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# Intuitionistic Fuzzy Soft Matrix Theory and Multi Criteria in Decision Making Based on T-Norm Operators

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**Abstract** The aim of this paper is to find multi criteria decision making problems to a selected project using intuitionistic fuzzy soft matrix based on generalized operators of t-norm and t-conorm. We use the concept of level operators of intuitionistic fuzzy sets [ K.T.Atanassov, On intuitionistic fuzzy sets theory , Springer – Verlag 2012 ] to define intuitionistic fuzzy soft level matrix. Finally, we give an application of decision making problem by using the operators of t-norm and t-conorm .

**Keywords** Soft Sets, Fuzzy Soft Matrices , Intuitionistic Fuzzy Soft Matrices , T-Norm Operators

## 1. Introduction

Most of our traditional tools for formal modeling, reasoning, and computing are crisp, deterministic, and precise in character. However, in real life, there are many complicated problems in engineering, economics, environment, social sciences , medical sciences etc. that involve data which are not all always crisp, precise and deterministic in character because of various uncertainties . Such uncertainties are being dealing with the help of the theories, like theory of probability, fuzzy sets, intuitionistic fuzzy sets, interval mathematics , rough sets etc. Molodtsov [1] was the first to described the concept of "Soft Set Theory" having parameterization tools for dealing with uncertainties . Maji et al.[2] defined different types of soft sets. Researchers on soft set theory have received much attention in recent years because it is easy to understand and membership is decided by adequate parameters. Maji and Roy [3] first introduced soft set into decision making problems. Maji et al.[6] introduced the concept of fuzzy soft sets by combining soft sets and fuzzy sets. Cagman and Enginoglu [4] defined soft matrices which were a matrix representation of the soft sets and constructed a soft max-min decision making method . Mondal and Pal[12]described the operations of soft matrices. Borah et al.[8] extended fuzzy soft matrix theory and its application. Maji et al.[14] introduced the concept of intuitionistic soft sets . Deli and Cagmam[9] introduced intuitionistic fuzzy parameterized soft sets. They have also applied to the problems that contain uncertainties based on intuitionistic fuzzy parameterized soft sets. Chetia and Das [5] defined five types of products of intuitionistic fuzzy soft matrices . Babitha and John[11] described generalized intuitionistic fuzzy soft sets and solved multi criteria decision making problem in generalized intuitionistic fuzzy soft sets . Rajarajeswari and Dhanalakshmi[10] described intuitionistic fuzzy soft matrix with some traditional operations.

In this paper , we will propose definition of intuitionistic fuzzy soft level matrix . We will also discuss their properties . Finally we will give an application of multi criteria decision making based on t- norm and t-conorm operators.

## 2. Definition and Preliminaries

**Soft set** [1] Let U be an initial universe , P(U) be the power set of U , E be the set of all parameters and A⊆ E. A soft set ($f_A$,E) on the universe U is defined by the set of order pairs

$$(f_A,E) = \{(e , f_A(e)) : e\in E , f_A(e) \in P(U) \}$$

where

$$f_A : E \rightarrow P(U) \text{ such that } f_A(e) = \phi \text{ if } e \notin A.$$

Here $f_A$ is called an approximate function of the soft set ($f_A$,E). The set $f_A$(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter e ∈ E.

**Example 1.** let U = { $u_1$ , $u_2$ , $u_3$ , $u_4$} be a set of four shirts and E = { white( $e_1$), red ($e_2$) ,blue ($e_3$) } be a set of parameters. If A =$\{e_{11}$ , $e_2\}$ ⊆ E. Let $f_A(e_1)$= { $u_1$ , $u_2$ , $u_3$ , $u_4$} and $f_A(e_2)$= { $u_1$ , $u_2$ , $u_3$} , then we write the soft set ($f_A$,E)= {($e_1$,{ $u_1$ , $u_2$ , $u_3$ , $u_4$}), ($e_2$,{ $u_1$ , $u_2$ , $u_3$})}

over U which describe the "colour of the shirts" which Mr. X is going to buy.

**Fuzzy set** [6] Let U be an initial universe , E be the set of parameters and A⊆ E . A pair ( F, A ) is called a fuzzy set over U where F : A → $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U) , where $\tilde{P}$(U) denotes the collection of all subsets of U.

**Example 2.** Consider the example 1, here we can not express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1].Then

$$(f_A,E) = \{\ f_A(e_1) = \{ (\ u_1,.7) ,(u_2,.5) ,(u_3,.4) ,(u_4,.2) \},\ f_A(e_2) = \{ (u_1,.5) , (u_2,.1) , (u_3,.5)\} \}$$

is the fuzzy soft set representing the "colour of the shirts" .

**Fuzzy Soft Matrices (FSM)** [5] Let ($f_A$, E) be fuzzy soft set over U. Then a subset of U x E is uniquely defined by

$$R_A = \{ ( u , e) : e\in A , u \in f_A(e) \},$$

which is called relation form of ($f_A$, E) .
The characteristic function of $R_A$ is written by
$\mu_{R_A}$ : U x E → [ 0 , 1] , where $\mu_{R_A}$(u , e ) ∈ [ 0,1] is the membership value of u∈U for each e ∈U.
If $\mu_{ij} = \mu_{R_A}(u_i,e_j)$ , we can define a matrix

$$[\mu_{ij}]_{mXn} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

which is called an m x n soft matrix of the soft set ($f_A$,E) over U.
Therefore we can say that a fuzzy soft set ($f_A$,E) is uniquely characterized by the matrix $[\mu_{ij}]_{mXn}$ and both concepts are interchangeable.
**Example 3.** Assume that U = { $u_1$ , $u_2$ , $u_3$ , $u_4$, $u_5$, $u_6$} is a universal set and E = { $e_1$ , $e_2$ , $e_3$, $e_4$} is a set of all parameters. If A⊆ E = { $e_1$ , $e_2$ , $e_3$} and

$f_A(e_1)$= { ($u_1$ ,.3), ($u_2$,.4) , ($u_3$,.6) , ($u_4$,.1),( $u_5$,.6), ($u_6$,.5 }

$f_A(e_2)$= { ($u_1$,.2) , ($u_2$,.5) , ($u_3$,.7) , ($u_4$,.3) ,( $u_5$,.7), ($u_6$,.1)}

$f_A(e_3)$= { ($u_1$,.5) , ($u_2$,.2) , ($u_3$,.5) , ($u_4$,.6),( $u_5$,.7), ($u_6$,.3) }

Then the fuzzy soft set ($f_A$,E) is a parameterized family { $f_A(e_1)$ , $f_A(e_2)$ , $f_A(e_3)$} of all fuzzy sets over U.

Hence the fuzzy soft matrix $[\mu_{ij}]$ can be written as

$$[\mu_{ij}] = \begin{bmatrix} .3 & .2 & .5 & 0 \\ .4 & .5 & .2 & 0 \\ .6 & .7 & .5 & 0 \\ .1 & .3 & .6 & 0 \\ .6 & .7 & .7 & 0 \\ .5 & .1 & .3 & 0 \end{bmatrix}$$

# 3. Intuitionistic Fuzzy Soft Matrix Theory

### 3.1. Intuitionistic Fuzzy Soft Set ( IFSS) [14]

Let U be an initial universe , E be the set of parameters and A⊆ E. Then (***f_A***,E) is called an Intuitionistic fuzzy soft set ( IFSS) over U where $\boldsymbol{f_A}$ is a mapping given by $\boldsymbol{f_A}$ : A → $\boldsymbol{I^U}$ , $\boldsymbol{I^U}$ denotes the collection of intuitionistic fuzzy subsets of U .

### 3.2. Intuitionistic Fuzzy Soft Matrix ( IFSM) [5]

Let U be an initial universe , E be the set of parameters and A⊆ E. Let ($f_A$,E) be an Intuitionistic fuzzy soft set ( IFSS) over U. Then a subset of U x E is uniquely defined by

$$R_A = \{ ( u , e) : e\in A , u \in f_A(e) \},$$

which is called relation form of ($f_A$,E) . The membership and non-membership functions of are written by
$\mu_{R_A}$ : U x E → [ 0 , 1] and $\nu_{R_A}$ : U x E → [ 0 , 1] where $\mu_{R_A}$: (u , e ) ∈ [ 0,1] and $\nu_{R_A}$ : (u , e ) ∈ [ 0,1] are the membership value and non membership value of u∈U for each e $\in E$.
If $(\mu_{ij}, \nu_{ij}) = ( \mu_{R_A}(u_i,e_j) , \nu_{R_A}(u_i,e_j) )$, we can define a matrix

$$[\mu_{ij}, \nu_{ij}]_{m\ X\ n} = \begin{bmatrix} (\mu_{11},\nu_{11}) & (\mu_{12},\nu_{12}) & \cdots & (\mu_{1n},\nu_{1n}) \\ (\mu_{21},\nu_{21}) & (\mu_{22},\nu_{22}) & \cdots & (\mu_{2n},\nu_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1},\nu_{m1}) & \mu_{m2},\nu_{m2}) & \cdots & (\mu_{mn},\nu_{mn}) \end{bmatrix},$$

which is called an m x n IFSM of the IFSS($f_A$,E) over U. Therefore, we can say that IFSS( $f_A$ ,E) is uniquely characterized by the matrix $[\mu_{ij}, \nu_{ij}]_{m\ X\ n}$ and both concepts are interchangeable. The set of all m x n IFS matrices will be denoted by $\text{IFSM}_{m\ X\ n}$.
**Example 4.** Let U = { $u_1$ , $u_2$ , $u_3$ , $u_4$ , $u_5$ , $u_6$} is a universal set and E = { $e_1$ , $e_2$ , $e_3$ , $e_4$ } is a set of parameters. If A = { $e_1$ , $e_2$ , $e_3$} ⊆ E and

$f_A(e_1)$= { ($u_1$,.3,.4) , ($u_2$,.5,.4) , ($u_3$,.4 , .5) , ($u_4$,.6,.3) ,( $u_5$,.8,.1) ,($u_6$, .7,.2) }

$f_A(e_2)$= {( $u_1$,.4,.5) ,($u_2$,.6 ,.2),($u_3$,.1,0) , ($u_4$,.6,.2),( $u_5$,.3,.4),( $u_6$,.5,.4) }

$f_A(e_3)$= { ($u_1$,.6 ,.2),( $u_2$,.1,0) ,( $u_3$,.9 ,.1), ($u_4$,.4,.2) ,( $u_5$,.6,.4), ($u_6$,.7,.3) }

Then the IFS set ($f_A$,E) is a parameterized family { $f_A(e_1)$ , $f_A(e_2)$ , $f_A(e_3)$} of all IFS sets over U.
Hence IFSM [($\mu_{ij}$ , $\nu_{ij}$ )] can be written as

$$[(\mu_{ij}, \nu_{ij})] = \begin{bmatrix} (.3,.4) & (.4,.5) & (.6,.2) & (0,0) \\ (.5,.4) & (.6,.4) & (1,0) & (0,0) \\ (.4,.5) & (1,0) & (.9,.1) & (0,0) \\ (.6,.3) & (.6,.2) & (.4,.2) & (0,0) \\ (.8,.1) & (.3,.4) & (.6,.4) & (0,0) \\ (.7,.2) & (.5,.4) & (.7,.3) & (0,0) \end{bmatrix}$$

### 3.3. Intuitionistic Fuzzy Soft Zero Matrix[5]

Let $\tilde{A} = [( \mu_{ij}^{\tilde{A}} , \nu_{ij}^{\tilde{A}} )] \in \text{IFSM}_{m\ X\ n}$ . Then $\tilde{A}$ is called a Zero IFSM denoted by $\tilde{0}$ = [(0,0)], if $\mu_{ij}^{\tilde{A}} = 0$ and $\nu_{ij}^{\tilde{A}} = 0$ for all i and j .

### 3.4. Intuitionistic Fuzzy Soft μ-Universal Matrix

An Intuitionistic fuzzy soft matrix of order m x n is said to be an Intuitionistic Fuzzy Soft μ-Universal Matrix if $\mu_{ij}^{\tilde{A}} = 1$ and $\nu_{ij}^{\tilde{A}} = 0$ for all i and j. It is denoted by $\tilde{I}$ .

### 3.5. Intuitionistic Fuzzy Soft ν -Universal Matrix

An Intuitionistic fuzzy soft matrix of order m x n is said to be an Intuitionistic Fuzzy Soft ν -Universal Matrix if $\mu_{ij}^{\tilde{A}} = 0$ and $\nu_{ij}^{\tilde{A}} = 1$ for all i and j. It is denoted by $\underbrace{I}$ .

### 3.6. Intuitionistic Fuzzy Soft Sub Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then $\tilde{A}$ is said to be intuitionistic fuzzy soft sub matrix of $\tilde{B}$ denoted by $\tilde{A} \subseteq \tilde{B}$ if $\mu_{ij}^{\tilde{A}} \le \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} \ge \nu_{ij}^{\tilde{B}}$ for all i and j .

### 3.7. Intuitionistic Fuzzy Soft Super Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then $\tilde{A}$ is said to be intuitionistic fuzzy soft super matrix of $\tilde{B}$ denoted by $\tilde{A} \supseteq \tilde{B}$ if $\mu_{ij}^{\tilde{A}} \ge \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} \le \nu_{ij}^{\tilde{B}}$ for all i and j .

### 3.8. Intuitionistic Fuzzy Soft Equal Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then $\tilde{A}$ is said to be equal $\tilde{B}$ denoted by $\tilde{A} = \tilde{B}$ if $\mu_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} = \nu_{ij}^{\tilde{B}}$ for all i and j .

### 3.9. Union of Intuitionistic Fuzzy Soft Matrices

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then Union $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A}\ \tilde{\cup}\ \tilde{B}$ is defined as $\mu_{ij}^{\tilde{C}}\ \tilde{A}\ \tilde{\cup}\ \tilde{B}$= [max $\{\mu_{ij}^{\tilde{A}}\ ,\mu_{ij}^{\tilde{B}}\}$ , min $\{\nu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{B}}\}$ ] for all i and j .

### 3.10. Intersection of Intuitionistic Fuzzy Soft Matrices

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then intersection $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A}\ \tilde{\cap}\ \tilde{B}$ is defined as $\tilde{A}\ \tilde{\cap}\ \tilde{B}$ = [min $\{\mu_{ij}^{\tilde{A}}\ ,\mu_{ij}^{\tilde{B}}\}$ , max$\{\nu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{B}}\}$ ] for all i and j .

### 3.11. Complement of Intuitionistic Fuzzy Soft Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$ . Then Complement of $\tilde{A}$ denoted by $\tilde{A}^0$ is defined as $\tilde{A}^0= [(\nu_{ij}^{\tilde{A}},\mu_{ij}^{\tilde{A}})]$ for all i and j .

**Proposition1.** Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then De Morgan's type results are true which can be written as:

$$\text{a) } (\tilde{A}\ \tilde{\cup}\ \tilde{B})^0 = \tilde{A}^0\ \tilde{\cap}\ \tilde{B}^0$$

$$\text{b) } (\tilde{A}\ \tilde{\cap}\ \tilde{B})^0 = \tilde{A}^0\ \tilde{\cup}\ \tilde{B}^0$$

Proof: a) For all i and j we have,

$$(\tilde{A}\ \tilde{\cup}\ \tilde{B})^0 = ([(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]\ \tilde{\cup}\ [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})])^0$$
$$=[\max\{\mu_{ij}^{\tilde{A}}\ ,\mu_{ij}^{\tilde{B}}\}, \min\{\nu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{B}}\}]^0$$
$$= [\min\{\nu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{B}}\}\ ,\ \max\{\mu_{ij}^{\tilde{A}}\ ,\mu_{ij}^{\tilde{B}}\}]$$
$$= [(\nu_{ij}^{\tilde{A}},\mu_{ij}^{\tilde{A}})]\ \tilde{\cap}\ [(\nu_{ij}^{\tilde{B}},\mu_{ij}^{\tilde{B}})]$$
$$= \tilde{A}^0\ \tilde{\cap}\ \tilde{B}^0\ \square$$

The result b) can be proved in similar way.

**Proposition2.** Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then

a) $((\tilde{A}^0))^0 = \tilde{A}$ f) $\tilde{A}\ \tilde{\cap}\ \underbrace{I} = \tilde{A}$
b) $(\tilde{I})^0 = \underbrace{I}$ g) $\tilde{A}\ \tilde{\cap}\ \tilde{A} = \tilde{A}$
c) $(\underbrace{I})^0 = \tilde{I}$ h) $\tilde{A}\ \tilde{\cap}\ \tilde{I} = \tilde{A}$
d) $\tilde{A}\ \tilde{\cup}\ \tilde{A} = \tilde{A}$ i) $\tilde{A}\ \tilde{\cap}\ \underbrace{I} = \underbrace{I}$
e) $\tilde{A}\ \tilde{\cup}\ \tilde{I} = \tilde{I}$

### 3.12. Intuitionisti6.8c Fuzzy Soft Square Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then $\tilde{A}$ is said to be Intuitionistic Fuzzy Soft Square Matrix if m=n for all i and j .

### 3.13. Intuitionistic Fuzzy Soft Row Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then $\tilde{A}$ is said to be Intuitionistic Fuzzy Soft Row Matrix if n=1 for all i and j.

### 3.14. Intuitionistic Fuzzy Soft Column Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then $\tilde{A}$ is said to be Intuitionistic Fuzzy Soft Column matrix if m=1 for all i and j.

### 3.15. Intuitionistic Fuzzy Soft Diagonal Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then $\tilde{A}$ is said to be intuitionistic Fuzzy Soft Diagonal Matrix if m=n and i= j.

### 3.16. Max-Min Product of Intuitionistic Fuzzy Soft Matrices

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{n\,X\,p}$. Then the Max-Min Product of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A} * \tilde{B}$ is defined as $\tilde{A} * \tilde{B}$ = [ max-min($\mu_{ij}^{\tilde{A}}$ , $\mu_{ij}^{\tilde{B}}$ ) , min-max($\nu_{ij}^{\tilde{A}}$ , $\nu_{ij}^{\tilde{B}}$ )] for all i and j .

### 3.17. Scalar Multiplication of Intuitionistic Fuzzy Soft Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$ and k be a scalar .Then the Scalar Multiplication of Intuitionistic Fuzzy Soft Matrix $\tilde{A}$ by the scalar k denoted by k $\tilde{A}$ defined as k $\tilde{A}$ = [( k $\mu_{ij}^{\tilde{A}}$, k $\nu_{ij}^{\tilde{A}}$ )] ,where $0 \le k \le 1$, for all i and j .

### 3.18. Intuitionistic Fuzzy Soft Transpose Matrix

Let $\tilde{A}=[(\mu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{m\,X\,n}$. Then $\tilde{A}$ is said to be Intuitionistic Fuzzy Soft Transpose Matrix denoted by $\tilde{A}^T$ and is defined as $\tilde{A}^T$ =$[(\mu_{ji}^{\tilde{A}},\nu_{ji}^{\tilde{A}})] \in \mathrm{IFSM}_{n\,X\,m}$.

**Example 5.** Let $\tilde{A} = \begin{bmatrix} (.8,.2) & (.3,.1) \\ (.6,.3) & (.4,.3) \end{bmatrix} \in \mathrm{IFSM}_{2\,X\,2}$. Then

$\tilde{A}^T = \begin{bmatrix} (.8,.2) & (.6,.3) \\ (.3,.1) & (.4,.3) \end{bmatrix}$

**Proposition3.** Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\ X\ n}$. Then

a) $\tilde{A}\ \tilde{\cup}\ \tilde{A}^T = \tilde{A}^T\ \tilde{\cup}\ \tilde{A}$

b) $\tilde{A}\ \tilde{\cap}\ \tilde{A}^T = \tilde{A}^T\ \tilde{\cap}\ \tilde{A}$

### 3.19. Intuitionistic Fuzzy Soft Level Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\ X\ n}$ and $\alpha$ , $\beta \in [0,1]$ are two fixed number such that $\alpha + \beta \leq 1$. Then Intuitionistic Fuzzy Soft Level Matrix denoted by $N_{\alpha,\beta}(\tilde{A})$ is defined as $N_{\alpha,\beta}(\tilde{A}) = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ ,where $\mu_{ij}^{\tilde{A}} \geq \alpha$ and $\nu_{ij}^{\tilde{A}} \leq \beta$ , for all i and j .

### 3.20. Intuitionistic Fuzzy Soft Membership Level Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\ X\ n}$ and $\alpha \in [0,1]$ is a fixed number. Then Intuitionistic Fuzzy Soft Membership Level Matrix denoted by $N_{\alpha}(\tilde{A})$ is defined as $N_{\alpha}(\tilde{A}) = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ ,where $\mu_{ij}^{\tilde{A}} \geq \alpha$ for all i and j .

### 3.21. Intuitionistic Fuzzy Soft Non-Membership Level Matrix

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\ X\ n}$ and $\alpha \in [0,1]$ is a fixed number. Then Intuitionistic Fuzzy Soft Non-Membership Level Matrix denoted by $N^{\alpha}(\tilde{A})$ is defined as $N^{\alpha}(\tilde{A}) = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ ,where $\nu_{ij}^{\tilde{A}} \leq \alpha$ for all i and j .

**Proposition4.** Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\ X\ n}$. Then

a) $N_{\alpha,\beta}(\tilde{A}) \subset \begin{cases} N^{\beta}(\tilde{A}) \\ N_{\alpha}(\tilde{A}) \end{cases} \subset \tilde{A}$

b) $N_{\alpha,\beta}(\tilde{A}) = N_{\alpha}(\tilde{A}) \cap N^{\beta}(\tilde{A})$

**Example 6.** Let $\tilde{A} = \begin{bmatrix} (.8,.2) & (.3,.4) \\ (.6,.3) & (.2,.3) \end{bmatrix} \in \text{IFSM}_{2\ X\ 2}$. Then

$N_{.5,.3}(\tilde{A}) = \begin{bmatrix} (.8,.3) & (.5,.3) \\ (.6,.3) & (.5,.3) \end{bmatrix}$ , $N_{.5}(\tilde{A}) = \begin{bmatrix} (.8,.2) & (.5,.4) \\ (.6,.3) & (.5,.3) \end{bmatrix}$

$N^{.3}(\tilde{A}) = \begin{bmatrix} (.8,.2) & (.3,.3) \\ (.6,.3) & (.2,.3) \end{bmatrix}$

### 3.22. t-norm [13]

Let T : [0,1] x [0,1] → [0,1] be a function satisfying the following axioms:

i. T( a , 1 ) = a , ∀ a ∈ [0,1] ( Identity )
ii. T(a, b) = T(b ,a) , ∀ a ,b ∈ [0,1] ( Commutativity)
iii. if $b_1 \leq b_2$ , then T( a, $b_1$) ≤ T( a, $b_2$) , ∀ a , $b_1$ , $b_2$ ∈ [0,1] ( Monotonicity)
iv. T ( a, T( b,c) ) = T(T (a,b) , c) , ∀ a ,b ∈ [0,1] (Associativity)

Then T is called a t-norm.

A t-norm is said to be continuous if T is continuous function in [0,1].

An example of continuous t- Norm = a b .

N.B. :The functions used for intersection of fuzzy sets are called t-norms.

### 3.23. t-conorm[13]

Let S : [0,1] x [0,1] → [0,1] be a function satisfying the following axioms:

i. S( a , 0 ) = a , ∀ a ∈ [0,1] ( Identity)
ii. S(a,b) = S(b,a) , ∀ a ,b ∈ [0,1] ( Commutativity )
iii. if $b_1 \leq b_2$ , then S( a, $b_1$) ≤ S( a, $b_2$) , ∀ a , $b_1$ , $b_2$ ∈ [0,1] ( Monotonicity )
iv. S ( a, S( b,c) ) = S(S (a,b) , c) ∀ a ,b ∈ [0,1] (Associativity )

Then S is called t-conorm.

A t-conorm is said to be continuous if S is continuous function in [0,1].

N.B. :The functions used for union of fuzzy sets are called t-conorms.

An example of continuous t- Conorm = a + b − a .b .

### 3.24. Operators of Intuitionistic Fuzzy Soft Matrices

Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}\ ,\ \nu_{ij}^{\tilde{A}})]$ , $\tilde{B} = [(\mu_{ij}^{\tilde{B}}\ ,\ \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m\ X\ n}$. Then IFSM $\tilde{C} = [(\mu_{ij}^{\tilde{C}}\ ,\ \nu_{ij}^{\tilde{C}})]$ is called

a) the "."(product) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A}\ .\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \mu_{ij}^{\tilde{A}} .\ \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{C}} = \nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\ \nu_{ij}^{\tilde{B}}$ for all i and j.

b) the "+"(Probabilistic sum) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A} + \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}} .\ \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{C}} = \nu_{ij}^{\tilde{A}}\ .\ \nu_{ij}^{\tilde{B}}$ for all i and j.

c) the "@"(Arithmetic Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A}\ @\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \frac{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}{2}$ and $\nu_{ij}^{\tilde{C}} = \frac{\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}}}{2}$ for all i and j.

d) the "$@^{w}$" ( Weighted Arithmetic Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A}\ @^{w}\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \frac{w_1 \mu_{ij}^{\tilde{A}} + w_2 \mu_{ij}^{\tilde{B}}}{w_1 + w_2}$ , $\nu_{ij}^{\tilde{C}} = \frac{w_1 \nu_{ij}^{\tilde{A}} + w_2\ \nu_{ij}^{\tilde{B}}}{w_1 + w_2}$ , for all i and j. $w_1 > 0$ , $w_2 > 0$

e) the "\$" (Geometric Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A}\ \$\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \sqrt{\mu_{ij}^{\tilde{A}} .\ \mu_{ij}^{\tilde{B}}}$ and $\nu_{ij}^{\tilde{C}} = \sqrt{\nu_{ij}^{\tilde{A}}\ .\ \nu_{ij}^{\tilde{B}}}$ for all i and j.

f) the "$\$^{w}$" (Weighted Geometric Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A}\ \$^{w}\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = ((\mu_{ij}^{\tilde{A}})^{w_1} . (\mu_{ij}^{\tilde{B}})^{w_2})^{\frac{1}{w_1 + w_2}}$ and $\nu_{ij}^{\tilde{C}} = ((\nu_{ij}^{\tilde{A}})^{w_1} . (\mu_{ij}^{\tilde{B}})^{w_2})^{\frac{1}{w_1 + w_2}}$ for all i and j. $w_1 > 0$ , $w_2 > 0$

g) the "⋈"(Harmonic Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A} \bowtie \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = 2.\frac{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}$ and

$\nu_{ij}^{\tilde{C}} = 2.\frac{\nu_{ij}^{\tilde{A}}.\nu_{ij}^{\tilde{B}}}{\nu_{ij}^{\tilde{A}}+\nu_{ij}^{\tilde{B}}}$ for all i and j. $w_1 > 0$ , $w_2 > 0$

h) the "$\bowtie^{w}$" ( Weighted Harmonic Mean) operation of $\tilde{A}$ and $\tilde{B}$ ,denoted by $\tilde{C} = \tilde{A} \bowtie^{w} \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \left( \frac{w_1+w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}}+\frac{w_2}{\mu_{ij}^{\tilde{B}}}} \right.$ , $\nu_{ij}^{\tilde{C}} = \frac{w_1+w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}}+\frac{w_2}{\nu_{ij}^{\tilde{B}}}}$ for all i and j.

# 4. Operators of T-Norm and T-Conorm

For any n membership functions $\mu^1$ , $\mu^2$ , … ,$\mu^n$ ( or n non-membership functions $\nu^1$, $\nu^2$, … ,$\nu^n$ ) four basic t-norms and t- conorms are :

$$T_M( a^1 , a^2 ,\ldots\ldots,a^n ) = \min \{ a^1 , a^2 ,\ldots,a^n \}$$

$$T_P( a^1 , a^2 ,\ldots,a^n) = \prod_{i=1}^{n} a^i$$

$$T_L(a^1 , a^2 ,\ldots,a^n) = \max \{( \textstyle\sum_{i=1}^{n} a^i - n + 1) , 0 \}$$

$$T_D\{ a^1 , a^2 ,\ldots,a^n )= \begin{cases} 0 \; if \; S_M\{ a^1 , a^2 ,\ldots, a^n \} < 1 \\ T_M\{ a^1 , a^2 ,\ldots, a^n\} \text{ if } S_M\{ a^1 , a^2 ,\ldots, a^n \} = 1 \end{cases}$$

$$S_M (a^1 , a^2 ,\ldots,a^n) = \max \{ a^1 , a^2 ,\ldots,a^n \}$$

$$S_P(a^1 , a^2 ,\ldots,a^n) = 1 - \prod_{i=1}^{n}(1 - a^i)$$

$$S_L(a^1 , a^2 ,\ldots,a^n) = \min ( \textstyle\sum_{i=1}^{n} a^i , 1 )$$

$$S_D(a^1 , a^2 , \ldots,a^n)= \begin{cases} 1 \; if \; T_M\{a^1 , a^2 ,\ldots, a^n\} > 0 \\ S_M\{ a^1 , a^2 ,\ldots, a^n \} \text{ if } T_M\{ a^1 , a^2,\ldots, a^n\} = 0 \end{cases}$$

where $a^1$ , $a^2$ , … ,$a^n$ = $\mu^1$ , $\mu^2$ , … ,$\mu^n$ ( or $\nu^1$, $\nu^2$,… ,$\nu^n$ )

# 5. Generalized Operators of Intuitionistic Fuzzy T-Norm and T-Conorm

Let $A_r$= [( $\mu_{ij}^r$ , $\nu_{ij}^r$)] ∈ IFSM$_{m \, X \, n}$ for r= 1,2,…n . Then

a) A ($T_M$ , $S_M$) = [ ( min ($\mu_{ij}^r$ ) , max ( $\nu_{ij}^r$ ) ) ]
b) A ($T_P$ , $S_P$) = [ ( $\prod_{r=1}^{n} \mu_{ij}^r$ , 1 − $\prod_{r=1}^{n}(1 - \nu_{ij}^r)$ ) ]
c) A ($T_L$ , $S_L$) = [ (max {( $\sum_{r=1}^{n} \mu_{ij}^r$ − n + 1) , 0 } , min ( $\sum_{r=1}^{n} \nu_{ij}^r$ , 1 ) ) ]
d) A ($T_D$ , $S_D$) =

$$[ ( \begin{cases} 0 \; if \; S_M\{ \mu_{ij}^1,\ldots,\mu_{ij}^n \} < 1 \\ T_M\{ \mu_{ij}^1,\ldots,\mu_{ij}^n\} \text{ if } S_M\{ \mu_{ij}^1,\ldots,\mu_{ij}^n \} = 1 \end{cases}, \begin{cases} 1 \; if \; T_M\{\nu_{ij}^1,\ldots,\nu_{ij}^n\} > 0 \\ S_M\{\nu_{ij}^1,\ldots,\nu_{ij}^n\} \text{ if } T_M\{\nu_{ij}^1,\ldots,\nu_{ij}^n \} = 0 \end{cases} ) ]$$

### 5.1. Arithmetic Mean

Let $A_r$= [( $\mu_{ij}^r$ , $\nu_{ij}^r$)] ∈ IFSM$_{m \, X \, n}$ for r= 1,2,...,n. Then Arithmetic Mean of $A_r$ denoted by $A_{AM}$ and is defined as

$$A_{AM} = [ ( \tfrac{1}{n} (\textstyle\sum_{r=1}^{n} \mu_{ij}^r ) , \tfrac{1}{n} (\sum_{r=1}^{n} \nu_{ij}^r ) ) ]$$

# 6. Intuitionistic Fuzzy Soft Matrix in Decision Making Based on T- Norm and T-Conorm Operators

We have defined different types of Intuitionistic Fuzzy t-norm and t-conorm operators. On the basis of these operators, we will find out average of membership and non-membership value for decision making problems.

Input : Intuitionistic fuzzy soft sets with m objects , each of which has n parameters.
Output : An optimum result.
Algorithm:
Step 1: Choose the set of parameters.
Step 2: Construct the intuitionistic fuzzy soft matrices for each set of parameters.
Step 3: Compute A ($T_M$ , $S_M$) of intuitionistic fuzzy soft matrices as mentioned in ch.5 .
Step 4: Compute the arithmetic mean of membership and non-membership value of intuitionistic fuzzy soft matrices $A_{AM}(T_M , S_M)$ as mentioned in ch.5.1.
Step 5: Find the highest membership value.
Step 6: Find the lowest non-membership value.
Step 7: Find the decision. Decision is obtained by choosing highest membership value. In case of tie, decision is obtained by choosing highest membership value and lowest non-membership value simultaneously.

**Example 7.** Suppose a company advertises to fill the post of Chief Executive Officer in his office for which candidates apply. The expert committee requires the candidates to possess the certain qualities. The candidates must be confident, have computer knowledge and willing to take risk. The candidate with highest membership value will be considered as the best candidate for the job.

Let U = { $c_1$, $c_2$, $c_3$, $c_4$, $c_5$ } be the five candidates appearing in an interview for the appointment of Chief Executive Officer and E = { $e_1$(confident ) , $e_2$( computer knowledge) , $e_3$( willing to take risk )} be the set of parameters.

Suppose , three expert of the committee Mr. $A_1$ , Mr. $A_2$ and Mr. $A_3$ are taking the interview of the five candidates and give marks to the candidates on the basis of the interview. At the end of the interview, intuitionistic fuzzy soft matrices are constructed on the basis of the marks given by the expert as follows :

$$A_1 = \begin{bmatrix} (.5,.3) & (.7,.1) & (.6,.3) \\ (.8,.1) & (.6,.3) & (.7,.2) \\ (.6,.2) & (.7,.3) & (.5,.4) \\ (.7,.2) & (.5,.4) & (.4,.4) \\ (.7,.2) & (.5,.4) & (.8,.1) \end{bmatrix},\ A_2 = \begin{bmatrix} (.7,.3) & (.7,.3) & (.6,.2) \\ (.6,.3) & (.5,.4) & (.6,.3) \\ (.8,.1) & (.5,.4) & (.7,.2) \\ (.5,.3) & (.6,.3) & (.5,.4) \\ (.7,.2) & (.7,.2) & (.6,.2) \end{bmatrix},\ A_3 = \begin{bmatrix} (.7,.2) & (.8,.1) & (.6,.3) \\ (.8,.1) & (.6,.2) & (.5,.3) \\ (.8,.1) & (.8,.2) & (.8,.2) \\ (.5,.2) & (.8,.1) & (.7,.2) \\ (.6,.2) & (.7,.1) & (.8,.1) \end{bmatrix}$$

$$A\,(T_M,\ S_M) = \begin{bmatrix} (.5,.3) & (.7,.3) & (.6,.3) \\ (.6,.3) & (.5,.4) & (.5,.3) \\ (.6,.2) & (.5,.4) & (.5,.4) \\ (.5,.3) & (.5,.4) & (.5,.4) \\ (.6,.2) & (.5,.4) & (.6,.2) \end{bmatrix},\ A_{AM}\ (T_M,\ S_M) = \begin{bmatrix} (.6,.3) \\ (.533,.033) \\ (.533,.033) \\ (.5,.367) \\ (.567,.267) \end{bmatrix} \quad \ldots(6.1)$$

It is obvious from the result (6.1) that $c_1$ candidate is suitable for the post of CEO of the company.

**Note 1:** If A $(T_P,\ S_P)$ is used instead of A $(T_M,\ S_M)$ , then we have

$$A\,(T_P,\ S_P) = \begin{bmatrix} (.245,.559) & (.392,.443) & (.216,.608) \\ (.384,.433) & (.18,.664) & (.21,.608) \\ (.384,.352) & (.28,.664) & (.28,.616) \\ (.175,.552) & (.24,.622) & (.14,.712) \\ (.294,.488) & (.245,.568) & (.384,.352) \end{bmatrix},\ A_{AM}\ (T_P,\ S_P) = \begin{bmatrix} (.284,.592) \\ (.258,.488) \\ (.315,.544) \\ (.172,.629) \\ (.307,.469) \end{bmatrix}$$

and so decision will go in favour of $c_3$ .

**Note 2:** If A $(T_L,\ S_L)$ is used , then we have

$$A\,(T_L,\ S_L) = \begin{bmatrix} (.0,.8) & (.2,.5) & (.0,.8) \\ (.2,.5) & (.0,.9) & (.0,.8) \\ (.2,.4) & (.0,.9) & (.0,.8) \\ (.0,.7) & (.0,.8) & (.0,1) \\ (.0,.6) & (.0,.7) & (.2,.4) \end{bmatrix},\ A_{AM}\ (T_L,\ S_L) = \begin{bmatrix} (.067,.7) \\ (.067,.733) \\ (.067,.7) \\ (.0,.833) \\ (.067,.567) \end{bmatrix}$$

From the above result , it is clear that $c_1$ , $c_2$ , $c_3$ and $c_5$ candidates secure highest membership value , but $c_5$ is preferable for the post of CEO of the company due to lowest non-membership value.

**Note 3:** If A $(T_D,\ S_D)$ is used , then each entry of the matrix A $(T_D,\ S_D)$ becomes (0,0). So we can not make any conclusion.

# 7. Conclusion

In this paper, we give some basic properties of intuitionistic fuzzy soft matrix and use the concept of level operators of intuitionistic fuzzy set to define intuitionistic soft level matrix and its properties with examples. We use generalized operators of intuitionistic fuzzy t-norm and t-conorm for decision making problem. Finally, we give a numerical example for decision making on the basis of the operators which shows that decision varies for different operators .This method can also be applied on other decision making problems with uncertain parameters.

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