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Solution of DGLAP Evolution Equation for xF_3 Structure Function in Leading and Next-to-Leading Order at Small- x

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Abstract This report attempts to the phenomenological study of the charged-current neutrino deep-inelastic scattering (DIS) within the perturbative QCD framework. The study is based on the solution of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation in leading and next-to-leading order at small-x for parity-violating DIS structure function $xF_3(x, Q^2)$ by means of Taylor expansion method. The solutions are analyzed phenomenologically in comparison with the experimental data taken from CCFR, NuTeV, CHORUS and CDHSW collaborations.

Keywords Neutrino-Nucleon Scattering, DGLAP Evolution Equation, $xF_3(x, Q^2)$ Structure Function

1. Introduction

Considerable effort has been devoted to obtain clear and reliable quantitative information about the QCD observables such as scaling violation, strong coupling constant, QCD sum rules and distribution of quarks and gluons in the nucleons by means of lepton-nucleon deep inelastic scattering (DIS) for the last three decades. Leptons used in deep inelastic processes are either charged leptons (electron or muon) or neutrinos which scatter off the target nucleons via the electromagnetic or weak interactions respectively. As neutrinos interact weakly with other particles, due to parity violation in the weak interaction the $xF_3(x, Q^2)$ structure function appears in neutrino DIS. The $xF_3(x, Q^2)$ structure function receives contributions from non-singlet part of the co-efficient function only and reflects only the valence quark distributions. It is free from sea quark and gluon densities about which we have very poor information in particular in the small-x region [1, 2]. So it helps in proper interpretation of experimental data which can provide valuable insights in to the origin of nuclear force and helps us to understand various nuclear effects[1-7]. Therefore the studies on neutrino-DIS as well as $xF_3(x, Q^2)$ structure function have been the most active frontiers in both theoretical and experimental particle physics. Recently available differential cross-sections and structure functions in neutrino-DIS experiments are from the experiments in CCFR [8] and NuTeV [9] at Fermilab, and CHORUS [10] and CDHSW [11] at CERN. These measurements have been done for heavy nuclear targets of iron and lead. Theoretically there are various QCD evolution equations to obtain the quark and gluon distribution functions such as Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP), Balitsky-Fadin-Kuraev-Lipatov(BFKL), Ciafaloni-Catani-Fiorani-Marchesini (CCFM), Balitsky-Kovchegov (BK), Gribov-Levin-Ryskin (GLR) etc., in different kinematical regions. Among these evolution equations, BFKL or GLR equations are more appealing at small-x, but still the DGLAP evolution equation is used to study various structure functions because this equation is a simple perturbative tool which is relevant for the presently accessible x-Q^2 range of structure functions. In this paper we have obtained the $xF_3(x, Q^2)$ structure function by solving the DGLAP evolution equation analytically in leading-order (LO) and next-to-leading order (NLO) at small-x using the Taylor series expansion method which was developed in Ref. [17-23]. Application of Tailor series expansion method reduces the integro-differential DGLAP equation to the first-order linear differential equation, which in turn can be solved easily. The solutions are then analyzed in comparison with experimental data taken from CCFR, NuTeV and CHORUS collaborations. It is observed that our theoretical results agree well with these experimental data. The agreement of the theoretical as well as experimental data reflects that the Taylor series method is a significant method in order to study the small- x behavior of the $xF_3(x, Q^2)$ structure functions.

2. Theory

The DGLAP evolution equation for $xF_3(x,Q^2)=F(x,Q^2)$ structure function in standard form is given by

$$\frac{\partial F(x,Q^2)}{\partial Q^2}=\int\frac{d\omega}{\omega}F\left(\frac{\omega}{x},Q^2\right)P(\omega) \quad (1)$$

where$P(\omega)$ is the splitting function defined by

$$P(\omega)=\frac{\alpha(Q^2)}{2\pi}P^0(\omega)+\left(\frac{\alpha(Q^2)}{2\pi}\right)^2P^1(\omega)+\cdots$$

Here $P^0(\omega)$ and$P^1(\omega)$are the splitting functions in LO and NLO respectively [24, 25]. Substituting the splitting functions in "(1)", the DGLAP equation in LO and NLO becomes

$$\frac{\partial F(x,t)}{\partial t}=\frac{\alpha(Q^2)}{2\pi}\left[\frac{2}{3}\{3+4\ln(1-x)\}F(x,t)+I_1(x,t)\right] \quad (2a)$$

and

$$\frac{\partial F(x,t)}{\partial t}=\frac{\alpha(Q^2)}{2\pi}\left[\frac{2}{3}\{3+4\ln(1-x)\}F(x,t)+I_1(x,t)\right]+\left(\frac{\alpha(t)}{2\pi}\right)^2I_2(x,t) \quad (2b)$$

respectively. Where $t=\ln\frac{Q^2}{\Lambda^2}$and Λ is the QCD cut-off parameter. The integral functions are given by

$$I_1(x,t)=\int_x^1\frac{d\omega}{(1-\omega)}\left\{\frac{1+\omega^2}{\omega}F\left(\frac{x}{\omega},t\right)-2F(x,t)\right\}$$

and

$$I_2(x,t)=k.F(x,t)+\int_x^1\frac{d\omega}{\omega}f(\omega)F\left(\frac{x}{\omega},t\right)$$

with

$$k=\left\{C_F^2\left[\frac{3}{8}-\frac{1}{2}\pi^2+\varsigma(3)-8\tilde{S}(\infty)\right]+\frac{1}{2}C_FC_A\left[\frac{17}{12}-\frac{11}{9}\pi^2-\varsigma(3)+8\tilde{S}(\infty)\right]+C_FT_RN_F\left[\frac{1}{6}+\frac{2}{9}\pi^2\right]\right\}$$

and

$f(\omega)=\left[C_F^2\{P_F(\omega)-P_A(\omega)\}+\frac{1}{2}C_FC_A\{P_G(\omega)+P_A(\omega)\}+C_FT_RN_FP_{N_F}(\omega)\right]$, which are obtained from the splitting functions [24, 25].

Now let us consider a new variable u such that $u=1-\omega$ and $\frac{x}{\omega}=\frac{x}{1-u}=x+\frac{xu}{1-u}=x+x\sum_{i=0}^{\propto}u^i$ as discussed elsewhere earlier [17-23]. we can expand the term $F\left(\frac{x}{\omega},t\right)$ applying Taylor expansion method as

$$F\left(\frac{x}{\omega},t\right)=F\left(\frac{x}{1-u},t\right)=F\left(x+\frac{xu}{1-u},t\right)$$

$$=F\left(x+x\sum_{i=0}^{\propto}u^i,t\right)$$

$$=F(x,t)+\left(x\sum_{i=1}^{\propto}u^i\right)\frac{\partial F(x,t)}{\partial x}+\left(x\sum_{i=1}^{\propto}u^i\right)^2\frac{\partial^2F(x,t)}{\partial^2x}+\cdots\ldots\ldots\ldots$$

$$=F(x,t)+\left(x\frac{u}{1-u}\right)\frac{\partial F(x,t)}{\partial x}+\left(x\frac{u}{1-u}\right)^2\frac{\partial^2F(x,t)}{\partial^2x}+\cdots\ldots\ldots\ldots$$

Again it has been observed that when we solve DGLAP evolution equation like second order partial differential equation by Monges method, which is produced by introducing the second order term in Taylor expansion above, it becomes ultimately the first order partial differential equation as before due to the form of the DGLAP equation[23]. Similarly by introducing more terms of Taylor expansion, we hope the same. Moreover, for the smaller values of x, the terms in the expansion containing x^2 and higher powers of x can be neglected [17-23]. Thus using the first two terms of the Taylor expansion series we can write

$$F\left(\frac{x}{\omega},t\right)=F(x,t)+\left(x\frac{u}{1-u}\right)\frac{\partial F(x,t)}{\partial x}.$$

Substituting this expansion in "(2a)" and "(2b)" and performing u-integrations we get these equations as

$$\frac{\partial F(x,t)}{\partial t} = \frac{a}{t}\left[P(x)F(x,t) + Q(x)\frac{\partial F(x,t)}{\partial x}\right] \quad (3a)$$

$$\frac{\partial F(x,t)}{\partial t} = \frac{a}{t}\left[1 - b\frac{\ln t}{t}\right]\left[R(x)F(x,t) + S(x)\frac{\partial F(x,t)}{\partial x}\right] \quad (3b)$$

Where,

$$P(x) = \frac{2}{3}(1 + 2x - 2\ln x + 4\ln(1-x)),$$

$$Q(x) = \frac{4}{3}(1 - x^2),$$

$$R(x) = P(x) + T_0\left\{\left(\int_x^1 \frac{d\omega}{\omega} f(\omega)\right) + k\right\},$$

$$S(x) = Q(x) + T_0 x\left\{\left(\int_x^1 f(\omega)(1-\omega)\frac{d\omega}{\omega^2}\right)\right\},$$

$a = \frac{2}{\beta_0}$, and $b = \frac{\beta_1}{{\beta_0}^2}$. Here $\beta_0 = 11 - \frac{2}{3}N_F$ and $\beta_1 = \frac{34}{3}C_G^2 - \frac{10}{3}C_G N_F - 2C_F N_F$ are the one-loop and two-loop corrections to the QCD β function with $C_G = C_A = N_C = 3$, $C_F = \frac{4}{3}$, $T_F = \frac{1}{2}N_F$, N_F being the number of flavor. Here we have considered $N_F = 3$.

Again, here we have considered a numerical (not arbitrary) parameters T_0such that $T^2(t) = T_0.T(t)$ with $T(t) = \frac{\alpha(t)}{2\pi}$ (see [17-23] for more details). The values of the parameters T_0 is chosen such that the differences between $T^2(t)$ and $T_0.T(t)$is very small in the range of consideration. Within the range $0 < Q^2 \leq 30$ of our consideration it is observed to be $T_0 = 0.05$. Due to the introduction of these parameters the numerical error is very much less as compared to other errors especially in NLO.

The general solution of "(3a)"is $(U,V) = 0$, where $F(U,V)$ is an arbitrary function. Here $U(x,t,F(x,t)) = C_1$ and $V(x,t,F(x,t)) = C_2$ are two independent solutions of the Lagrange's equation

$$\frac{dx}{aQ(x)} = \frac{dt}{-t} = \frac{dF(x,t)}{-a\,P(x)\,F(x,t)} \quad (4a)$$

which are given by

$$U(x,t,F(x,t)) = t.exp\left(\int_x^1 \frac{1}{a\,Q(x)}\,dx\right)$$

and

$V(x,t,F(x,t)) = F(x,t)\;exp\left(\int_x^1 \frac{P(x)}{Q(x)}\,dx\right)$ respectively. It is known that the "(4a)" has no unique solution and the simplest possible solution is the linear combination $\alpha\,U + \beta\,V = 0$ satisfying $F(U,V) = 0$, where α, β are two arbitrary constants. Thus the simplest solution can be written as

$$F(x,t) = xF_3(x,t) = \gamma\;t\;exp\left[\int_x^1\left(\frac{1}{a\,Q(x)} - \frac{P(x)}{Q(x)}\right)dx\right] \quad (5)$$

with $\gamma = -\frac{\alpha}{\beta}$. Now defining an input point

$$xF_3(x,t_0) = \gamma\;t_0\;exp\left[\int_x^1\left(\frac{1}{a\,Q(x)} - \frac{P(x)}{Q(x)}\right)dx\right]$$

at $Q^2 = Q_0^2$, for which $t = t_0$, we get from "(5)"

$$xF_3(x,t) = xF_3(x,t_0)\left(\frac{t}{t_0}\right) \quad (6a)$$

This gives the t- evolution of $xF_3(x,t)$structure function in LO.
Proceeding in the same manner, the solution of the Lagrange's equations

$$\frac{dx}{aS(x)} = \frac{dt}{-t\left[1 - b\frac{\ln t}{t}\right]} = \frac{dF(x,t)}{-a\,R(x)\,F(x,t)}$$

obtained from the "(3b)" lead towards the t-evolution of $xF_3(x,t)$structure function in NLO which is given by

$$xF_3(x,t) = xF_3(x,t_0)\left(\frac{t^{\left(1+\frac{b}{t}\right)}}{t_0^{\left(1+\frac{b}{t_0}\right)}}\right)exp\left\{b\left(\frac{1}{t} - \frac{1}{t_0}\right)\right\}. \quad (6b)$$

Thus we have obtained $xF_3(x,t)$structure function by solving DGLAP evolution equations in LO and NLO analytically.

3. Result and Discussion

The results of the calculation of $xF_3(x,t)$ structure function using"(6a)" and "(6b)" are depicted in figures Fig.1 and Fig. 2 in comparison with the experimental data taken from CCFR[8], NuTeV[9], CHORUS[10] and CDHSW[11] collaborations. As NuTeV, CHORUS and CDHSW data have the similar x bins (within our range $x < 0.1$ of consideration), therefore they are combined in Fig. 2, and the CCFR data, at different x, plotted separately in Fig. 1. One important point to be noted is that one cannot make absolute predictions for structure functions in QCD. The DGLAP equation can only be solved once an initial distribution or an input point is given [26] i.e. if, for example,

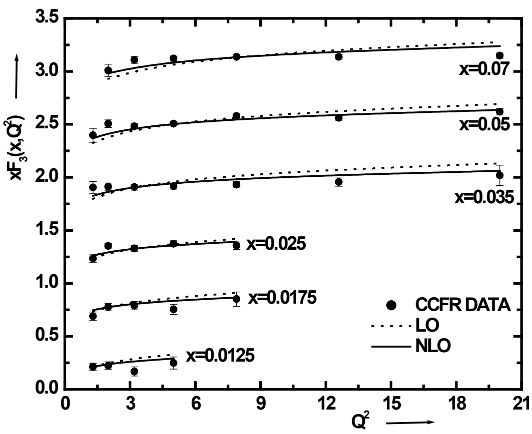

Figure 1. Q^2-evolution of $xF_3(x,Q^2)$structure function in LO and NLO in comparison with experimental data taken from CCFR. For clarity, data are scaled up by +0.5i (i = 0, 1, 2 …) starting from the bottom of all graphs.

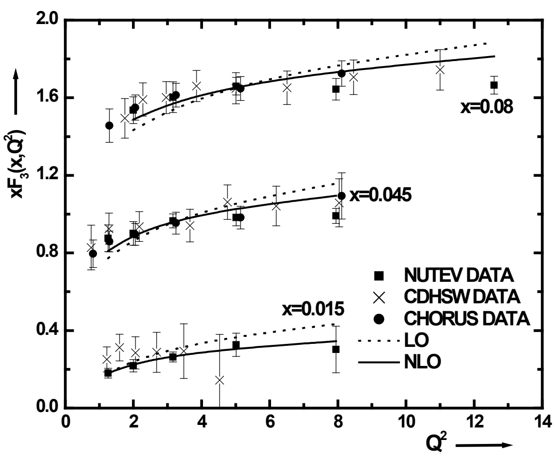

Figure 2. Q^2-evolution of $xF_3(x,Q^2)$ structure function in LO, and NLO in comparison with experimental data taken from NuTeV, CHORUS and CDHSW collaborations. For clarity, data are scaled up by +0.5i (i = 0, 1, 2…) starting from the bottom.

$xF_3(x,Q_0^2)$is given for all values of x at a given Q_0^2, the $xF_3(x,Q^2)$ may be calculated by "(6a)". Here we have considered the points with minimum experimental errors as input points at different values of x and evolved the $xF_3(x,t)$ structure functions against this points. Form Fig. 1 and Fig. 2, it is observed that our theoretical results, particularly when NLO effect is considered, agree well with the experimental data for smaller values of x.

4. Conclusion

We have calculated the structure functions $xF_3(x,Q^2)$ in neutrino-nucleon DIS at small-x by solving the DGLAP evolution equation applying Taylor series expansion method and studied phenomenologically in comparison with experimental data taken from CCFR, NuTeV, CHORUS and CDHSW collaborations. As the applied method gives an arbitrary parameter free analytical solution of DGLAP evolution equation which agrees well with experimental results at small-x, it can be considered as an alternative among the various methods [27-29] in order to study the small-x behavior of the structure functions.

REFERENCES

[1] L. F. Abbott, and R Michael Barnett, The effect of $1/Q^2$ and α_s correction on tests of QCD, *Annals of Physics,* Vol. 125, pp. 276-308, 1980 (and the references therein).

[2] Janet M. Conrad, Michael H. Shaevitz and Tim Bolton, "Precision measurements with high-energy neutrino beams," *Review of Modern Physics*, Vol. 70, 1341-1392, 1998 (and the references therein).

[3] S AKulagin, Nuclear effects in the F_3 structure function for finite and asymptotic Q^2, *Nuclear Physics A*Vol. 640, pp. 435-456, 1998.

[4] S A Kulagin, Nuclear effects and higher twists in F_3 structure function, *European Physics Journal A* Vol. 9, pp. 261-267, 2000.

[5] S A Kulagin and R Petti, Neutrino inclusive inelastic scattering off nuclei, *Phys. Rev. D*Vol. 76, pp. 094023(1-24), 2007.

[6] Athar M Sajjad et al, Nuclear effects in F3 structure function of nucleon, *PhysicsLetters B,* Vol. 668, pp. 133-142, 2008.

[7] M Hirai, S Kumano and K Saito, Nuclear effect in neutrino-nucleus DIS,*arxiv:hep-ph/0909.2329.*

[8] W. G. Seligman et al., Improved determination of α_s from neutrino nucleon scattering, *Physical Review Letters,* Vol. 79, pp. 1213-1216, 1997.

[9] M. Tzanov et al., Precise Measurement of neutrino and anti-neutrino differential cross section, *Physical Review D*, Vol. 74, Article ID 012008, 16 pages, 2006.

[10] G. Onengut et al., Measurements of nucleon structure functions in neutrino scattering, *Physics Letters B,* Vol. 632, pp. 65-75, 2006.

[11] J. P. Berge et al., A measurement of differential cross-sections and nucleon structure functions in charged current neutrino interactions on iron,*Zeitschrift für Physik C*,Vol. 49, pp. 187-224, 1991.

[12] A D Martin, W J Stirling, R S Thorne and G Watt,Parton distributions for the LHC,*EuropeanPhysics Journal C,* Vol. 63, p. 189, 2009.

[13] V. N. Gribov and L. N. Lipatov, Deep inelastic ep scattering in perturbation theory, *Soviet Journal of Nuclear Physics*, Vol. 15, p. 438, 1972.

[14] L. N. Lipatov, The parton model and perturbation theory, *Soviet Journal of Nuclear Physics*, Vol. 20, p. 94, 1975.

[15] Yu. L. Dokshitzer, Calculation of the structure functions for deep inelastic scattering and e+ e− annihilation by perturbation theory in Quantum Chromodynamics, *Soviet Physics Journal of Experimental and Theoretical Physics*, Vol. 46, pp. 641–653, 1977.

[16] G. Altarelli and G. Parisi, Asymptotic freedom in parton language, *Nuclear Physics B*, Vol. 126, no. 2, pp. 298–318, 1977.

[17] M. Devee, R. Baisya and J. K. Sarma, Evolution of singlet structure functions from DGLAP equation at next-to-next-to-leading order at small-*x,EuropeanPhysics Journal C,*Vol. 72, Article ID. 2036, 11 pages, 2012.

[18] R. Baishya, U. Jamil, and J. K. Sarma, J. K. Evolution of spin-dependent structure functionsfrom DGLAP equations in leadingorder and next to leadingorder,*Physical Review D*, Vol. 79, Article ID. 034030, 6 pages, 2009.

[19] R. Baishya, and J. K. Sarma,Method of characteristics and solution of DGLAP evolutionequation in leading and next to leadingorderatsmall*x*, *Physical Review D*, Vol. 79, Article ID. 107702, 3 pages, 2006.

[20] J. Baishya, and J. K. Sarma, Semi numerical solution of non-singlet Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolutionequation up to next-to-next-to-leadingorderatsmall*x*, *EuropeanPhysics Journal C,*Vol. 60, pp. 585-591, 2009.

[21] D K Choudhury and Saiful Islam, An analysis of non-singlet structure function in next-to-next-to-leading order at small-*x,Indian Journal of Physics,*Vol. 85, pp. 319-328, 2011.

[22] N. N. K. Borah, D. K. Choudhury, and P. K. Sahariah, "Comparison of Analytical Solution of DGLAP Equations for $F_2^{NS}(x,t)$at Small x by Two Methods, *Advances in High Energy Physics,* Vol. 2013, Article ID 829803, 10 pages, 2013.

[23] D. K. Choudhury, J. K. Sarma and G. K. Medhi, *t* and *x*-Evolution of gluon structure functions at low-*x, Gauhati University Journal of Science*, Golden Jubilee volume, pp. 39-53, 1998.

[24] S. Moch, J. A. M. Vermaseren and A. Vogt, The three-loop splitting functions in QCD: the non-singlet case,*NuclearPhysics B,*Vol. 688, pp. 101-134, 2004.

[25] R. T. Herrrod and S. Wada, Altarelli-Parisi Equation in the Next-to-LeadingOrder,*PhysicsLetters B,* Vol. 96, p. 195, 1980.

[26] Y. L. Dokshitzer, V. A. Khoze, A. H. Mueller and S. I. Troyan, *Basics of Perturbative QCD*, Frontierseditions, 1991.

[27] R. Toldra, A C++ code to solve the DGLAP equations applied to Ultra High Energy Cosmic Rays, *Computational Physics Communication,* Vol. 143, pp. 287-304, 2002.

[28] N. Cabibbo and R. Petronzio, Two-stage model of hadron structure: Parton distributions and their Q^2 dependence, *Nucearl Physics B* Vol. 137, pp. 395-412, 1978.

[29] M. Gluck, E. Reya, and A. Vogt, Radiatively generated parton distributions for high energycollisions, *Zeitschrift für Physik C,* ,Vol. 48, pp. 471-482 1990.