Proof of the lemma follows from the obvious equality

$$\mathcal{E}\int_{\tau(k-1)}^{\tau(k)}\Gamma_t^{(\tau(k-1))}dt=$$

$$=\mathcal{E}\int_{\tau(k-1)}^{\tau(k)}\gamma_t^{(\tau(k-1))}dt+\int_0^\infty\rho(x)\cdot\left(\int_0^x(\alpha-\beta)tdt\right)^2dx,$$

from (2) and from the statement of ***Lemma 1***.

The next result gives a way for finding of $\Psi(\lambda)$ in terms of $\Phi(t)$ and stopping times $\tau(k)$:

Lemma 5. *For the loss process* Φ *and* $k=[\lambda\cdot T]$ *the following convergence takes place:*

$$\Psi(\lambda)=\lim_{k\to\infty}(\lambda/k)\cdot\mathcal{E}\Phi(\tau(k)).$$

Proof is similar to that of ***Lemma 2***.

The value of $\Psi(\lambda)$ in (19) is obtained in the following lemma:

Lemma 6. *Let* $h \geq 0,\ g > 0$. *Then for the function* $\Psi(\lambda)$ *holds*

$$\Psi(\lambda) = 2h \cdot (\alpha - \beta)^2 / \lambda^2 + h \cdot (\alpha + \beta) / \lambda + g \cdot \lambda. \quad (21)$$

Proof is similar to the proof of ***Lemma 3***.

The next result gives a way for solving the problem (20):

Theorem 2. *Let* $h \geq 0,\ g > 0$. *Then the value* Λ *in the problem (20) for the loss function (19) is a solution of the following equation:*

$$\lambda^3 - \lambda \cdot (\alpha + \beta) \cdot (h/g) - 4 \cdot (\alpha - \beta) \cdot (h/g) = 0.$$

The value of loss function $\Psi(\Lambda)$ is defined by (21).

Proof is analogous to that of **Theorem 1** and follows from the requirement $d\Psi(\lambda)/d\lambda = 0$.

3. Conclusion

The method discussed in the paper is based on the semimartingale approach developed for the random walks of a general type in [2]. This approach was shown to be useful for the limit theorems and for the problems of estimation. The problems of the optimal intensity of observations of the processes are developed mostly for the Gaussian or for the stationary systems (see, e.g. [4-6]) but not for random walks yet. Nevertheless it is possible to receive new results by means of this (martingale) approach. The investigated in the article simple case of the random walk is interesting because it demonstrates the method developed for a non-stationary (and non-ergodic) case. Thus it can be easily applied to a variety of processes (including the case of random walks in the conditionally-Markov random environments, considered as an example of the martingale approach in [2]) and point counting processes for the numbers of observation. The main result of the paper, stated in the *Theorem 1*, permits to consider the optimal control problems for the rate of instant observations of nonstationary systems (e.g. streams of data in queueing systems similar to [7]). The problems of comparison of optimal intensities of observations (for estimation) and optimal rates of approximation are of especial interest, and the statement of the *Theorem 2* (along with the results of *Theorem 1*) can be considered as a simple approach to the problems of such type.

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