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On N-Fold Positive Implicative Artinian and Positive Implicative Notherian BCK-Algebras

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Abstract The notion of BCK-algebras was initiated by Imai and Iseki in 1966 as a generalization of both classical and non-classical positional calculus. In 1999, Huang and Chen introduced the notion of n-fold positive implicative ideals in BCK-algebras. In 2011, Satyanarayana and Durga Prasad introduced foldness of intuitionistic fuzzy positive implicative ideals in BCK-algebras. In this paper, we introduce the notion of n-fold positive implicative ideals, n-fold positive implicative Artinian (shortly, PI^n-Artinian) and n-fold positive implicative Noe-therian (shortly, PI^n -Noetherian) BCK-algebras and study some of its properties.

Keywords BCK-Algebras, N-Fold Positive Implicative Ideal, Artinian and Noetherian BCK-Algebras

1. Introduction

It is known that mathematical logic is a discipline used in sciences and humanities with different point of view. Non-classical logic takes the advantage of the classical logic (two-valued logic) to handle information with various facts of uncertainty. The non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. The notion of logical algebra: BCK-algebras [7] were initiated by Imai and Iseki in 1966 as a generalization of both classical and non-classical positional calculus. In the same year, Iseki introduced BCI-algebras [8] as a super class of the class of BCK-algebras. In 1983, Hu and Li introduced BCH-algebras [6]. They demonstrated that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. For the general development of BCK-algebras, the ideal theory plays an important role. In 1999, Huang and Chen [5] introduced the notion of n-fold positive implicative ideals in BCK-algebras. In [9], Jun and Kim consider the fuzzification of n-fold positive implicative ideals in BCK-algebras and Satyanarayana and Durga Prasad [12] studied foldness of intuitionistic fuzzy positive impli-cative ideals in BCK-algebras. In this paper, we introduce the notion of n-fold positive implycative Artinian (shortly, PI^n-Artinian) and n-fold positive implicative Noetherian (shortly, PI^n -Noetherian) BCK-algebras and give its characterization.

2. Preliminaries

In this section we include some elementary aspects of BCK-algebras that are necessary for this paper. By a BCK-algebra (see [12, 13]) we mean algebra $(X;\ *,\ 0)$ of type $(2,0)$ satisfying the following axioms:

(BCK-1) $(x*y)*(x*z)\le(z*y)$,

(BCK-2) $(x*(x*y))\le y$,

(BCK-3) $x\le x$,

(BCK-4) $x\le y, y\le x\Rightarrow x=y$,

(BCK-5) $0\le x$, for all $x,y,z\in X$.

We can define a binary relation $\le$ on X by letting $x\le y$ if and only if $x*y=0$. In any BCK-algebra X the following hold: (P1) $x*0=x$, (P2) $x*y\le x$, (P3) $(x*y)*z=(x*z)*y$, (P4) $(x*z)*(y*z)\le x*y$, (P5) $x*(x*(x*y))=x*y$, (P6) $x\le y\Rightarrow x*z\le y*z$ and $z*y\le z*x$ for all $x,y,z\in X$. Throughout this paper X will always mean a BCK-algebra unless otherwise specified. A BCK-algebra X is said to be positive implicative if $(x*y)*z=(x*z)*(y*z)$ for every $x,y,z\in X$. A non-empty subset I of X is said to be an n-fold positive implicative ideal if (I_1) $0\in I$ and there exists a fixed $n\in N$ such that (I_2) $(x*y)*z^n\in I$ and $y*z^n\in I\Rightarrow x*z^n\in I$ for $x,y,z\in X$. For any elements x and y of X, $x*y^n$ denotes $(.....((x*y)*y)*.....)*y$ in which 'y' occurs n-times.

A fuzzy set in a set X is a function $\mu : X \to [0,1]$ and the complement of μ, denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x) = 1 - \mu(x)$, for all $x \in X$. Let μ and λ be the fuzzy sets of X. For $s, t \in [0,1]$ the set $U(\mu; s) = \{x \in X \mid \mu(x) \geq s\}$ is called upper s- level cut of μ and the set $L(\lambda; t) = \{x \in X \mid \lambda(x) \leq t\}$ is called lower t-level cut of λ.

As an important generalization of the notion of fuzzy sets in X, Atanassove [1, 2] introduced the concept of an intuitionistic fuzzy set (IFS, in short) defined by "An intuitionistic fuzzy setA in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$, where the function $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denoted the degree of membership (namely $\mu_A(x)$) and the degree of non member-ship (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, for all $x \in X$. For the sake of simplicity, we use the symbol $A = (X, \mu_A, \lambda_A)$.

Definition 2.2. [12] An IFS $A = (X, \mu_A, \lambda_A)$ in X is an intuitionistic fuzzy ideal (IF-ideal) of X if it satisfies

(IF-1) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$,

(IF-2) $\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(x)\}$ and

(IF-3) $\lambda_A(x) \leq \max\{\lambda_A(x * y), \lambda_A(y)\}$ for ll $x, y \in X$.

Definition 2.3. [12] An IFS $A = (X, \mu_A, \lambda_A)$ in X is an intuitionistic fuzzy subalgebra of X,
if it satisfies

i. $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$
ii. $\lambda_A(x * y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ for all $x, y \in X$.

3. Intuitionistic Fuzzy N-Fold Positive Implicative Ideals and Chain Conditions

Definition 3.1 [13] An IFS $A = (X, \mu_A, \lambda_A)$ in X is an intuitionistic fuzzy n-fold positive implicative ideal $(\mathrm{IFPI}^n\text{ - ideal})$ of X if it satisfies

$(\mathrm{IFPI}^n 1)$ $\mu_A(0) \geq \mu_A(x)$, $\lambda_A(0) \leq \lambda_A(x)$,

there exists a fixed $n \in N$ such that

$(\mathrm{IFPI}^n 2)$

$\mu_A(x * z^n) \geq \min\{\mu_A((x * y) * z^n),\ \mu_A(y * z^n)\}$ and

$(\mathrm{IFPI}^n 3)$

$\lambda_A(x * z^n) \leq \max\{\lambda_A((x * y) * z^n),\ \lambda_A(y * z^n)\}$ for all $x, y, z \in X$.

Definition 3.2 A BCK- algebra X is said to satisfies the PI^n -ascending (resp., PI^n - descending) chain condition (briefly, PI^n -ACC (resp., PI^n -DCC)) if every ascending (resp., descending) sequence $A_1 \subseteq A_2 \subseteq \ldots.. (A_1 \supseteq A_2 \supseteq \ldots..)$ of n-fold positive implicative ideals of X , there exists a natural number k such that $A_r = A_k$ for all $r \geq k$.

i. If X satisfies PI^n -DCC, we say that X is a PI^n -Artinian BCK-algebra.
ii. If X satisfies PI^n -ACC, we say that X is a PI^n -Noetherian BCK-algebra.

Theorem 3.3 Let X be a BCK- algebra. If every intuitionistic fuzzy n-fold positive implicative ideal of X has finite number of values, then X is a PI^n -Artinian.

Proof. Suppose that every intuitionistic fuzzy n-fold positive implicative ideal of X does not satisfy PI^n -DCC then there exists a strictly descending chain $A_0 \supset A_1 \supset A_2 \supset \ldots..$ of n-fold positive implicative ideals of X which does not terminates at finite steps. We consider IFS $A = (X, \mu_A, \lambda_A)$ in X by

$$\mu_A(x) = \left\{ \begin{array}{ll} r/_{r+1}, & if\ x \in A_r \backslash A_{r+1},\ for\ r = 1,2,\ldots.. \\ 1, & if\ x \in \bigcap_{r=0}^{\infty} A_r \end{array} \right\},$$

$$\lambda_A(x) = \left\{ \begin{array}{ll} 1/_{r+1}, & if\ x \in A_r \backslash A_{r+1},\ for\ r = 1,2,\ldots.. \\ 0, & if\ x \in \bigcap_{r=0}^{\infty} A_r \end{array} \right\},$$

where A_0 stands for X. Let us prove that $A = (X, \mu_A, \lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of X. Clearly $\mu_A(0) = 1 \geq \mu_A(x)$ and $\lambda_A(0) = 0 \leq \lambda_A(x)$ for all $x \in X$. Let $x, y, z \in X$. Assume that $(x * y) * z^n \in A_r \setminus A_{r+1}$ and $y * z^n \in A_k \setminus A_{k+1}$ for $r = 0,1,2,\ldots$; $k = 0, 1, 2,..$ Without loss of generality, we may assume that $r \leq k$. Then obviously $(x * y) * z^n$ and $y * z^n \in A_r$ and so $x * z^n \in A_r$, because A_r is an n-fold positive implicative ideal of X. Thus

$$\mu_A\left(x * z^n\right) \geq \frac{r}{r+1} = \min\left\{ \mu_A((x * y) * z^n), \mu_A(y * z^n) \right\},$$

$$\lambda_A\left(x*z^n\right) \le \frac{1}{r+1} = \max\left\{ \lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

If $(x*y)*z^n \in \bigcap_{r=0}^{\infty} A_r$ and $y*z^n \in \bigcap_{r=0}^{\infty} A_r$, then $x*z^n \in \bigcap_{r=0}^{\infty} A_r$. Thus

$$\mu_A\left(x*z^n\right) = 1 = \min\left\{ \mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\},$$

$$\lambda_A\left(x*z^n\right) = 0 = \max\left\{ \lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

If $(x*y)*z^n \notin \bigcap_{r=0}^{\infty} A_r$ and $y*z^n \in \bigcap_{r=0}^{\infty} A_r$, then there exists $i \in N$ such that $((x*y)*z^n) \in A_i \setminus A_{i+1}$.

It follows that $x*z^n \in A_i$, so that $\mu_A\left(x*z^n\right) \ge \frac{i}{i+1}$

$$= \min\left\{ \mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\}$$

$$\lambda_A\left(x*z^n\right) \le \frac{1}{i+1} = \max\left\{ \lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

Finally, suppose that $(x*y)*z^n \in \bigcap_{r=0}^{\infty} A_r$ and $y*z^n \notin \bigcap_{r=0}^{\infty} A_r$. Then there exists $j \in N$ such that $y*z^n \in A_j \setminus A_{j+1}$. Hence $x*z^n \in A_j$, and so

$$\mu_A\left(x*z^n\right) \ge \frac{j}{j+1}$$

$$= \min\left\{ \mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\}$$

$$\lambda_A\left(x*z^n\right) \le \frac{1}{j+1} = \max\left\{ \lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

Consequently we conclude that $A=(X,\mu_A,\lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of *X* and $A=(X,\mu_A,\lambda_A)$ has an infinite number of different values. This is a contradiction and hence X is a PI^n -Artinian.

Theorem 3.4 Let *X* be a PI^n-Artinian BCK-algebra and $A=(X,\mu_A,\lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of X. If a sequence of elements of $\mathrm{Im}(A)$ is strictly intuitionistic increasing, that is, a sequence of elements of $\mathrm{Im}(\mu_A)$ is strictly increasing and a sequence of elements of $\mathrm{Im}(\lambda_A)$ is strictly decreasing, then $A=(X,\mu_A,\lambda_A)$ has finite number of intuitionistic values, that is, μ_A and λ_A has finite number of values.

Proof: Suppose that $\mathrm{Im}(\mu_A)$ is not finite.

Let $\{s_n\}$ be a strictly increasing sequence of elements of $\mathrm{Im}(\mu_A)$. Define $U(\mu_A;s_r)=\{x\in X \ / \ \mu_A(x)\ge s_r\}$, for $r=2,3,4,\ldots$. Then $U(\mu_A;s_r)$ is an n-fold positive implicative ideal of *X*. Let $x\in U(\mu_A;s_r)$ then $\mu_A(x)\ge$, $s_r > s_{r-1}$ which implies that $x\in U(\mu_A;s_{r-1})$. Hence $U(\mu_A;s_r)\subseteq.\ U(\mu_A;s_{r-1})$. Since $s_{r-1}\in \mathrm{Im}(\mu_A)$, there exists $x_{r-1}\in X$ such that $\mu_A(x_{r-1})=s_{r-1}$. It follows that $x_{r-1}\in U(\mu_A;s_{r-1})$, but $x_{r-1}\notin U(\mu_A;s_r)$. Thus $U(\mu_A;s_r)$ is a proper sub set of $U(\mu_A;s_{r-1})$ and so we can obtain a strictly descending chain $U(\mu_A;s_1)\supset U(\mu_A;s_2)\supset U(\mu_3;s_3)\supset .....$ of n-fold positive implicative ideals of *X* which is not terminating, which is a contradiction. Let $\{t_n\}$ be a strictly decreasing sequence of elements of $\mathrm{Im}(\lambda_A)$. Then $0\le .....<t_2<t_1\le 1.$ Define $L(\mu_A;t_k)=\{x\in X \ / \ \lambda_A(x)\le t_k\}$ for $k=2,3,4,\ldots\ldots$

Then $L(\mu_A;t_k)$ is an n-fold positive implicative ideal of *X*. If $y\in L(\lambda_A;t_k)$, then $\lambda_A(y)\le t_k<t_{k-1}$ and so $y\in L(\lambda_A;t_{k-1})$. This shows that $L(\lambda_A;t_k)\subseteq L(\lambda_A;t_{k-1})$. Since $t_{k-1}\in\mathrm{Im}(\lambda_A)$, there exists $y_{k-1}\in X$ such that $\lambda_A(y_{k-1})=t_{k-1}$. It follows that $y_{k-1}\in L(\lambda_A;t_{k-1})$ but $y_{k-1}\notin L(\lambda_A;t_k)$. Therefore $L(\lambda_A;t_k)$ is a proper sub set of $L(\lambda_A;t_{k-1})$ and thus we can obtain a strictly descending chain $L(\lambda_A;t_1)\supset L(\lambda_A;t_2)\supset.\ L(\lambda_A;t_3)\supset ....$ of n-fold positive implicative ideals of *X*, which is not terminating, which is again contradiction. Thus $A=(X,\mu_A,\lambda_A)$ has finite number of intuitionistic values.

Theorem 3.5 Let *X* be a PI^n -Noetherian BCK-algebra and $A=(X,\mu_A,\lambda_A)$ be an intuitionistic fuzzy n-fold positive implicative ideal of *X*. If a sequence of elements of $\mathrm{Im}(A)$ is strictly intuit-tionistic decreasing, that is, a sequence of elements of $\mathrm{Im}(\mu_A)$ is strictly decreasing and a sequence of elements of $\mathrm{Im}(\lambda_A)$ is strictly increasing. Then $A=(X,\mu_A,\lambda_A)$ has finite number of intuitionistic values, that is, μ_A and λ_A has finite number of values.

Corollary 3.6 Let *X* be a PI^n -Artinian and PI^n -Noetherian BCK-algebra and $A=(X,\mu_A,\lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of *X*. If a sequence of elements of $\mathrm{Im}(\mu_A)$ and $\mathrm{Im}(\lambda_A)$ is strictly decreasing. Then A has finite number of intuitionistic values, that is μ_A and λ_A has finite number of values.

Proof: The proof is straight forward.

Theorem 3.7 The following conditions are equivalent.

X is a PI^n-Noetherian BCK- algebras.

The set of values of any intuitionistic fuzzy n-fold positive implicative ideal of X is a well-ordered sub-set of [0, 1].

Proof: (i) $\Rightarrow$ (ii) Let $A=(X,\mu_A,\lambda_A)$ be an intuit-tionistic fuzzy n-fold positive implicative ideal of X. Suppose that the set of values of A is not a well- order sub set of [0, 1]. Then there exist a strictly decreasing sequence $\{s_n\}$ such that $\mu_A(x)=s_n$ (elements of $\mathrm{Im}(\mu_A)$). Define $U(\mu_A;s_r)=\{x\in X \ / \ \mu_A(x)\geq s_r\}$, for $r=2,3,4,.....$. Then $U(\mu_A;s_r)$ is a n-fold positive implicative ideal of X and thus we can obtain a strictly ascending chain $U(\mu_A;s_1)$Ì$U(\mu_A;s_2)$Ì$U(\mu_3;s_3)$Ì.... of n-fold positive implicative ideals of X which is not terminates. Thus we arrive a contradiction. If there exist a strictly increasing sequence $\{t_n\}$ such that $\lambda_A(x)=t_n$ (elements of $\mathrm{Im}(\lambda_A)$. That is, $0\leq t_1<t_2<.....\leq 1$. Define $L(\mu_A;t_k)=\{x\in X \ / \ \lambda_A(x)\leq t_k\}$ for $k=2,3,4,.....$ .. Then $L(\mu_A;t_k)$ is an n-fold positive implicative ideal of X and thus we get a strictly ascending chain $L(\lambda_A;t_1)\subset L(\lambda_A;t_2)\subset L(\lambda_A;t_3)\subset .....$ of n-fold positive implicative ideals of X which is not terminating. Thus we arrive a contradiction the assumption that X satisfies the PI^n-ACC.

(ii) $\Rightarrow$ (i), Suppose that there exists a strictly ascending chain $G_1\subset G_2\subset G_3\subset .....(*)$ of n-fold positive implicative ideals of X, Which does not terminates at finite step.

Define IFS $A=(X,\mu_A,\lambda_A)$ in X by

$$\mu_A(x)=\begin{cases}\frac{1}{k}, & where\ k=\min\{r\in N \ / \ x\in G_r\}\\ 0, & if\ x\notin G_r\end{cases}$$

$$\lambda_A(x)=\begin{cases}\frac{1}{k}, & where\ k=\max\{n\in N \ / \ x\in G_n\}\\ 1, & if x\notin G_n\end{cases}$$

Where $X=\overset{\infty}{\underset{r=0}{\cup}} G_r$. Now, we prove that $A=(X,\mu_A,\lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of X. Since $0\in G_r$, for all $r=1,2,3,....$ We have $\mu_A(0)=1\geq \mu_A(x)$ and $\lambda_A(0)=0\leq \lambda_A(x)$ for all $x\in X$. Let $x,y,z\in X$. Assume that $(x*y)*z^n\in G_r\setminus G_{r-1}$ and $y*z^n\in G_r\setminus G_{r-1}$, r=2,3,4,..... , then $x*z^n\in G_r$, because G_r is an n-fold positive implicative ideal of X. It follows that

$$\mu_A\left(x*z^n\right)\geq \frac{1}{r}=\min\left\{\mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\},$$

$$\lambda_A\left(x*z^n\right)\leq \frac{1}{r}=\max\left\{\lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

Assume that $(x*y)*z^n\in G_r$ and $y*z^n\in G_r\setminus G_s$ for all $s<r$. Since G_r is an n-fold positive implicative ideal of X, therefore $x*z^n\in G_r$. Thus

$$\mu_A\left(x*z^n\right)\geq \frac{1}{r}\geq \frac{1}{s+1}\geq \mu_A(y*z^n)$$

$$\lambda_A\left(x*z^n\right)\leq \frac{1}{r}\leq \frac{1}{s+1}\leq \lambda_A(y*z^n).$$

Hence $\mu_A\left(x*z^n\right)\geq \min\left\{\mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\}$, $\lambda_A\left(x*z^n\right)\leq \max\left\{\lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}$.

Similarly, for the case $(x*y)*z^n\in G_r\setminus G_s$ and $y*z^n\in G_r$. We have

$$\mu_A\left(x*z^n\right)\geq \min\left\{\mu_A((x*y)*z^n),\ \mu_A(y*z^n)\right\}$$

$$\lambda_A\left(x*z^n\right)\leq \max\left\{\lambda_A((x*y)*z^n),\ \lambda_A(y*z^n)\right\}.$$

Hence $A=(X,\mu_A,\lambda_A)$ is an intuitionistic fuzzy n-fold positive implicative ideal of X. Since the chain (*) is not terminating, A has strictly decreasing sequence of values. This contradicts that the value set of an intuitionistic fuzzy n-fold positive implicative ideals of X is well-ordered.

Notation: Let $A=(X,\mu_A,\lambda_A)$ be an intuitionistic fuzzy n-fold positive implicative ideal of X, "u_{μ_A}" denotes the family of all upper level n-fold positive implicative ideals of X with respect to μ_A and "v_{λ_A}" denotes the family of all lower level n-fold positive implicative ideals of X with respect to λ_A.

Theorem 3.8 Let X be a PI^n-Artinian BCK-algebra and $A=(X,\mu_A,\lambda_A)$ be an intuitionistic fuzzy n-fold positive implicative ideal of X. If a sequence of elements of $\mathrm{Im}(A)$ is strictly intuitionistic increasing, then $\left|u_{\mu_A}\right|=\left|\mathrm{Im}(\mu_A)\right|$ and $\left|v_{\lambda_A}\right|=\left|\mathrm{Im}(\lambda_A)\right|$.

Theorem 3.9 Let $A=(X,\mu_A,\lambda_A)$ and $B=(X,\mu_B,\lambda_B)$ be intuitionistic fuzzy n-fold positive implicative ideals of PI^n-Artinian BCK-algebra X. If sequence of elements of $\mathrm{Im}(A)$ and $\mathrm{Im}(B)$ are strictly intuitionistic increasing, then

(i) $u_{\mu_A}=u_{\mu_B}$ and $\mathrm{Im}(A)=\mathrm{Im}(B)\Leftrightarrow \mu_A=\mu_B$,

(ii) $v_{\lambda_A} = v_{\lambda_B}$ and $\mathrm{Im}(\lambda_A) = \mathrm{Im}(\lambda_B) \Leftrightarrow \lambda_A = \lambda_B$.

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REFERENCES

[1] Atanassov, K.T., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20:87-96 (1983).

[2] Atanassov, K.T., New operations defined over the intuitionistic fuzzy Sets, Fuzzy Sets and Systems, 61(2):137-142(1994).

[3] Atanassov, K., Intuitionistic fuzzy sets, Theory and applications, Studies in Fuzziness and Soft Computing, 35. Heidelberg Physica- Verlag, 1990.

[4] Atanassov. K., Intuitionistic fuzzy systems, Springer Physica-Verlag, Berlin, 1999

[5] Huang, Y., and Chen.Z., On ideals in BCK-algebras, Math. Japan, 50:211-226(1999).

[6] Hu Q. P. and Li X., On BCH-algebras, Math. Seminar Notes, 11:313-320(1983).

[7] Imai Y. and Iseki K., On axiom system of propositional calculi XIV, Proc. Japanica Acad, 42:19-22(1966).

[8] Iseki K., An algebra related with a propositional calculus, Proc. Japan Acad, 42:26-29(1966).

[9] Jun, Y.B., and Kim, K, H., on n-fold fuzzy positive implicative ideals of BCK- algebras, Hindawi Publishing corp. 26(9):525-537(2001).

[10] Satyanarayana, B., Bindu Madhavi.U and Durga Prasad, R., On foldness of intuitionistic fuzzy H-ideals in BCK-algebras, International Mathematical forum, 5(45):2205-2211(2010).

[11] Satyanarayana, B., and Durga Prasad, R., On Intuitionistic fuzzy ideals in BCK- algebras, International J.of Math.Sci and Appls, 5(1):283-294(2011).

[12] Satyanarayana, B and Durga Prasad, R., on foldness of intuitionistic fuzzy positive implicated ideals of BCK-algebras, Research Journal of Pure Algebra,1: 40-51(2011).

[13] Satyanarayana, B and Durga Prasad, R., Positive implicative Artinian BCK-algebra and Positive implicative Noetherian BCK-algebra (Communicated).

[14] Zhan, J. and Tan, Z.: Characterizations of Doubt fuzzy in H-ideals in BCK-algebras, Soochow Journal of Mathematics 29:293-298(2003).

[15] Zadeh, L.A., Fuzzy sets, Information Control, 8:338-353(1965).