Advances in Signal Processing 2(2): 46-49, 2014
DOI: 10.13189/asp.2014.020202 http://www.hrpub.org

Diffraction of Sound Signals at Elastic Shell of Non-analytical Form Put in Plane Waveguide

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Abstract With the help of the method of imaginary sources and imaginary scatterers, of the method of integral equations and of the Fourier transform is solved the problem of the diffraction of the pulse sound signal at elastic body of the non-analytical form, put in the plane waveguide.

Keywords Integral Equation, Non-Analytical Form, Waveguide, Imaginary Scatterer, Diffraction

1. Introduction

At the basis of the method of the imaginary sources and imaginary scatterers and of the method of integral equations is solved the problem of scattering of the pulse signals of the elastic shell of the non-analytical form, accommodated in the plane waveguide with the ideal boundary conditions. The impulse signal put the energy, therefore they are propagating with the group velocity, which lie in the principles of the method of the imaginary sources and imaginary scatterers.

2. The Method of Imaginary Sources and Imaginary Scatterers for the Problem of the Sound Diffraction at the Elastic Shell of the Non-Analytical Form, Put in the Plane Waveguide

The scattering of sound by the bodies, placed in the waveguide or in the sound shannel, are investigated in papers [1-9]. In the paper [4] were calculated the spectral characteristics of the idial spheroid, placed in the sound channel, by the pulse irradiation; in the papers [2] and [3] with the help of the method of the imaginary sources and scatterers are found the vertical distributions of the scattered sound field of the ideal soft spheroid, placed in the plane waveguide, at the irradiation his by the harmonic signal. In papers [10, 11] with the help of the Fourier transform and characteristics of the stationary (continuos) sound signal are calculated the pulses, scattered by the ideal prolate spheroid. In the article [12] is solved the problem of the scattering of the pulse sound signal by the elastic spheroidal shell, put in the plane waveguide.

Let's put the he elastic elastic shell into the liquid layer with the thickness H and the constant sound velocity. At the upper boundary of the waveguide is fulfilled Dirichle condition, at the lower boundary – Neiman condition (Fig. 1).

The dimehsions of the scatterer, distance from it to the boundaries and the thickness of the waveguide H are supposed to be such that we can do without taking into consideration the scattering of the second order of the waves reflected from the boundaries of waveguide are not taken into account in the further process of the diffraction [3, 10 – 16].

The centre of the scatterer is fixed at the distance z_1 from the bottom, at the horizontal distance R from it and on the depth $H - z_0$ (Fig. 1) is placed the point-source Q of the impulse sound signal. Using the method of the imaginary sources and scatterers [2. 3], are found the scattered pulse signal in the point Q. The sound pulse signals were the two appearance: with the harmonic filling.

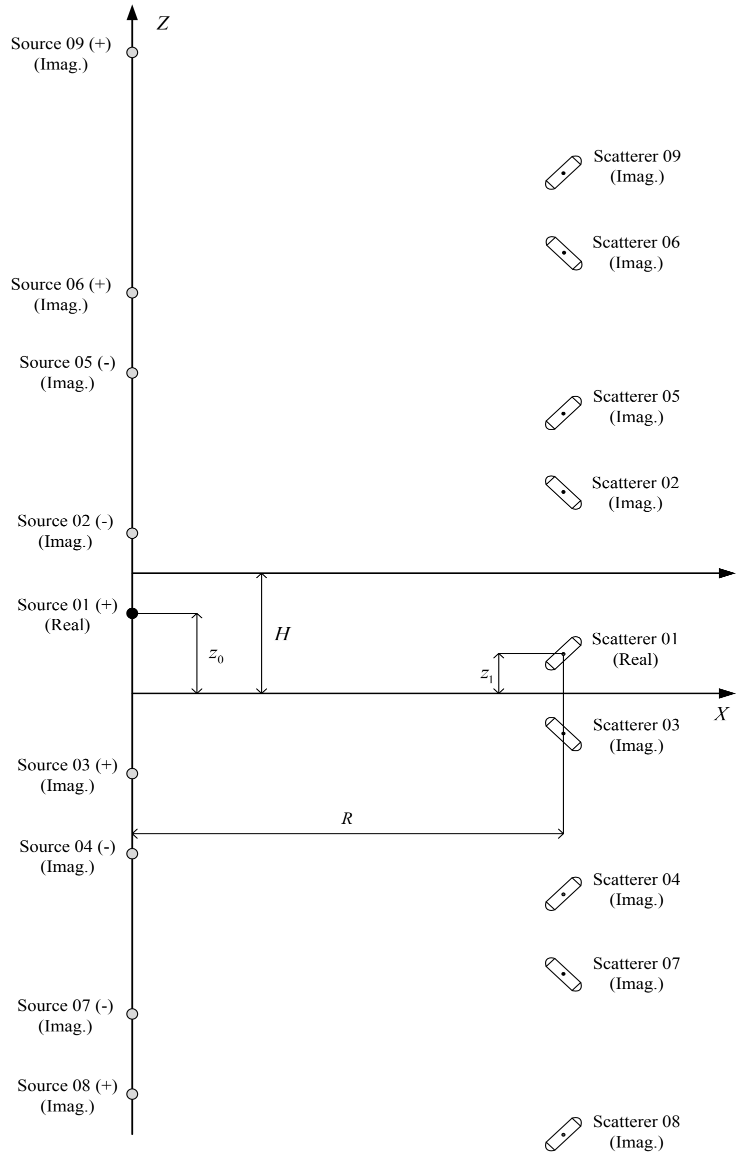

Figure 1. The mutual disposition of the impulse point-sources and scatterers in the plane waweguide

The spectrum $S_0(2\pi\nu)$ of the sound pulse of the source $\Psi_i(t)$ with the harmonic filling has the appearance [17] :

$$S_0(2\pi\nu) = \frac{i\nu_0}{\pi(\nu_0^2 - \nu^2)}(-1)^n \sin(\pi n \frac{\nu}{\nu_0}), \quad (1)$$

where: ν_0 – the frequency of the filling of the impulse; n – the number of the oscillation periods of the harmonic signal in the pulse; ν – the circular frequency.

The spectrum $S_0(2\pi\nu)$ is connected with $\Psi_i(t)$ by the return Fourier transform:

$$\Psi_i(t) = (\pi)^{-1} \operatorname{Re} \int_0^{\infty} S_0(2\pi\nu) \exp(+i2\pi\nu) d(2\pi\nu) \quad (2)$$

The spectrum of the reflected signal $S_s(2\pi\nu)$ is by the product of the spectrum $S_0(2\pi\nu)$ at the corresponding meanings of the angular characteristic of the scattering of the elastic shell $D(\eta,\varphi,\nu)$ (η and φ – the angular coordinates of the point of the observation).

3. Method of Integral Equations In Problem of Sound Diffraction on Bodies of Non – Analytical Form

The scattered field for the elastic shell of the non – analytical form was determined either with the help of the method of the method of the integral equations [3, 18]. In the quality of the such scatterer we are going to consider the terminal isotropic elastic cylindrical shell with the semi – spheres on its ends (see Fig. 2). The density of the material of the shell is ρ_1, the Lame's coefficients - λ and μ. The shell was filled in the internal liquid medium with the density ρ_2

and the sound velocity C_3 and it was placad in the external liquid medium with the density ρ_0 and the sound velocity C_0. At the shell falls the plane harmonic wave with pressure p_i under the angle Θ_0 and with the wave vector $\vec{k}$.

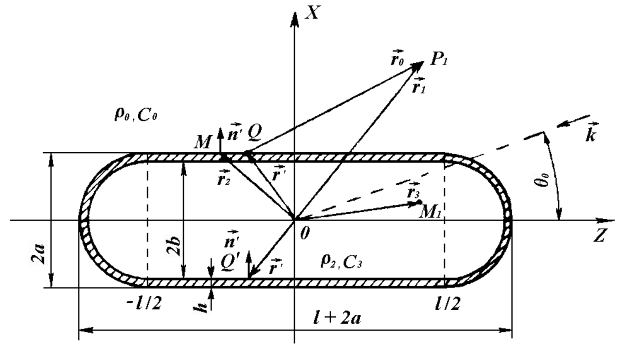

Figure 2. The elastic shell in the form of the terminal cylinder with the semi – spheres.

As was shown in [3, 19 – 21], the initial equation is integral equation , having the sense of the generalized Huygen's principle, for the displacement vector $\vec{u}(\vec{r})$ of the elastic shell:

$$\vec{u}(\vec{r}) = \iint_S \left\{ \vec{t}(\vec{r}') G(\vec{r}';\vec{r}) - \vec{u}(\vec{r}') \left[\hat{n}' \sum(\vec{r}';\vec{r}) \right] \right\} dS(\vec{r}'), \quad \vec{r} \in V, \tag{3}$$

where $\vec{t}(\vec{r}') = \hat{n}' T(\vec{r}')$ is the stress vector; $\hat{n}' \equiv \hat{n}'(\vec{r}') = \vec{n}'(\vec{r}')$ is the single vector of the external along the relation to S normal; $T(\vec{r}')$ is the stress tensor of the isotropic material; $G(\vec{r}';\vec{r})$ is the displacement Green's tensor; $\sum(\vec{r}';\vec{r})$ is the stress Green's tensor; if $\vec{r}$ concerns to the point of the surface S, in the left part of the equation (3) will stand $\vec{u}(\vec{r}')/2$.

The displacement vector $\vec{u}(\vec{r})$, the stress tensor $T(\vec{r})$, the displacement Green's tensor $G(\vec{r}';\vec{r})$ and the stress Green's tensor $\Sigma(\vec{r}';\vec{r})$ were connected between them selves by the following correlations [3, 16 – 18]:

$$T(\vec{r}) = \lambda I \nabla \vec{u}(\vec{r}) + \mu(\nabla \vec{u} + \vec{u}\nabla), \tag{4}$$

where $I = I_L + I_T$; $I_L = (\nabla\nabla)/\nabla^2$; $I_L \cdot I_T = 0$; $I_T = -[\nabla(\nabla I)]/\nabla^2$, I_L and I_T are the longitudinal and transverse single tensors for the Hamilton's operator ∇;

$$\sum(\vec{r}';\vec{r}) = \lambda I \nabla G(\vec{r}';\vec{r}) + \mu[\nabla G(\vec{r}';\vec{r}) + G(\vec{r}';\vec{r})\nabla]; \tag{5}$$

$$G(\vec{r}';\vec{r}) = (1/4\pi\rho_t\omega^2)\{k_2 I\, g(k_2|\vec{r}' - \vec{r}|) + \\ + \nabla'[g(k_1|\vec{r}' - \vec{r}|) - g(k_2|\vec{r}' - \vec{r}|)]\nabla\}, \tag{6}$$

where k_1 and k_2 are the wave numbers of the longitudinal and transwerse waves in the material of the shell; $g(k_2|\vec{r}' - \vec{r}|) = exp(ik_2|\vec{r}' - \vec{r}|)/4\pi|\vec{r}' - \vec{r}|$ is the Green's function.

The second integral equation presents the Kirchhoff integral for the diffracted pressure $p_\Sigma(P_1)$ in the external medium [3, 22]:

$$C(P_1)p_\Sigma(P_1) = -\iint_{S_a} \{p_\Sigma(Q)(\partial/\partial n')[exp(ikr_0/r_0)] - \\ -[exp(ikr_0/r_0)]\rho_0\omega^2(\vec{u}\vec{n}')\}dS_a + 4\pi\, p_i(P_1), \tag{7}$$

where $p_\Sigma(P_1) = p_i(P_1) + p_s(P_1)$; $p_s(P_1)$ is the scattered pressure in the point P_1; $C(P_1)$ is the numerical coefficient, equal 2π, if $P_1 \in S_a$ and 4π, if P_1 out S_a; S_a is the external surface of the shell; Q is the point of the external surface of the shell.

For the pressure $p_2(M_1)$ in the internal liquid medium in the point M_1 is got the third integral equation:

$$C(M_1)p_2(M_1) = \iint_{S_b} \{p_2(Q')(\partial/\partial n')[exp(ikr_3)/r_3] - \\ -[exp(ikr_3)/r_3]\rho_0\omega^2(\vec{u}\,\vec{n}')\}dS_b, \tag{8}$$

where Q' is the point of the internal surface of the shell;

$$C(M_1) = \begin{cases} 4\pi, & if \quad M_1 \quad out \quad S_b; \\ 2\pi & if \quad M_1 \in S_b; \end{cases}$$

S_b is the internal surface of the shell.

To the integral equations (3), (7) and (8) are added the boundary conditions on the external (S_a) and internal (S_b) surfaces of the shell:

1. at the both surfaces of the shell the tangent stresses are equally null:

$$\tau_i|_{S_a} = 0; \quad \tau_i|_{S_b} = 0; \quad i = 1, 2; \tag{9}$$

2. the normal stress $\sigma_{n'}$ at the external surface of the shell is equally the diffracted pressure pΣ, but at the internal surface is equally the pressure p2

$$\sigma_{n'}|_{S_a} = p_\Sigma; \quad \sigma_{n'}|_{S_b} = p_2; \tag{10}$$

In the conformity with the conditions (9) and (10) the stress vector $\vec{t}(\vec{r}')$ in the equation (3) is equal:

$$\vec{t}(\vec{r}') = p_\Sigma \vec{n}'|_{S_a}; \quad \vec{t}(\vec{r}') = p_2 \vec{n}'|_{S_b}; \tag{11}$$

3. the continuity of the normal component of the displacement at the both boundaries of the shell:

$$\left.\begin{aligned} u_{n'} &= (1/\rho_0\omega^2)(\partial p_\Sigma/\partial n')|_{S_a}; \\ u_{n'} &= (1/\rho_2\omega^2)(\partial p_2/\partial n')|_{S_b}. \end{aligned}\right\} \tag{12}$$

The substitution of the integral equations (8), (3) and (7) in the boundary conditions (9) – (11) gives the system of equations in terms of unknown functions p_Σ, p_2 and the components of the displacement vector $\vec{u}$ at the both surfaces of the shell. To obtain numerical solution of this system the integral equations are replaced the quadrature formulas and the grid of the nodal points is chosen at both surfaces of the shell as well as it has be done for the ideal non – analytical scatterers [3, 21].

For choosing boundary conditions we will have the integrals of the two types: the integrals with the isolated special point and the integrals which are considered of the sence of the principal meaning. The method of the calculation of the second types was described in [3].

3. Conclusions

In the paper is shown the effectiveness of the method of integral equations and of the method of imaginary sources and imaginary scatterers for the pulse sequence, got from body of non-analytical form and based at the use of the group velocity of the sound.

Acknowledgments

This work was supported as part of research under State Contract no P242 of April 21, 2010, within the Federal Target Program "Scientific and scientific – pedagogical personnel of innovative Russia for the years 2009 – 2013".

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