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Solution of Problem of Sound Scattering on Bodies of Non-analytical Form with Help of Method of Green's Functions

S. L. Ilmenkov, A. A. Kleshchev*

Saint-Petersburg State Navy Technical University, Saint-Petrsburg, Lotsmanskaya 3, St. Petersburg, 190008, Russia
*Corresponding Author: alexalex-2@yandex.ru

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Abstract The real scatterers have non – analytical form and therefore, the method of separation of variables for calculation of the reflected sound field does not apply to them. In the article is presented the method of Green's functions for the solution of the problem of the sound diffraction on the ideal non – analytical scatterers. In detail is giving the analysis of the solutions and are calculating the modules of the angular characteristics of the sound scattering.

Keywords Diffraction, Green's Functions, Non – Analytical Form, Boundary Conditions

1. Introduction

There are quite a number of task solving methods for reflecting and scattering of sound bodies with non – analytical surface. Let's call the most well-known and frequently used in studies of these methods: methods of finite and boundary elements, method Kupradze, method of the T - matrix, the method of geometrical theory of diffraction, method of integral equations, method of Green's functions and so on [1 – 24]. In this paper the method of Green's functions to applies [22 – 24].

2. The Ideal Scatterer of the Non-analytical Form

We consider non-analytical body, the surface of which does not apply to coordinate systems ones with divided variables in the scalar Helmholtz equation. We examine this non-analytical scatterer in the form of a finite circular cylinder, bounded on the sides of the hemispheres (Fig.1).

Sound pressure, scattered by this body, can be found one of the numerical methods for the solution of diffraction problems [1 - 24]. The method of Green's f functions [22 - 24], based on the use of mathematical formulation of the principle of Helmholtz-Huygens (Kirchhoff integral), one of the most convenient methods [25 – 27]. The algorithm of calculation requires knowledge of the amplitude-phase distribution of the sound pressure and the normal component of oscillatory velocity on some closed surface integration of *S,* that includes the lateral surface of the cylinder S_2 and the surface of hemispheres S_1 and S_3 (Fig. 1):

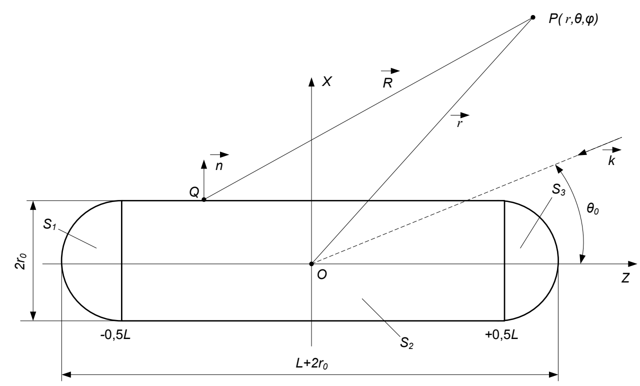


Figure 1. Non-analytical scatterer in the form of a cylinder with hemispheres.

$$p_s(P) = \frac{-1}{4\pi}\int_S [p_s(Q)\frac{\partial}{\partial n}G(P,Q) - \frac{\partial p_s(Q)}{\partial n}G(P,Q)]dS \quad (1)$$

where $p_s(P)$ – the sound pressure scattered by the body, P –the point of observation, which has a spherical coordinates: r, θ, φ; Q – the point of the surface S; $p_s(Q)$ - the sound pressure in the point Q ; $G(P,Q)$ - Green's function of the free space, satisfying the inhomogeneous Helmholtz equation.

In the (1) Green's function is selected as a potential point source:

$$G(P,Q) = \frac{e^{ikR}}{R}, \quad (2)$$

where $k = 2\pi/\lambda$ – the wave number, λ – the length of a sound wave in the liquid environment, R - the distance between the points *P* and *Q*.

Using relative arbitrariness of the choice of green's function, you can get the Kirchhoff formula options, consisting of a single member:

$$p_S(P) = \frac{-1}{4\pi}\int_S p_S(Q)\frac{\partial}{\partial n}G^{(1)}(P,Q)dS \tag{3}$$

$$p_S(P) = \frac{1}{4\pi}\int_S \frac{\partial p_S(Q)}{\partial n}G^{(2)}(P,Q)dS \tag{4}$$

In the system of circular cylindrical and spherical coordinates connected with the corresponding parts of the surface S (Fig.1), the expressions for the Green's functions $G^{(1)}(P,Q)$ and $G^{(2)}(P,Q)$ can be written as [28]:

$$G_{ц}^{(2)}, \frac{\partial G_{ц}^{(1)}}{\partial r_{ц}'}(r,z,\varphi,r',z',\varphi') = \frac{e^{ik(R-z'\cos\theta)}}{R}\sum_{n=-\infty}^{n=+\infty} e^{in(\varphi-\varphi'-\pi/2)}\Big[J_n(kr'\sin\theta) - \Omega Jnkr0 sin\theta \Omega Hn1kr0 sin\theta Hn1kr' sin\theta, \tag{5}$$

$$G_{сф}^{(2)}, \frac{\partial G_{сф}^{(1)}}{\partial r_{сф}'}(\theta,R,\varphi,\theta',r',\varphi') = ik\sum_{n=0}^{\infty}(2n+1 m=-nnn-m!n+m! ein\varphi-\varphi' Pnmcos\theta Pnmcos\theta\, jnkr' - \Omega jnkr0\Omega hn1kr0 hn1kr' hn1kR, \tag{6}$$

$$\Omega = \begin{cases} 1, \text{для } G_{ц,сф}. \\ \frac{\partial}{\partial r_{ц,сф}'} \Big| \, r' = r_0, \text{ для } \frac{\partial G_{ц,сф}}{\partial r_{ц,сф}'} \end{cases} \tag{7}$$

By using formulas (3), (4) is considerably simplified computational procedure: you want to define only one of the parameters ($p_s(Q)$ or $\partial p_s(Q)/\partial n$) on the surface *S*. However, in this case, the match of the surface *S* with a coordinate the surface one of coordinate systems in which it is possible separation of variables is necessary. Thus, application of Green's functions for analytical surfaces (infinite cylinder and sphere) faces of these surfaces, interconnected is the main feature of this method.

The possibility of such a method and test calculations of the scattered field were considered in [29, 30]. For example, an experiment at the decision of a test problem [29] for the calculation of the far field of a point source (group of point sources) directly and through (3), (4) has shown that in the considered range of wave sizes of the results obtained by these two methods, good enough coincide. When solving the problem of diffraction to determine the values of $p_s(Q)$ and $\partial p_s(Q)/\partial n$ on the surface S you can use the following expression:

1. for the homogeneous Dirichlet conditions (ideally soft body), pressure scattered waves on the surface S has the form :

$$p_s(Q) = -\,p_i(Q)\,, \tag{8}$$

2. for the homogeneous Neumann conditions (ideally rigid body):

$$\frac{\partial\, p_s(Q)}{\partial n} = -\frac{\partial\, p_i(Q)}{\partial n}\,, \tag{9}$$

where $p_i(Q)$ – the sound pressure of the incident wave in point Q. When determining the values $p_i(Q)$ you can use the expression for the scalar potential of the plane monochromatic wave single amplitude of the incident on the body from a source located at infinity.

This potential for a perfectly reflective sphere is natural functions in solving the Helmholtz equation in a spherical coordinate system has the following form [13]:

$$p_i(r,\theta,\varphi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\varepsilon_n\, i^{-n}(2n+1)\frac{(n-m)!}{(n+m)!}cosm\varphi P_n^m(cos\theta)j_n(kr); \tag{10}$$

$$\varepsilon_n = 1(n=0);\ \varepsilon_n = 2(n \neq 0);$$

The expression (10) is simplified when considering the axis-symmetric problem (dependence on the coordinate φ missing):

$$p_i(r,\theta) = \sum_{m=0}^{\infty} i^{-m}(2m+1)P_m(cos\theta)j_m(kr); \tag{11}$$

For scatterer in the form of a perfectly reflecting cylinder scalar potential incident plane harmonic waves unit amplitude of the wave vector $\vec{k}$, aimed at the angle θ_0 to the *z* axis of the cylinder, folding natural functions solutions to the Helmholtz equation in a circular cylindrical coordinate system:

$$p_i(r,\varphi,z) = -e^{ikcos\,\theta_0 z_0}\sum_{m=0}^{\infty}\varepsilon_m(-i)^m H_m^{(1)}(kr)cosm\varphi\frac{\Omega J_m(kr_0\sin\theta_0)}{\Omega H_n^{(1)}(kr_0 sin\,\theta_0)}; \tag{12}$$

In the case of the plane problem of the wave vector $\vec{k}$ perpendicular to the *z* axis of the cylinder and expression (12) is simplified [13]:

$$p_i(r,\varphi) = -\sum_{m=0}^{\infty}\varepsilon_m(-i)^m H_m^{(1)}(kr)cosm\varphi\frac{\Omega J_m(kr_0)}{\Omega H_n^{(1)}(kr_0)}; \tag{13}$$

3. The Results of Numerical Experiment for Determination of the Angular Characteristics of the Sound Scattering

For calculation of integrals (3.3), (3.4) on the surface *S* the quadrature formulas is used. Step integration over the surface S in the axial and circumferential directions $(dz_0, d\varphi_0, d\theta_0)$ in the system of nodal points must not exceed 0,5 λ (Fig. 2, 3).

In numerical integration over the surface S_2 element surface of a cylinder with a radius r_0 will be equal $dS = r_0 d\varphi_0 dz_0$ (see Fig. 2), item surface hemispheres S_1 and S_3 in spherical coordinates is equal $dS = r_0{}^2 sin\theta_0 d\theta_0 d\varphi_0$ (see Fig. 3).

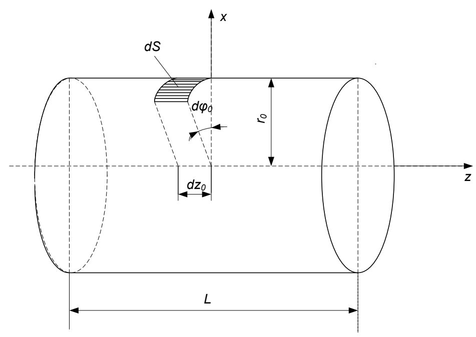


Figure 2. The coordinate system associated with the cylinder.

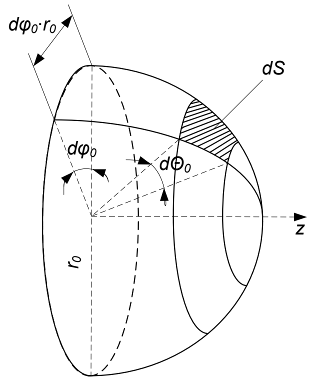
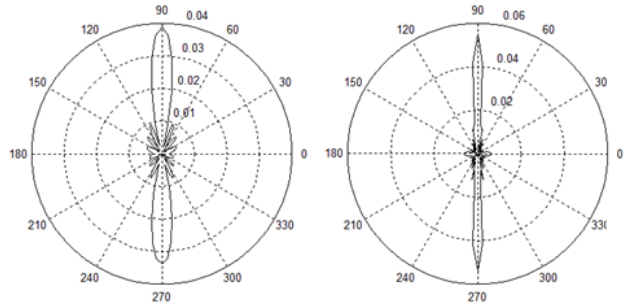


Figure 3. The coordinate system associated with the hemispheres.

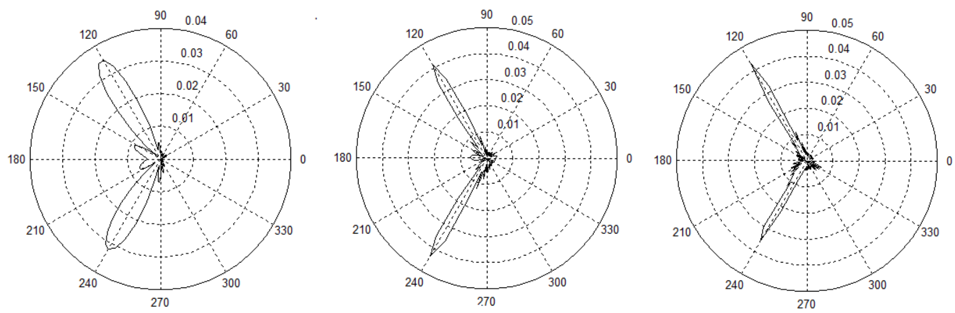
In the figures 4 - 7 for the non – analytical scatterer with the ratio of body length L with the radius r_0 equal to 25.8 at different angles of incidence of a plane wave and different wave sizes of the scatterer modules angular characteristics of scattering shows.



a) b)

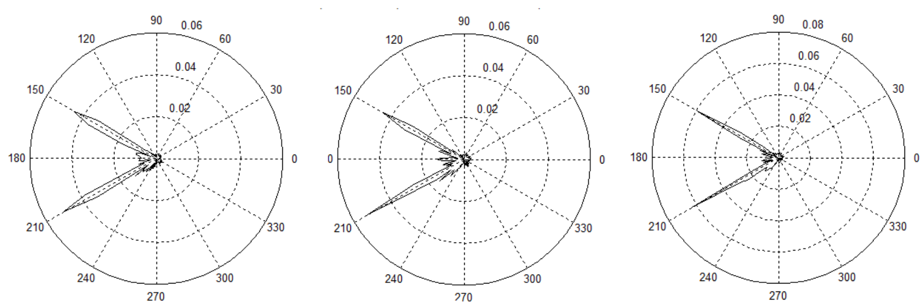
Figure 4. Modules $|D(\theta)|$ when $\theta_0 = 90^0 : a) kr_0 = 1,05; b) kr_0 = 2,09.$

At all figures clearly observed diffraction (shadow) petal, and it grows and shrinks with increasing frequency. On the Fig. 4 - 6 the mirror petal is shows, which is similar to the shadow petal with increasing frequency, but in contrast, limited asymptotically. You may notice that the angular diagrams of the non – analytical scatterer are very similar to the angular characteristics of the scattering elongated spheroids (ideal and elastic) with the ratio of the semi-axes 1:10 [13, 24, 30, 31]. In contrast to works [12 - 14], which used a method of integral equations and were calculated for non – analytical body with short cylindrical insert, in this study cylindrical insert was much longer.



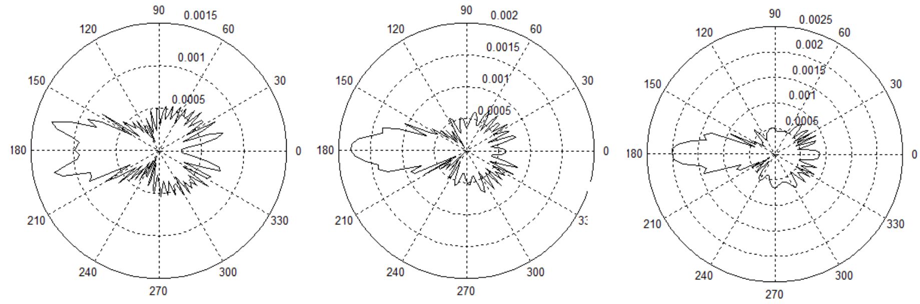
a) b) c)

Figure 5. Modules $|D(\theta)|$ when $\theta_0 = 60^0 : a) kr_0 = 1,05; b) kr_0 = 2,09; c) kr_0 = 3,14.$



a) b) c)

Figure 6. Modules $|D(\theta)|$ when $\theta_0 = 30^0 : a) kr_0 = 4,19; b) kr_0 = 5,24; c) kr_0 = 6,28.$



a) b) c)

Figure 7. Modules $|D(\theta)|$ when $\theta_0 = 0^0 : a)kr_0 = 4,19; b)kr_0 = 5,24; c)kr_0 = 6,28.$

4. Conclusions

Based on the method of Green's functions for the solution of problems of sound diffraction on bodies with mixed boundary conditions, in this work the problem of sound scattering on an elongated (with a ratio of length of a body to its radius) ideal non – analytical forms body is solved. In addition to the analytical solution is made by calculation of the angular scattering characteristics of a wide frequency range (wave sizes) and angles of a plane wave on a non – analytical body.

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