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On Prime and Semiprime Rings with Additive Mappings and Derivations

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Abstract Let R be an associative ring. A mapping $f : R \longrightarrow R$ is said to be additive if $f(x+y) = f(x) + f(y)$ holds for all $x, y \in R$. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. In this paper, we investigate commutativity of prime and semiprime rings satisfying certain identities involving additive mappings and derivations. Moreover, some results have also been discussed.

Keywords Prime Ring, Semiprime Ring, Ideal, Additive Mapping, Commuting Mapping, Derivation, Generalized Derivation

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1 Introduction

Let R be an associative ring with center $Z(R)$ and extended centroid C. For $x, y \in R$, the symbol $[x, y]$ will denote the commutator $xy - yx$ and the symbol $x \circ y$ will denote the anti-commutator $xy + yx$. A ring R is called 2-torsion free, if $2x = 0$, $x \in R$, implies $x = 0$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = (0)$ implies $a = 0$ or $b = 0$, and is semiprime if for any $a \in R$, $aRa = (0)$ implies $a = 0$. For any subset S of R, we will denote by $r_R(S)$ the right annihilator of S in R, that is, $r_R(S) = \{x \in R \mid Sx = 0\}$ and by $l_R(S)$ the left annihilator of S in R, that is, $l_R(S) = \{x \in R \mid xS = 0\}$. If $r_R(S) = l_R(S)$, then $r_R(S)$ is called an annihilator ideal of R and is written as $ann_R(S)$. We know that if R is a semiprime ring and I is an ideal of R, then $r_R(I) = l_R(I)$. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Let $F, d : R \longrightarrow R$ be two mappings such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. If F is additive and d is a derivation of R, then F is said to be a generalized derivation of R. The notion of generalized derivation was introduced by Brešar [13]. In [23], Hvala gave the algebraic study of generalized derivations of prime rings. Recall that if R has the property $Rx = (0)$ implies $x = 0$ and $F : R \to R$ is an additive mapping such that $F(xy) = F(x)y + xh(y)$ for all $x, y \in R$ and some function $h : R \to R$, then F is uniquely determined by h. Moreover, h must be a derivation by [13, Remark 1]. Obviously, every derivation is a generalized derivation of R. Thus, generalized derivations cover both the concept of derivations and the concept of left centralizers. An additive mapping $F : R \longrightarrow R$ is a left centralizer (multiplier) if $F(xy) = F(x)y$ for all $x, y \in R$ (see [33] and [34] for details).

Let S be a nonempty subset of R. A mapping $f : R \longrightarrow R$ is called centralizing on S if $[f(x), x] \in Z(R)$ for all $x \in S$ and is called commuting on S if $[f(x), x] = 0$ for all $x \in S$. The study of such mappings were initiated by Posner (Posner second theorem). In [30, Theorem 2], proved that if a prime ring R admits a nonzero derivation d

such that $[d(x), x] \in Z(R)$ for all $x \in R$, then R is commutative. An analogous result for centralizing automorphisms on prime rings was obtained by Mayne [28]. A number of authors have extended these theorems of Posner and Mayne; they have showed that derivations, automorphisms, and some related maps cannot be centralizing on certain subsets of noncommutative prime (and some other) rings. For these results we refer the reader to ([10], [12], [15], [25], where further references can be found). In [14], the description of all centralizing additive maps of a prime ring R of characteristic not 2 was given and subsequently in [4] the characterization for semiprime rings of characteristic not 2 was given. It was shown that every such map f is of the form $f(x) = \lambda x + \mu(x)$, where $\lambda \in C$, the extended centroid of R, and μ is an additive map of R into C (see also [12] where similar results for some other rings are presented). Recently, some authors have obtained commutativity of prime and semiprime rings with derivations and generalized derivations satisfying certain polynomial identities (viz., [1], [2], [3], [5], [6], [7], [8], [9], [16], [17], [18], [19], [20], [24], [25], [27], [29], [31] and [32]). The main objective of the present paper is to investigate commutativity of prime and semiprime rings satisfying certain identities involving additive mappings and derivations.

2 Some Preliminaries

We shall do a great deal of calculations with commutators and anti-commutators, routinely using the following basic identities: For all $x, y, z \in R$;

$$[xy, z] = x[y, z] + [x, z]y \quad \text{and} \quad [x, yz] = [x, y]z + y[x, z]$$

$$x \circ (yz) = (x \circ y)z - y[x, z] = y(x \circ z) + [x, y]z$$

$$(xy) \circ z = x(y \circ z) - [x, z]y = (x \circ z)y + x[y, z].$$

The next statements are well-known results which we will use in the next section.

Lemma 2.1 [9, Lemma 1].

(*a*) *If R is a semiprime ring, then the center of a nonzero one-sided ideal is contained in the center of R. In particular, any commutative one-sided ideal is contained in the center of R.*

(*b*) *If R is prime with a nonzero central ideal, then R is commutative.*

Lemma 2.2 [10, Theorem 3]. *Let R be a semiprime ring and I a nonzero left ideal of R. If R admits a derivation d which is nonzero on I and centralizing on I, then R contains a nonzero central ideal.*

Lemma 2.3 [10, Theorem 4]. *Let R be a prime ring and I a nonzero left ideal of R. If R admits a nonzero derivation d which is centralizing on I, then R is commutative.*

Lemma 2.4 [22, Corollary 2]. *If R is a semiprime ring and I is an ideal of R, then $I \bigcap ann_R(I) = 0$.*

Lemma 2.5 [26, Theorem 2]. *Let R be a prime ring and ρ a nonzero right ideal of R. Then every centralizing additive mapping $f : \rho \to R$ is of the form $f(x) = \lambda x + \mu(x)$ for all $x \in \rho$, where $\lambda \in C$ and $\mu : \rho \to C$, unless $[\rho, \rho]\rho = 0$.*

Lemma 2.6 [29, Lemma 4]. *Let R be a prime ring, $a \in R$ and $0 \neq z \in Z(R)$. If $az \in Z(R)$, then $a \in Z(R)$.*

3 The Main Results

We begin our discussion with the following theorem which extends some known results obtained in [8].

Theorem 3.1. *Let R be a semiprime ring, I a nonzero ideal of R, and let $F, d : R \to R$ be two additive mappings such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. If $F(x)F(y) \pm xy \in Z(R)$ for all $x, y \in I$, then $[d(x), x] = 0$ for all $x \in I$. Moreover, if d is a derivation such that $d(I) \neq (0)$, then R contains a nonzero central ideal.*

Proof. We begin with the situation

$$F(x)F(y) - xy \in Z(R) \tag{1}$$

for all $x, y \in I$. Replacing y with yz, $z \in I$, we have

$$F(x)F(yz) - x(yz) \in Z(R) \tag{2}$$

which gives

$$F(x)(F(y)z + yd(z)) - xyz \in Z(R) \tag{3}$$

for all $x, y \in I$. Commuting (3) with z and then using (1), we get

$$[F(x)yd(z), z] = 0 \tag{4}$$

for all $x, y, z \in I$. Putting $y = zy$ in above relation we obtain

$$[F(x)zyd(z), z] = 0 \tag{5}$$

for all $x, y, z \in I$. Now putting $x = xz$ in (4), we get

$$[F(x)zyd(z), z] + [xd(z)yd(z), z] = 0 \tag{6}$$

for all $x, y, z \in I$. In view of (5), above relation reduces to

$$[xd(z)yd(z), z] = 0 \tag{7}$$

for all $x, y, z \in I$. Substituting $d(z)x$ for x in (7) and using it, we obtain

$$[d(z), z]xd(z)yd(z) = 0 \tag{8}$$

for all $x, y, z \in I$. This implies $[d(z), z]x[d(z), z]y[d(z), z] = 0$ that is $(I[d(z), z])^3 = (0)$ for all $z \in I$. Since R is semiprime, it contains no nonzero nilpotent left ideals, implying $I[d(z), z] = (0)$ for all $z \in I$. Thus, $[d(z), z] \in ann_R(I)$ for all $z \in I$. On the other hand, since I is an ideal of R, $[d(z), z] \in I$ for all $z \in I$. Thus $[d(z), z] \in I \cap ann_R(I)$ for all $z \in I$. In view of Lemma 2.4, $[d(z), z] = 0$ for all $z \in I$. Moreover, if d is a derivation such that $d(I) \neq (0)$, then by Lemma 2.2, R contains a nonzero central ideal.

By the same argument, we may obtain the same conclusion in case $F(x)F(y) + xy \in Z(R)$ for all $x, y \in I$. This completes the proof of theorem.

Following corollaries are the immediate consequences of the above theorem.

Corollary 3.2 [8, Theorem 2.5]. *Let R be a prime ring and I a nonzero ideal of R. If R admits a generalized derivation F associated with a nonzero derivation d such that $F(x)F(y) \pm xy \in Z(R)$ for all $x, y \in I$, then R is commutative.*

Corollary 3.3. *Let R be a prime ring, I a nonzero ideal of R, and let $F, d : R \to R$ be two additive mappings such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. If $F(x)F(y) \pm xy \in Z(R)$ for all $x, y \in I$, then $[d(x), x] = 0$ for all $x \in I$. Moreover, if d is a derivation, then one of the following holds:*

1. *$d = 0$ and $F(x) = \lambda x + \zeta(x)$ for all $x \in I$, where $\lambda \in C$ and $\zeta : I \to C$ is a left multiplier mapping.*
2. *R is commutative.*

Proof. By Theorem 3.1, we obtain $[d(x), x] = 0$ for all $x \in I$. If d is a derivation, then by Lemma 2.3, either $d = 0$ or R is commutative. If R is commutative, we obtain our conclusion (2). So we assume that R is noncommutative. Then $d = 0$. In this case, for all $r, s \in R$, we get $F(rs) = F(r)s + rd(s) = F(r)s$. In other words, F is a left multiplier on R. We have

$$F(x)F(y) \pm xy \in Z(R) \tag{9}$$

for all $x, y \in I$. Replacing y with yz, and using the fact that F is left multiplier on R, we find that

$$(F(x)F(y) \pm xy)z \in Z(R) \tag{10}$$

for all $x, y, z \in I$. If for some $x, y \in I$, $0 \neq F(x)F(y) \pm xy \in Z(R)$, then by Lemma 2.6, we get $I \subseteq Z(R)$. Hence, R is commutative by Lemma 2.1(b), a contradiction. Thus we are forced to conclude that

$$F(x)F(y) \pm xy = 0 \tag{11}$$

for all $x, y \in I$. Replacing x by xy in (11), we get

$$F(x)yF(y) \pm xy^2 = 0 \tag{12}$$

for all $x, y \in I$. Right multiplying (11) by y and then subtracting from (12), we have $F(x)[F(y), y] = 0$ for all $x, y \in I$. Putting xz for x in the last relation, we obtain $F(x)z[F(y), y] = 0$ for all $x, y, z \in I$. This implies $[F(x), x]z[F(x), x] = 0$ for all $x, z \in I$, that is, $[F(x), x]I[F(x), x] = (0)$ for all $x \in I$. Since R is prime, it follows that $[F(x), x] = 0$ for all $x \in I$. Then by Lemma 2.5, $F(x) = \lambda x + \zeta(x)$ for all $x \in I$, where $\lambda \in C$ and $\zeta : I \to C$. Since F is additive, ζ is also additive map. Moreover, since F is left multiplier, so the map $\zeta(x) = F(x) - \lambda x$ is also a left multiplier on I. This implies that $F(x) = \lambda x + \zeta(x)$ for all $x \in I$. Hence, we obtain our conclusion (1).

Theorem 3.4. *Let R be a semiprime ring, I a nonzero ideal of R, and let $F, d : R \to R$ be two additive mappings such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. If $F(x)F(y) \pm yx \in Z(R)$ for all $x, y \in I$, then $[d(x), x] = 0$ for all $x \in I$. Moreover, if d is a derivation such that $d(I) \neq (0)$, then R contains a nonzero central ideal.*

Proof. First we consider the case

$$F(x)F(y) - yx \in Z(R) \tag{13}$$

for all $x, y \in I$. Replacing y by yz, we get

$$F(x)(F(y)z + yd(z)) - yzx \in Z(R) \tag{14}$$

for all $x, y, z \in I$. This implies that

$$(F(x)F(y) - yx)z + y[x, z] + F(x)yd(z) \in Z(R) \tag{15}$$

for all $x, y, z \in I$. Application of (13) yields that

$$[y[x, z], z] + [F(x)yd(z), z] = 0 \tag{16}$$

for all $x, y, z \in I$. Substituting xz for x in (16), we obtain

$$[y[x, z], z]z + [(F(x)z + xd(z))yd(z), z] = 0 \tag{17}$$

for all $x, y, z \in I$. Putting $y = zy$ in (16), we get

$$z[y[x, z], z] + [F(x)zyd(z), z] = 0 \tag{18}$$

for all $x, y, z \in I$. Subtracting (18) from (17), we have

$$[[y[x, z], z], z] + [xd(z)yd(z), z] = 0 \tag{19}$$

for all $x, y, z \in I$. Replacing x by xz in (19), we obtain

$$[[y[x, z], z], z]z + [xzd(z)yd(z), z] = 0 \tag{20}$$

for all $x, y, z \in I$. Right multiplying (19) by z and then subtracting from (20), we get

$$[x[d(z)yd(z), z], z] = 0 \tag{21}$$

for all $x, y, z \in I$. Now we substitute $d(z)yd(z)x$ for x in (21) and get

$$\begin{aligned} 0 &= [d(z)yd(z)x[d(z)yd(z), z], z] \\ &= d(z)yd(z)[x[d(z)yd(z), z], z] \\ &+ [d(z)yd(z), z]x[d(z)yd(z), z]. \end{aligned} \tag{22}$$

By using (21), it reduces to

$$[d(z)yd(z), z]x[d(z)yd(z), z] = 0 \tag{23}$$

for all $x, y, z \in I$. Since I is an ideal, it follows that $x[d(z)yd(z), z]Rx[d(z)yd(z), z] = (0)$ and hence

$$x[d(z)yd(z), z] = 0 \tag{24}$$

for all $x, y, z \in I$ that is,

$$x\{d(z)yd(z)z - zd(z)yd(z)\} = 0 \tag{25}$$

for all $x, y, z \in I$. Now we put $y = yd(z)u$ and then obtain

$$x\{d(z)yd(z)ud(z)z - zd(z)yd(z)ud(z)\} = 0 \tag{26}$$

for all $x, y, z, u \in I$. By (25), this can be written as

$$x\{d(z)yzd(z)ud(z) - d(z)yd(z)zud(z)\} = 0 \tag{27}$$

that is,

$$xd(z)y[d(z), z]ud(z) = 0 \tag{28}$$

for all $x, y, z, u \in I$. This implies $x[d(z), z]y[d(z), z]u[d(z), z] = 0$ for all $x, y, z, u \in I$ and so $(I[d(z), z])^3 = (0)$ for all $z \in I$. Since semiprime ring contains no nonzero nilpotent left ideals, it follows that $I[d(z), z] = (0)$ for all $z \in I$. Therefore, for all $x \in I$ we have $[d(x), x] \in I \bigcap ann_R(I)$. Since R is semiprime, by Lemma 2.4, we conclude that $[d(x), x] = 0$ for all $x \in I$, as desired.

Moreover, if d is a derivation such that $d(I) \neq (0)$, then by Lemma 2.2, R contains a nonzero central ideal.

In the same manner the conclusion can be obtained in case $F(x)F(y) + yx \in Z(R)$ for all $x, y \in I$. Hence, the theorem is now proved.

Corollary 3.5 [8, Theorem 2.6]. *Let R be a prime ring and I be a nonzero ideal of R. If R admits a generalized derivation F associated with a nonzero derivation d such that $F(x)F(y) \pm yx \in Z(R)$ for all $x, y \in I$, then R is commutative.*

Corollary 3.6. *Let R be a prime ring, I a nonzero ideal of R and let $F, d : R \to R$ be two additive mappings such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. If $F(x)F(y) \pm yx \in Z(R)$ for all $x, y \in I$, then $[d(x), x] = 0$ for all $x \in I$. Moreover, if d is a derivation, then R is commutative.*

Proof. In a view of Theorem 3.4 we have $[d(x), x] = 0$ for all $x \in I$. Moreover, if d is a derivation, then by Lemma 2.3, either $d = 0$ or R is commutative. If R is commutative, then we are done. So, let R be noncommutative. Then $d = 0$. In this case, for any $r, s \in R$, we have $F(rs) = F(r)s + rd(s) = F(r)s$. In other words, F is a left multiplier on R. Replacing y with yx, we get by our hypothesis that $(F(x)F(y) \pm yx)x \in Z(R)$ for all $x, y \in I$. This implies by Lemma 2.6 that for each $x \in I$, either $F(x)F(y) \pm yx = 0$ for all $y \in I$ or $x \in Z(R)$. The sets $x \in I$ for which these two conditions hold are additive subgroups of I whose union is I, therefore $F(x)F(y) \pm yx = 0$ for all $x, y \in I$ or $I \subseteq Z(R)$. The last condition implies by Lemma 2.1(b) that R is commutative, a contradiction. Hence we conclude that $F(x)F(y) \pm yx = 0$ for all $x, y \in I$. Now replacing y with yx and x with x^2, the last expression yields $F(x)F(y)x \pm yx^2 = 0$ for all $x, y \in I$ and $F(x)xF(y) \pm yx^2 = 0$ for all $x, y \in I$, respectively. Subtracting one from another implies that $F(x)[F(y), x] = 0$ for all $x, y \in I$. Substituting $yF(z)$ for y in the last relation, we get $F(x)[F(y)F(z), x] = 0$ for all $x, y, z \in I$, which gives by our hypothesis that $F(x)[zy, x] = 0$ for all $x, y, z \in I$. Now putting y with yr, where $r \in R$, we have $0 = F(x)[zyr, x] = F(x)[zy, x]r + F(x)zy[r, x] = F(x)zy[r, x]$ for all $x, y, z \in I$. Since R is prime, for each $x \in I$, either $F(x) = 0$ or $[R, x] = 0$. Since both of these two conditions form two additive subgroups of I whose union is I, we conclude that either $F(I) = 0$ or $I \subseteq Z(R)$. The last condition implies that R is commutative, a contradiction. Hence $F(I) = 0$. This case gives $0 = F(RI) = F(R)I$, implying $F(R) = 0$. In this case, again our hypothesis yields that $I^2 \subseteq Z(R)$. Since I^2 is a nonzero central ideal of R, so R must be commutative by Lemma 2.1(b). This completes the proof.

Corollary 3.7. *Let R be a semiprime ring, I a nonzero ideal of R, and let $F : R \to R$ be a generalized derivation of R induced by a nonzero derivation d. If $F(x^2) = ax^2$ for all $x \in I$, where $a \in \{0, 1, -1\}$, then $[I, I]d(I) = (0)$. Moreover, if R is prime, then R is commutative.*

Proof. Linearizing $F(x^2) = ax^2$, we get $F(x^2+y^2+xy+yx) = a(x^2+y^2+xy+yx)$ for all $x, y \in I$. By the hypothesis, we obtain $F(x \circ y) = a(x \circ y)$ for all $x, y \in I$. Thus, by Theorem 2.1 in [21], we conclude that $[I, I]d(I) = (0)$. Further, if R is prime, from the last relation, we have $[I, I]Id(R) = (0)$. This implies that $[I, I]RId(R) = (0)$. The primeness of R forces that either $[I, I] = (0)$ or $Id(R) = (0)$. If $[I, I] = (0)$, then $[I, R]I = (0)$. Since R is prime and I a nonzero ideal of R, the last expression yields that $[I, R] = (0)$ that is, I is a central ideal of R. Thus, by Lemma 2.1(b), R is commutative. On the other hand, if $Id(R) = (0)$, then $IRd(R) = (0)$. Since R is prime, we conclude that $d(R) = (0)$, which is a contradiction. This completes the proof.

Corollary 3.8. *Let R be a 2-torsion free ring with identity 1, and let $F : R \to R$ be a generalized derivation induced by d. If $F(x)F(x) = \pm x^2$ for all $x \in R$, then there exists $c \in Z(R)$ such that $c^2 = \pm 1$ and $F(x) = cx$ for all $x \in R$.*

Proof. Linearization of $F(x)F(x) = \pm x^2$ gives $F(x)F(x)+F(y)F(y)+F(x)F(y)+F(y)F(x) = \pm(x^2+y^2+xy+yx)$ for all $x, y \in R$. Our hypothesis yields that $F(x) \circ F(y) = \pm(x \circ y)$ for all $x, y \in R$. Hence, by Theorems 4.2 and 4.3 in [11], we get required result.

We can conclude this paper with an example which show that the restriction imposed on the hypothesis of several results are not superfluous.

Example 3.9. Let S be any ring. Then $R = \left\{ \left(\begin{array}{ccc} 0 & a & b \\ 0 & c & w \\ 0 & 0 & 0 \end{array} \right) \middle| \, a, b, c, w \in S \right\}$ is a ring with center $Z(R) = \left\{ \left(\begin{array}{ccc} 0 & 0 & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \middle| \, b \in S \right\}$.

Next, let $I = \left\{ \left(\begin{array}{ccc} 0 & a & b \\ 0 & 0 & w \\ 0 & 0 & 0 \end{array} \right) \middle| \, a, b, w \in S \right\}$, and define maps $F,\ d : R \longrightarrow R$ as follows:

$$F \left(\begin{array}{ccc} 0 & a & b \\ 0 & c & w \\ 0 & 0 & 0 \end{array} \right) = \left(\begin{array}{ccc} 0 & a & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{array} \right), \quad d \left(\begin{array}{ccc} 0 & a & b \\ 0 & c & w \\ 0 & 0 & 0 \end{array} \right) = \left(\begin{array}{ccc} 0 & 0 & -b \\ 0 & 0 & -w \\ 0 & 0 & 0 \end{array} \right).$$

It can be easily seen that I is a nonzero ideal of R, d is a nonzero derivation and F is additive mapping such that $F(xy) = F(x)y+xd(y)$ for all $x, y \in R$. For any two elements $x, y \in I$, the following conditions: (i) $F(x)F(y) \pm xy \in Z(R)$ (ii) $F(x)F(y) \pm yx \in Z(R)$ are satisfied. However, neither R is commutative nor $[d(x), x] = 0$ for all $x \in I$. Hence, in Corollaries 3.2, 3.5, & 3.6, the hypothesis of primeness is crucial.

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