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New Extended (G'/G)-Expansion Method and its Application in the (3+1)-Dimensional Equation to Find New Exact Traveling Wave Solutions

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Abstract A new extended (G'/G)-expansion method is presented in this paper to construct more general type and new traveling wave solutions of nonlinear partial differential equations. To illustrate the novelty and advantage of the proposed method, we solve the (3+1)-dimensional Jimbo-Miwa equation. Abundant exact traveling wave solutions of this equation is obtained, which successfully recover most of the previously published solutions. Many of those solutions are found for the first time. Furthermore, the results reveal that the proposed method is very elementary, effective and can be used for many other nonlinear partial differential equations.

Keywords New Extended (G'/G)-Expansion Method, the (3+1)-Dimensional Jimbo-Miwa Equation, Soliton Solutions, Traveling Wave Solutions

1 Introduction

Many complex real world problems in nature are due to nonlinear phenomena. Nonlinear processes are one of the biggest challenges and not easy to control because the nonlinear characteristic of the system abruptly changes due to some small changes of valid parameters including time. Seeking the exact solutions of nonlinear partial differential equations plays an significant role, when we want to understand the physical mechanism of the phenomena such as the wave phenomena observed in fluid dynamics [7,14], plasma and elastic media [8,13] and optical fibers [12,19] etc. Recently, a number of prominent mathematicians and physicists have devoted considerable efforts on this interesting area of research to obtain exact solutions of nonlinear partial differential equations using symbolical computer programs such as Maple, Matlab, Mathematica that facilitate complex and tedious algebraical computations.

In recent years, there has been significant progression in the development of obtaining various powerful and effective methods for exact traveling in this fields such as the inverse scattering method [1], Backlund transformation method [11], Darboux Transformations [10], tanh-function method [18], Exp-function method [9] and so on. Recently, Wang et al.[17] introduced the (G'/G)-expansion method which provides a straightforward and effective algorithm to look for traveling wave solutions of nonlinear evolution equations, where $G = G(\xi)$ satisfies the second-order linear ordinary differential equation $G'' + \lambda G' + \mu G = 0$ and λ and μ are arbitrary constants. Applying the (G'/G)-expansion method, many authors successfully obtained more exact traveling wave solutions to a large number of nonlinear partial differential equations like [4, 16]. Further the method has been extended by several researchers for its reliability, such as, Zhang el al. [23] presented an improved (G'/G) expansion method to seek general traveling wave solutions in which the power of (G'/G) may be any integer positive or negative, Guo and Zhou [6] presented the extended (G'/G)- expansion method in which solution is of the form $u(\xi) = a_0 + \sum_{i=1}^{n}(a_i(G'/G)^i + b_i(G'/G)^{i-1}\sqrt{\sigma[1 + \frac{(G'/G)^2}{\mu}]})$ and applying this method Zayed and EL-Malky [20], Zayed and Al-Joudi [21] constructed the traveling wave solutions of some nonlinear evolution equations. Roshid et. al.[15] also used this method to find new exact traveling wave solutions of nonlinear Klein-Gordon equation.

The objective of this article is to apply the proposed method to construct the exact solutions for nonlinear partial differential equations in mathematical physics via the (3+1)-dimensional Jimbo-Miwa equation.
The paper is arranged as follows. In section 2, we describe briefly the new extended (G'/G)-expansion method. In section 3, we apply the method to the (3+1)-dimensional Jimbo-Miwa equation. In section 4, Physical explanation of the solutions are given, In section 5, advantages and validity of the method over the (G'/G)-expansion method have been discussed. Lastly in section 6, some conclusions are given.

2 Materials and Method

Consider a nonlinear evolution equations with independent variables x, y, z and t, is of the form

$$F(u, u_t, u_x, u_y, u_z, u_{xy}, u_{xz}, u_{yz}, u_{tt}, ...) = 0 \quad (1)$$

By using traveling wave transformation

$$u(x, y, z, t) = u(\xi), \xi = x + y + z - Vt \quad (2)$$

where u is an unknown function depending on x, y, z, t and is a polynomial F in $u(\xi) = u(x, y, z, t)$ and its partial derivatives, V is a constant to be determined later. The existing steps of method are as follows :

Step 1: Using the Eq.(2) in Eq.(1), we can reduce Eq.(1) to an ordinary differential equation

$$Q(u, -Vu\prime, u\prime, u\prime, u\prime, u\prime\prime, u\prime\prime, u\prime\prime, V^2u\prime\prime, ...) = 0 \quad (3)$$

Step 2: Assume the solutions of Eq.(3) can be expressed in the form

$$u(\xi) = \sum_{i=-n}^{n} \{ \frac{a_i(G'/G)^i}{[1 + \lambda(G'/G)]^i} + b_i(G'/G)^{i-1}\sqrt{\sigma[1 + \frac{(G'/G)^2}{\mu}]}\} \quad (4)$$

with $G = G(\xi)$ satisfying the differential equation

$$G'' + \mu G = 0 \quad (5)$$

in which the value of σ must be ±1, $\mu \neq 0$, $a_i, b_i, (i = -n, ..., n)$ and λ are constants to be determined later. We can evaluate n by balancing the highest-order derivative term with the nonlinear term in the reduced equation (3).

Step 3: Inserting Eq.(4) into Eq.(3) and making use of Eq.(5) and then extracting all terms of like powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma[1 + \frac{(G'/G)^2]}{\mu}}$ together with each coefficients of them to zero yield a over-determined system of algebraic equations and then solving this system of algebraic equations for $a_i, b_i (i = -n, ..., n)$ and λ, V, we obtain several sets of solutions.

Step 4: For the general solutions of Eq.(5), we have when $\mu < 0$,then

$$\frac{G'}{G} = \sqrt{-\mu}(\frac{Asinh(\sqrt{-\mu}\xi)+Bcosh(\sqrt{-\mu}\xi)}{Acosh(\sqrt{-\mu}\xi)+Bsinh(\sqrt{-\mu}\xi)}) = f_2(\xi) \quad (6)$$

when $\mu > 0$,then

$$\frac{G'}{G} = \sqrt{\mu}(\frac{Acos(\sqrt{\mu}\xi)-Bsin(\sqrt{\mu}\xi)}{Asin(\sqrt{\mu}\xi)+Bcos(\sqrt{\mu}\xi)}) = f_1(\xi) \quad (7)$$

where A, B are arbitrary constants. At last, inserting the values of $a_i, b_i (i = -n, ..., n), \lambda, V$ and (6,7) into Eq.(4), we obtain required traveling wave solutions of Eq.(1).

Remark-1: It is remarkable to monitor that if we put $\lambda = 0$ in the Eq.(4), then the proposed new extended (G'/G)-expansion method agree with the Guo and Zhou's [6] extended (G'/G)-expansion. On the other hand if we put $b_i = 0$ and $\lambda = 0$ in the Eq.(4), then the proposed method is identical to the improved (G'/G)-expansion method presented by Zhang et al. [23]. Again if we set $b_i = 0$, $\lambda = 0$ and negative the exponents of (G'/G) are zero in Eq.(4), then the proposed method turn into the basic (G'/G)-expansion method introduced by Wang et al. [17]. Thus the methods presented in the Ref. [6, 17, 23] are only special cases of our proposed new extended (G'/G)-expansion method.

3 Application of our Method

Now, we are going to use our method in the (3+1)-dimensional Jimbo-Miwa equation. Let us consider the (3+1)-dimensional Jimbo-Miwa equation,

$$u_{xxxy} + 3u_yu_{xx} + 3u_xu_{xy} + 2u_{yt} - 3u_{xz} = 0 \quad (8)$$

Bring to bear the traveling wave transformation $u(x, y, z, t) = u(\xi)$, $\xi = x + y + z - Vt$, Eq. (8) is changing into the following ODE:

$$u^{(4)} + 6u'u'' - 2Vu'' - 3u'' = 0 \qquad (9)$$

Integrating (9) once and letting the integral constant be zero, we have

$$u''' + 3(u')^2 - 2Vu' - 3u' = 0 \qquad (10)$$

Substituting Eq. (4) into Eq. (10) and balancing the highest order derivative u''' with the nonlinear term of the highest order u'^2, we obtain $N = 1$.
Therefore, the solution takes the form

$$\begin{aligned} u(\xi) = a_0 + \frac{a_1(G'/G)}{1 + \lambda(G'/G)} + \frac{a_{-1}[1 + \lambda(G'/G)]}{(G'/G)} \\ +(b_0(G'/G)^{-1} + b_1 + b_{-1}(G'/G)^{-2}) \\ \times\sqrt{\sigma[1 + (G'/G)^2/\mu]} \end{aligned} \qquad (11)$$

where $G = G(\xi)$ satisfies Eq.(5). Substituting Eq.(11) and Eq.(5) into Eq.(10), collecting all terms with the like powers of $(G'/G)^j$ and $(G'/G)^j$ $\sqrt{\sigma[1 + (G'/G)^2/\mu]}$, and setting them to zero, we obtain a over-determined system that consists of thirty algebraic equations (we omitted these for convenience). Solving this over-determined system with the assist of Maple, we have the following results.

Cluster-1: $V = -2\mu - 3/2, \lambda = \lambda, a_0 = a_0, a_1 = 2\mu\lambda^2 + 2, a_{-1} = b_{-1} = b_0 = b_1 = 0$.
Now when $\mu > 0$, then using (7) and (11), we have

$$u(\xi) = a_0 + (2\mu\lambda^2 + 2)\frac{f_1(\xi)}{[1+\lambda f_1(\xi)]}, \qquad (12)$$

where $\xi = x + y + z + (2\mu + 3/2)t$
and when $\mu < 0$, then using (6) and (11), we have

$$u(\xi) = a_0 + (2\mu\lambda^2 + 2)\frac{f_2(\xi)}{[1+\lambda f_2(\xi)]}, \qquad (13)$$

where $\xi = x + y + z + (2\mu + 3/2)t$
Cluster-2: $V = -\mu/2 - 3/2, \lambda = 0, a_0 = a_0, a_1 = 1, b_1 = \pm\sqrt{\frac{\mu}{\sigma}}, a_{-1} = b_{-1} = b_0 = 0$.
Now when $\mu > 0$, then using (7) and (11), we have

$$u(\xi) = a_0 + f_1(\xi) \pm \sqrt{\tfrac{\mu}{\sigma}} \times \sqrt{\sigma[1 + \tfrac{f_1^2(\xi)}{\mu}]} \qquad (14)$$

where $x + y + z + (\mu/2 + 3/2)t$
and when $\mu < 0$, then using (6) and (11), we have

$$u(\xi) = a_0 + f_2(\xi) \pm \sqrt{\tfrac{\mu}{\sigma}} \times \sqrt{\sigma[1 + \tfrac{f_2^2(\xi)}{\mu}]} \qquad (15)$$

where $x + y + z + (\mu/2 + 3/2)t$
Cluster-3: $V = -2\mu - 3/2, \lambda = \lambda, a_0 = a_0, a_{-1} = -2\mu, a_1 = b_{-1} = b_0 = b_1 = 0$.
Now when $\mu > 0$, then using (7) and (11), we have

$$u(\xi) = a_0 - 2\mu f_1^{-1}(\xi)[1 + \lambda f_1(\xi)], \qquad (16)$$

where $\xi = x + y + z + (2\mu + 3/2)t$
and when $\mu < 0$, then using (6) and (11), we have

$$u(\xi) = a_0 - 2\mu f_2^{-1}(\xi)[1 + \lambda f_2(\xi)], \qquad (17)$$

where $\xi = x + y + z + (2\mu + 3/2)t$
Cluster-4: $V = -8\mu - 3/2, \lambda = 0, a_0 = a_0, a_{-1} = -2\mu, a_1 = 2, b_{-1} = b_0 = b_1 = 0$.
Now when $\mu > 0$, then using (7) and (11), we have

$$u(\xi) = a_0 + 2f_1(\xi) - 2\mu f_1^{-1}(\xi), \qquad (18)$$

where $\xi = x + y + z + (8\mu + 3/2)t$
and when $\mu < 0$, then using (6) and (11), we have

$$u(\xi) = a_0 + 2f_2(\xi) - 2\mu f_2^{-1}(\xi), \qquad (19)$$

where $\xi = x + y + z + (8\mu + 3/2)t$
Cluster-5: $V = -\mu/2 - 3/2, \lambda = \lambda, a_0 = a_0, a_{-1} = -\mu, b_0 = \pm\mu\sqrt{\frac{1}{\sigma}}, a_1 = b_{-1} = b_1 = 0$.
Now when $\mu > 0$, then using (7) and (11), we have

$$\begin{aligned} u(\xi) = a_0 - \mu f_1^{-1}(\xi)[1 + \lambda f_1(\xi)] \\ \pm\mu f_1^{-1}(\xi)\sqrt{1 + \tfrac{f_1^2(\xi)}{\mu}}, \end{aligned} \qquad (20)$$

where $\xi = x + y + z + (\mu/2 + 3/2)t$
and when $\mu < 0$, then using (6) and (11), we have

$$\begin{aligned} u(\xi) = a_0 - \mu f_2^{-1}(\xi)[1 + \lambda f_2(\xi)] \\ \pm\mu f_2^{-1}(\xi)\sqrt{1 + \tfrac{f_2^2(\xi)}{\mu}}, \end{aligned} \qquad (21)$$

where $\xi = x + y + z + (\mu/2 + 3/2)t$

Remark-2: It is shown that solutions (12) and (13) have been obtained in Ref.[16] when $\lambda = 0$. Solutions (5.16) and (5.18) of Ref. [16] are identical with our solutions (13) and (12) respectively. Other two soliton solutions can be obtained setting condition on the constants A, B simillar to the condition of constants C_1, C_2. Ref.[16]

4 Physical explanation

The graphical illustrations of the solutions are depicted in the figures from fig-1 to fig-10 with the aid of commercial software Maple 13, where all the figures

are estimated along $y = z = 0$.
Periodic solutions are traveling wave solutions that are periodic such as $cos(x - t)$. Solutions (12),(14),(16),(18) and (20) represent different types exact periodic traveling wave solutions of periodic wave. The graph of (12),(14),(16),(18) and (20) are described by fig.1, fig.2, fig.3, fig.4 and fig.5 respectively of the (3+1)-dimensional Jimbo-Miwa equation (8).

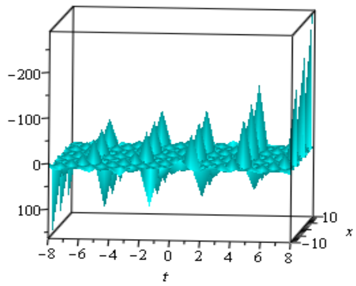

Fig-1: Periodic solution for A=1, B=4, a0= μ=2=λ, σ = -1

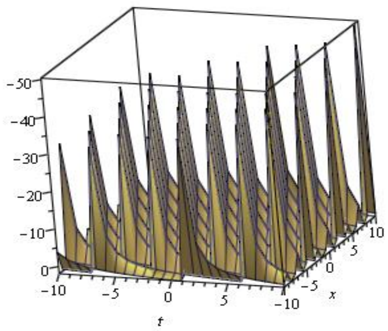

Fig-2: Periodic solution for A= -1, B=3, a0= μ=2, σ =1

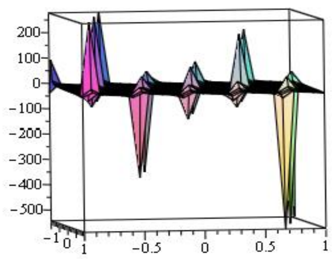

Fig-3: Periodic solution for A=1, B=3, λ= a0=μ=2, σ = -1

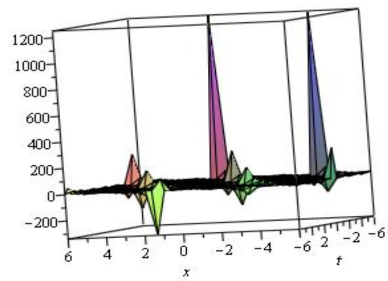

Fig-4: Periodic solution for A=1, B= a0=2, μ=1/2, σ = -1

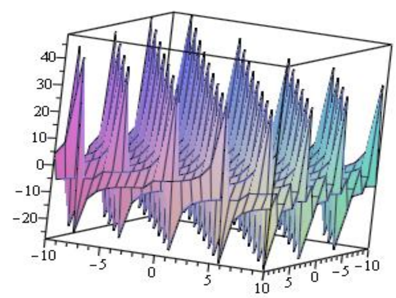

Fig-5: Periodic solution for A= σ =1, B =3, μ =λ =a0 =2

Solitons are special kinds of solitary waves. The soliton solution is a specially localized solution, hence $u'(\xi), u''(\xi), u'''(\xi) \to 0 as \xi \to \pm \propto, \xi = x - ct$. Solitons have a remarkable property that it keeps its identity upon interacting with other solitons. The fig.6 shows the shape of the exact singular soliton solution of Eq. (15) of the (3+1)-dimensional Jimbo-Miwa equation.

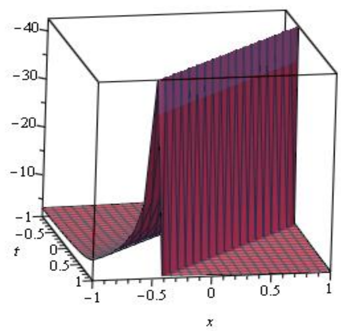

Fig-6: Singular soliton for A= μ= -1, σ = -1, a0= -2, B=2

Kink waves are traveling waves which arise from one asymptotic state to another. The kink solutions are approach to a constant at infinity. Fig.7,

Fig.8 and fig.9 bellow shows the shape of the exact Kink-type solution of Eq.(13), Eq. (17) and Eq. (21) respectively of the (3+1)-dimensional Jimbo-Miwa equation (8). Solutions (19) comes infinity as in hyperbolic function are singular Kink solution. The fig.10 shows singular Kink-type solution of Eq.(19).

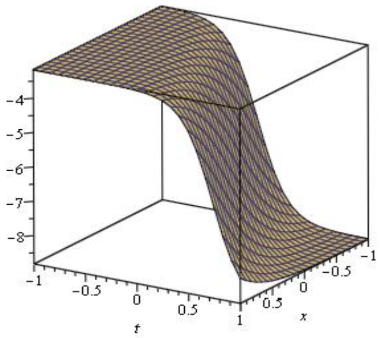

Fig-7: Kink solution for A=2, B=3, a0=2=λ, μ=- 2, σ =1

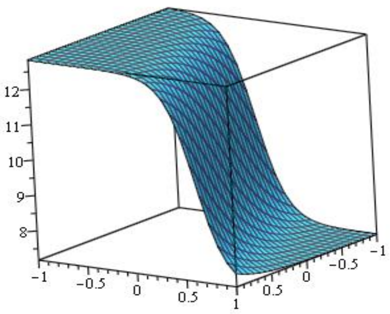

Fig-8: Kink solution for A=λ =a0=2 B=3, μ= -2, σ =1

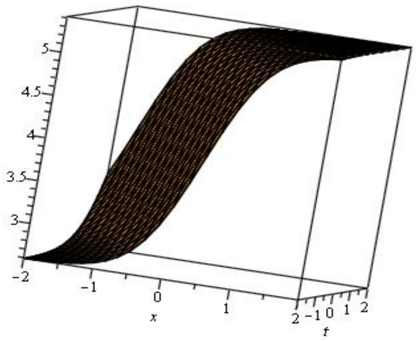

Fig-9: Kink solution for A=μ= -2, σ =λ=1, B=-3

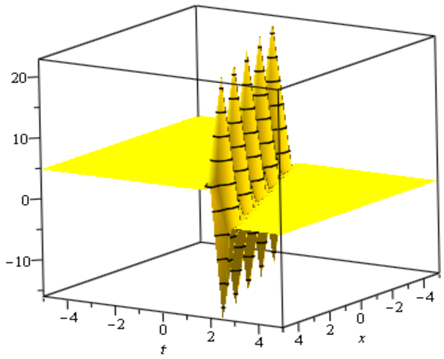

Fig-10: Singular Kink A=a0=2, B=3, σ =1, μ= -1/2

5 Discussions

The advantages and validity of the method over the basic (G'/G)-expansion method have been discussed in the following:

Advantages: The crucial advantage of the new approach against the basic (G'/G)-expansion method is that the method provides more general and large amount of new exact traveling wave solutions with several free parameters. The exact solutions have its great importance to expose the inner mechanism of the physical phenomena. Apart from the physical application, the close-form solutions of nonlinear evolution equations assist the numerical solvers to compare the accuracy of their results and help them in the stability analy.

Validity: In Ref.[16] Song and Ge used a linear ordinary differential equation $G'' + \lambda G' + \mu G = 0$ as an auxiliary equation and traveling wave solutions presented in the form $u(\xi) = a_0 + \sum_{i=1}^{n} a_i (G'/G)^i$ in which $a_n \neq 0, (i = 1, ..., n)$ are constants to be determined. It is noteworthy to point out that some of our solutions are coincided with already published results, if parameters taken particular values which authenticate our solutions. Moreover, in Ref. [16] Song and Ge investigated the well-established the (3+1)-dimensional Jimbo-Miwa equation to obtain exact solutions via the basic (G'/G)-expansion method and achieved only a set solutions (see in Ref [16]). Moreover, in this article five sets solutions of the (3+1)-dimensional Jimbo-Miwa equation are constructed by our proposed the new extended (G'/G)-expansion method.

6 Conclusions

Abundant traveling wave solutions of nonlinear partial differential equations have been found via presented (G'/G)-expansion method. We have used this method to the (3+1)-dimensional Jimbo-Miwa equation. As a result, we obtained plentiful new exact solutions including hyperbolic functions and trigonometric functions which might have significant impact on future researches. The obtained solutions with free parameters may be important to explain some physical phenomena. It is shown that the performance of this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations.

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