Universal Journal of Computational Mathematics 2(3): 39-47, 2014 http://www.hrpub.org
DOI: 10.13189/ujcmj.2014.020301

Solving Some Types of Integrals Using Maple

Chii-Huei Yu[1,*], Bing-Huei Chen[2]

[1]Department of Management and Information, Nan Jeon University of Science and Technology, Tainan City, 73746, Taiwan
[2]Department of Electrical Engineering, Nan Jeon University of Science and Technology, Tainan City, 73746, Taiwan
*Corresponding Author: chiihuei@mail.nju.edu.tw

Abstract This paper uses the mathematical software Maple for the auxiliary tool to study six types of integrals. We can obtain the infinite series forms of these six types of integrals by using integration term by term theorem. In addition, we propose some examples to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

Keyword sIntegrals, Infinite Series Forms, Integration Term By Term Theorem, Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, [1-7] can be adopted as references.

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on.In this paper, we study the following six types of integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int \exp(px)\cdot\sin^{-1}[\beta\exp(qx)]dx \tag{1}$$

$$\int \exp(px)\cdot\cos^{-1}[\beta\exp(qx)]dx \tag{2}$$

$$\int \exp(px)\cdot\tan^{-1}[\beta\exp(qx)]dx \tag{3}$$

$$\int \exp(px)\cdot\cot^{-1}[\beta\exp(qx)]dx \tag{4}$$

$$\int \exp(px)\cdot\csc^{-1}[\beta\exp(qx)]dx \tag{5}$$

$$\int \exp(px)\cdot\sec^{-1}[\beta\exp(qx)]dx \tag{6}$$

Where p, q, β are real numbers? We can obtain the infinite series forms of these six types of integrals by using integration term by term theorem; these are the major results of this paper (i.e., Theorems1-6). The study of related integral problems can refer to [8-22]. On the other hand, we provide some integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce some formulas used in this study.

2.1. Formulas ([23])

2.1.1. The Inverse Sine Function

$$\sin^{-1} y = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{4^n(2n+1)} y^{2n+1}$$,where y is a real number, $|y| \le 1$.

2.1.2. The Inverse Cosine Function

$$\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$$,where y is a real number, $|y| \le 1$.

2.1.3. The Inverse Tangent Function

$$\tan^{-1} y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} y^{2n+1}$$,where y is a real number, $|y| \le 1$.

2.1.4. The Inverse Cotangent Function

$$\cot^{-1} y = \frac{\pi}{2} - \tan^{-1} y$$,where y is a real number, $|y| \le 1$.

2.1.5. The Inverse Cosecant Function

$$\csc^{-1} y = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{4^n(2n+1)} y^{-(2n+1)}$$,where y is a real number, $|y| \ge 1$.

2.1.6. The Inverse Secant Function

$$\sec^{-1} y = \frac{\pi}{2} - \csc^{-1} y$$,where y is a real number, $|y| \ge 1$.

Next, we introduce an important theorem used in this paper.

2.2. Integration Term by TermTheorem ([24])

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an interval I . If $\sum_{n=0}^{\infty} \int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty} \int_I g_n$.

The following is the first result of this study, we obtain the infinite series form of the integral (1).

2.3. Theorem 1

Suppose p, q, β are real numbers, $q \ne 0$, $-\frac{q+p}{2q}$ is not a non-negative integer, and C is a constant. If $|\beta| \exp qx \le 1$, then the integral

$$\int \exp(px)\cdot\sin^{-1}[\beta\exp(qx)]dx = \sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)(2nq+q+p)}\exp[(2nq+q+p)x]+C \quad (7)$$

2.3.1. Proof

$$\int \exp(px)\cdot\sin^{-1}[\beta\exp(qx)]dx$$

$$=\int \exp(px)\sum_{n=0}^{\infty}\frac{\binom{2n}{n}}{4^n(2n+1)}[\beta\exp(qx)]^{2n+1}dx \text{ (By Formula 2.1.1)}$$

$$=\int\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)}\exp[(2nq+q+p)x]dx$$

$$=\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)}\int\exp[(2nq+q+p)x]dx \text{ (Using integration term by term theorem)}$$

$$=\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)(2nq+q+p)}\exp[(2nq+q+p)x]+C$$

q.e.d.

Next, we determine the infinite series form of the integral (2).

2.4. Theorem 2

Let the assumptions be the same as Theorem 1. If $|\beta|\exp qx \le 1$, then the integral

$$\int \exp(px)\cdot\cos^{-1}[\beta\exp(qx)]dx = \frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)(2nq+q+p)}\exp[(2nq+q+p)x]+C \quad (8)$$

2.4.1. Proof

Because $|\beta|\exp qx \le 1$, the integral

$$\int \exp(px)\cdot\cos^{-1}[\beta\exp(qx)]dx$$

$$=\int\exp(px)\left(\frac{\pi}{2}-\sin^{-1}[\beta\exp(qx)]\right)dx \text{ (Using Formula 2.1.2)}$$

$$=\frac{\pi}{2}\int\exp(px)dx-\int\exp(px)\sin^{-1}[\beta\exp(qx)]dx$$

$$=\frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{2n+1}}{4^n(2n+1)(2nq+q+p)}\exp[(2nq+q+p)x]+C \text{ (By Theorem 1)}$$

q.e.d.

The following is the third major result in this study, we obtain the infinite series form of the integral (3).

2.5. Theorem 3

Suppose the assumptions are the same as Theorem 1. If $|\beta|\exp qx \le 1$, then the integral

$$\int \exp(px)\cdot\tan^{-1}[\beta\exp(qx)]dx = \sum_{n=0}^{\infty}\frac{(-1)^n\beta^{2n+1}}{(2n+1)(2qn+q+p)}\exp[(2qn+q+p)x]+C \quad (9)$$

2.5.1. Proof

Because $|\beta|\exp qx \le 1$, the integral

$$\int \exp(px)\cdot\tan^{-1}[\beta\exp(qx)]dx$$

$$=\int \exp(px)\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}[\beta\exp(qx)]^{2n+1}dx \text{ (Using Formula 2.1.3)}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n\beta^{2n+1}}{2n+1}\int \exp[(2qn+q+p)x]dx \text{ (By integration term by term theorem)}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n\beta^{2n+1}}{(2n+1)(2qn+q+p)}\exp[(2qn+q+p)x]+C$$

q.e.d.

Next, we find the infinite series form of the integral (4).

2.6. Theorem 4

Let the assumptions be the same as Theorem 1. If $|\beta|\exp qx \le 1$, then the integral

$$\int \exp(px)\cdot\cot^{-1}[\beta\exp(qx)]dx = \frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{(-1)^n\beta^{2n+1}}{(2n+1)(2qn+q+p)}\exp[(2qn+q+p)x]+C \quad (10)$$

2.6.1. Proof

Because $|\beta|\exp qx \le 1$, the integral

$$\int \exp(px)\cdot\cot^{-1}[\beta\exp(qx)]dx$$

$$=\int \exp(px)\left(\frac{\pi}{2}-\tan^{-1}[\beta\exp(qx)]\right)dx \text{ (By Formula 2.1.4)}$$

$$=\frac{\pi}{2}\int \exp(px)dx-\int \exp(px)\tan^{-1}[\beta\exp(qx)]dx$$

$$=\frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{(-1)^n\beta^{2n+1}}{(2n+1)(2qn+q+p)}\exp[(2qn+q+p)x]+C \text{ (By Theorem 3)}$$

q.e.d.

In the following, we obtain the infinite series form of the integral (5).

2.7. Theorem 5

Suppose p,q,β are real numbers, $q,\beta \neq 0$, $-\dfrac{q-p}{2q}$ is not a non-negative integer, and C is a constant. If $|\beta|\exp qx \geq 1$, then the integral

$$\int \exp(px)\cdot\csc^{-1}[\beta\exp(qx)]dx = \sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{-(2n+1)}}{4^n(2n+1)(-2qn-q+p)}\exp[(-2qn-q+p)x]+C \quad (11)$$

2.7.1. Proof Because

$$\int \exp(px)\cdot\csc^{-1}[\beta\exp(qx)]dx$$

$$=\int \exp(px)\sum_{n=0}^{\infty}\frac{\binom{2n}{n}}{4^n(2n+1)}[\beta\exp(qx)]^{-(2n+1)}dx \text{ (Using Formula 2.1.5)}$$

$$=\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{-(2n+1)}}{4^n(2n+1)}\int \exp[(-2qn-q+p)x]dx \text{ (Using integration term by term theorem)}$$

$$=\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{-(2n+1)}}{4^n(2n+1)(-2qn-q+p)}\exp[(-2qn-q+p)x]+C \qquad \text{q.e.d.}$$

Finally, we determine the infinite series form of the integral (6).

2.8. Theorem 6

Let the assumptions be the same as Theorem 5. If $|\beta|\exp qx \geq 1$, then the integral

$$\int \exp(px)\cdot\sec^{-1}[\beta\exp(qx)]dx = \frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{-(2n+1)}}{4^n(2n+1)(-2qn-q+p)}\exp[(-2qn-q+p)x]+C \quad (12)$$

2.8.1. Proof Because

$$\int \exp(px)\cdot\sec^{-1}[\beta\exp(qx)]dx$$

$$=\int \exp(px)\left(\frac{\pi}{2}-\csc^{-1}[\beta\exp(qx)]\right)dx \text{ (By Formula 2.1.6)}$$

$$=\frac{\pi}{2}\int \exp(px)dx-\int \exp(px)\csc^{-1}[\beta\exp(qx)]dx$$

$$=\frac{\pi}{2p}\exp(px)-\sum_{n=0}^{\infty}\frac{\binom{2n}{n}\beta^{-(2n+1)}}{4^n(2n+1)(-2qn-q+p)}\exp[(-2qn-q+p)x]+C \text{ (By Theorem 5)}$$

q.e.d.

3. Examples

In the following, for the six types of integrals in this study, we provide some integrals and use our theorems to determine their infinite series forms. In addition, we evaluate some related definite integrals and employ Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

3.1. Example 1

If $\exp(3x) \le \frac{1}{2}$, i.e., $x \le -\frac{1}{3}\ln 2 \cong -0.231$. By Theorem 1, we obtain the integral

$$\int \exp(-4x)\sin^{-1}[2\exp(3x)]dx = \sum_{n=0}^{\infty} \frac{\binom{2n}{n} 2^{2n+1}}{4^n (2n+1)(6n-1)} \exp[(6n-1)x] + C \quad (13)$$

Therefore, we can evaluate the related definite integral from $x = -2$ to $x = -1$,

$$\int_{-2}^{-1} \exp(-4x)\sin^{-1}[2\exp(3x)]dx = \sum_{n=0}^{\infty} \frac{\binom{2n}{n} 2^{2n+1}}{4^n (2n+1)(6n-1)} [\exp(-6n+1) - \exp(-12n+2)] \quad (14)$$

We use Maple to verify the correctness of (14).
>evalf(int(exp(-4*x)*arcsin(2*exp(3*x)),x=-2..-1),18);

9.34333687809384508

>evalf(sum((2*n)!/(n!*n!)*2^(2*n+1)/(4^n*(2*n+1)*(6*n-1))*(exp(-6*n+1)-exp(-12*n+2)),n=0..infinity),18);

9.34333687809384508

3.2. Example 2

If $\exp(2x) \le \frac{1}{4}$, i.e., $x \le -\frac{1}{2}\ln 4 \cong -0.693$. Using Theorem 4, we obtain the following integral

$$\int \exp(6x)\cot^{-1}[-4\exp(2x)]dx = \frac{\pi}{12}\exp(6x) - \sum_{n=0}^{\infty} \frac{(-1)^n (-4)^{2n+1}}{(2n+1)(4n+8)} \exp[(4n+8)x] + C \quad (15)$$

Thus, we can evaluate the related definite integral from $x = -3$ to $x = -1$,

$$\int_{-3}^{-1} \exp(6x)\cot^{-1}[-4\exp(2x)]dx$$

$$= \frac{\pi}{12}[\exp(-6) - \exp(-18)] - \sum_{n=0}^{\infty} \frac{(-1)^n (-4)^{2n+1}}{(2n+1)(4n+8)} [\exp(-4n-8) - \exp(-12n-24)] \quad (16)$$

We also use Maple to verify the correctness of (16).
>evalf(int(exp(6*x)*arccot(-4*exp(2*x)),x=-3..-1),14);

0.00080697737112992

>evalf(Pi/12*(exp(-6)-exp(-18))-sum((-1)^n*(-4)^(2*n+1)/((2*n+1)*(4*n+8))*(exp(-4*n-8)-exp(-12*n-24)),n=0..infinity),14);

0.00080697737112992

3.3. Example 3

If $\exp(4x) \geq \frac{1}{5}$, i.e., $x \geq -\frac{1}{4}\ln 5 \cong -0.402$. By Theorem 6,the following integral

$$\int \exp(3x)\sec^{-1}[5\exp(4x)]dx = \frac{\pi}{6}\exp(3x) - \sum_{n=0}^{\infty} \frac{\binom{2n}{n} 5^{2n+1}}{4^n (2n+1)(-8n-1)} \exp[(-8n-1)x] + C \quad (17)$$

Hence, we can determine the related definite integral from $x = 4$ to $x = 7$,

$$\int_4^7 \exp(3x)\sec^{-1}[5\exp(4x)]dx$$

$$= \frac{\pi}{6}[\exp(21) - \exp(12)] - \sum_{n=0}^{\infty} \frac{\binom{2n}{n} 5^{-(2n+1)}}{4^n (2n+1)(-8n-1)} [\exp(-56n-7) - \exp(-32n-4)] \quad (18)$$

Using Maple to verify the correctness of (18) as follows:
>evalf(int(exp(3*x)*arcsec(5*exp(4*x)),x=4..7),18);

$$6.90445085602191943 \cdot 10^8$$

>evalf(Pi/6*(exp(21)-exp(12))-sum((2*n)!/(n!*n!)*5^(-2*n-1)/(4^n*(2*n+1)*(-8*n-1))*(exp(-56*n-7)-exp(-32*n-4)),n=0..infinity),18);

$$6.90445085602191948 \cdot 10^8$$

4. Conclusion

In this study, we provide a new technique to solve some types of integrals, and we hope this technique can be applied to evaluate another integral problem. On the other hand, we know the integration term by term theorem plays a significant role in the theoretical inferences of this study. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

REFERENCES

[1] C. Tocci and S. G. Adams, Applied Maple for Engineers and Scientists, Boston: Artech House, 1996.

[2] D. Richards, Advanced Mathematical Methods with Maple, New York: Cambridge University Press, 2002.

[3] F. Garvan, The Maple Book, London: Chapman & Hall/CRC, 2001.

[4] R. J. Stroeker and J. F. Kaashoek, Discovering Mathematics with Maple : An Interactive Exploration for Mathematicians, Engineers and Econometricians, Basel: Birkhauser Verlag, 1999.

[5] C. T. J. Dodson and E. A. Gonzalez, Experiments in Mathematics Using Maple, New York: Springer-Verlag, 1995.

[6] M. L. Abell and J. P. Braselton, Maple by Example, 3rd ed., New York: Elsevier Academic Press, 2005.

[7] J. S. Robertson, Engineering Mathematics with Maple, New York: McGraw-Hill, 1996.

[8] C. Oster, Limit of a definite integral, SIAM Review, Vol. 33, No. 1, pp. 115-116, 1991.

[9] A. A. Adams, H. Gottliebsen, S.A. Linton, and U. Martin,Automated theorem proving in support of computer algebra: symbolic definite integration as a case study,Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation,pp. 253-260, Vancouver,Canada, 1999.

[10] M.A. Nyblom, On the evaluation of a definite integral involving nested square root functions, Rocky Mountain Journal of Mathematics, Vol. 37, No. 4, pp. 1301-1304, 2007.

[11] C. -H. Yu, Using Maple to study two types of integrals, International Journal of Research in Computer Applications and Robotics, Vol. 1, Issue. 4, pp. 14-22, 2013.

[12] C. -H. Yu, Solving some integrals with Maple, International Journal of Research in Aeronautical and Mechanical Engineering, Vol. 1, Issue. 3, pp. 29-35, 2013.

[13] C. -H. Yu, A study on integral problems by using Maple, International Journal of Advanced Research in Computer Science and Software Engineering, Vol. 3, Issue. 7, pp. 41-46, 2013.

[14] C. -H. Yu, Evaluating some integrals with Maple, International Journal of Computer Science and Mobile Computing , Vol. 2, Issue. 7, pp. 66-71, 2013.

[15] C.-H. Yu, Application of Maple on the integral problem of some type of rational functions, Proceedings of the Annual Meeting and Academic Conference for Association of IE, D357-D362, 2012.

[16] C.-H. Yu, Application of Maple on evaluation of definite integrals, Applied Mechanics and Materials, in press, 2013.

[17] C. -H. Yu, Using Maple to study the integrals of trigonometric functions, Proceedings of the 6th IEEE/International Conference on Advanced Infocomm Technology, No. 00294, 2013.

[18] C.-H. Yu, Application of Maple on some type of integral problem, Proceedings of the Ubiquitous-Home Conference 2012, pp.206-210, 2012.

[19] C. -H. Yu, A study of the integrals of trigonometric functions with Maple, Proceedings of the Institute of Industrial Engineers Asian Conference 2013, Springer, Vol. 1, pp. 603-610, 2013.

[20] C.-H. Yu, Application of Maple on some integral problems, Proceedings of the International Conference on Safety & Security Management and Engineering Technology 2012, pp. 290-294, 2012.

[21] C.-H. Yu, Application of Maple on evaluating the closed forms of two types of integrals, Proceedings of the 17th Mobile Computing Workshop, ID16, 2012.

[22] C.-H. Yu,Application of Maple: taking two special integral problems as examples, Proceedings of the 8th International Conference on Knowledge Community, pp.803-811, 2012.

[23] Inverse trigonometric functions, online available from http://en.wikipedia.org/wiki/Inverse_trigonometric_functions

[24] T. M. Apostol, Mathematical Analysis, 2nd ed., Boston: Addison-Wesley, p269, 1975.